Section 5.7 Exponential Growth and Decay - Solutions

2a) $3\times10^4 = 2\times10^4 e^{8k} \Rightarrow \ln\frac{3}{2} = \ln e^{8k} \Rightarrow k = \frac{\ln 1.5}{8} \approx .05068$

b) $N = 2\times10^4 e^{(.05068)(2)} \approx 22133$ bacteria

5.079% is continuous growth rate

3. $3400 = 2000e^{k(3)} \Rightarrow \ln\frac{34}{20} = \ln e^{3k} \Rightarrow k = \frac{\ln(34/20)}{3} \approx .17876$

17.699% is the continuous growth rate

4. $e^k = -0.75$ is incorrect because e^k is always positive (the range of the exponential function regardless of the value of k is always >0)

7. Chad $N = 8e^{.033t}$ U.K $N = 59.8e^{.001t}$

$8e^{.033t} = 59.8e^{.001t} \Rightarrow \frac{e^{.033t}}{e^{.001t}} = \frac{59.8}{8} \Rightarrow \ln e^{.032t} = \ln\frac{59.8}{8} \Rightarrow$

$t = \frac{\ln(59.8/8)}{.032} \approx 62$ years. So by 2060 both countries will have the same population.

8a) Mali $N = 11.2e^{(.031)(15)} = 17.83$ million people

Cuba $N = 11.1e^{(.007)(15)} = 12.33$ million people

b) $20 = 11.12e^{.031t} \Rightarrow \ln\frac{20}{11.12} = \ln e^{.031t} \Rightarrow t = \frac{\ln(20/11.12)}{.031} \approx 18.7$ yrs

or 18 years and 8 months

$N = 11.1e^{(.007)(18.7038)} = 12.65$ million so in the same year Cuba's population will be 12.65 million people

13. a) U.S. $2 = e^{.006t} \Rightarrow t = \frac{\ln 2}{.006} \approx 115.5$ years.

b) Tajikistan $2 = e^{.016t} \Rightarrow t = \frac{\ln 2}{.016} \approx 43.3$ years.

c) Cambodia $2 = e^{.026t} \Rightarrow t = \frac{\ln 2}{.026} \approx 26.7$ years

d) Palestinian territory $2 = e^{.037t} \Rightarrow t = \frac{\ln 2}{.037} \approx 18.7$ years.

16.a) $N = 316.75e^{kt} \Rightarrow 368.37 = 316.75e^{k(39)} \Rightarrow \ln\frac{368.37}{316.75} = \ln e^{39k}$

$\Rightarrow k = \frac{\ln(368.37/316.75)}{39} \approx .00387$ so k is .387% per year

b) $N(25) = 316.75e^{(.00387)(25)} = 348.94$ ppm table 345.75

Error is $\frac{348.94 - 345.75}{345.75} = .009$ OR .9% error

16c $N(30) = 316.75e^{(.00387)(30)} = 355.76$ ppm table 354.04

ERROR $= \frac{355.76 - 354.04}{354.04} = .0048$ OR .48%

$N(35) = 362.7$ ppm $\frac{362.7 - 360.91}{360.91} = .004987$ OR .5%

d) Because we used 1999 to find k.

18. $t=10$ Iowa $N(10) = 2.926e^{(.0054)(10)} = 3.088$ million people

Arkansas $N(10) = 2.673e^{(.0137)(10)} = 3.065$ million people

Nevada $N(10) = 1.998e^{(.0663)(10)} = 3.877$ million people

New order: Nevada, Iowa, Arkansas

24. a) Polonium-210; half-life $= 138.4$ days $N(0) = .4$ $N(138.4) = .2$

$N(276.8) = .1$ $N(415.2) = .05$ $N(830.4) = .025$

b) Polonium-214; half-life 1.63×10^{-4} secs)

$N(0) = 0.1$ $N(3.26 \times 10^{-4}) = 0.05$ $N(6.5 \times 10^{-4}) = 0.025$ $N(13.04 \times 10^{-4}) = 0.0125$

32. radium-226 half life 1620 years $N = N_0e^{kt} \Rightarrow \frac{1}{2} = e^{k(1620)}$

$\Rightarrow \frac{\ln .5}{1620} = k \approx -.000427869$ a) $N = 2e^{(-.000427869)(100)} = 1.916$ grams.

b) $.4 = 2e^{(-.000427869)t} \Rightarrow \ln .2 = \ln e^{-.000427869t} \Rightarrow t = \frac{\ln .2}{-.000427869}$

$= 3761.5$ years.

57a $N(t) = \frac{5}{1+ae^{-bt}}$ $1 = \frac{5}{1+ae^{-b(0)}} \Rightarrow 1+a = 5 \Rightarrow a = 4$

b) $2 = \frac{5}{1+4e^{(-b)(1)}} \Rightarrow 1+4e^{-b} = 2.5 \Rightarrow 4e^{-b} = 1.5 \Rightarrow \ln e^{-b} = \ln .375$

$b = -\ln .375 = -\ln(3/8)$

58. a) $0.24 = \frac{50}{1+ae^{(-b)(0)}} \Rightarrow 1+a = \frac{50}{.24} \Rightarrow a = 207.\overline{33}$

$13.53 = \frac{50}{1+207.33e^{(-b)(2)}} \Rightarrow 1+207.33e^{-2b} = \frac{50}{13.53} \Rightarrow 2.7 = 207.33e^{-2b}$

$\Rightarrow \ln \frac{2.7}{207.33} = \ln e^{-2b} \Rightarrow b = 2.17$ $N(t) = \frac{50}{1+207.33e^{-2.17t}}$

b) $N(1) = 2.027 \frac{}{cm^2}$ $N(3) = 38.2 \frac{}{cm^2}$ $N(4) = 48.3 \frac{}{cm^2}$ $N(5) = 49.8 cm^2$

c) $10 = \frac{50}{1+207.33e^{-2.17t}} \Rightarrow 1+207.33e^{-2.17t} = 5 \Rightarrow \ln \frac{4}{207.33} = \ln e^{-2.17t}$

$t = \frac{\ln \frac{4}{207.33}}{-2.17} \approx 1.82$ days so $\approx$ 1 day 19.2 hours.