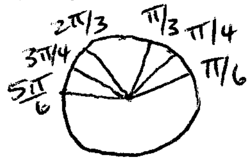
6.1 Radian Measure – Solutions.

4. $\theta = \frac{s}{r} \Rightarrow \theta = \frac{.0367 \times 100}{1.0013} = 3.665$ radians

6. a) $30° = \frac{\pi}{6}$ radians $\approx .52$ b) $150° = \frac{5\pi}{6}$ radians ≈ 2.62
   c) $300° = \frac{5\pi}{3}$ radians ≈ 5.24

12a) $\frac{5\pi}{6} = 150°$ b) $\frac{11\pi}{6} = 330°$ c) $0 = 0°$

16. $\angle A = \frac{\pi}{5}$ $\angle B = \frac{\pi}{6}$ $\frac{\pi}{5} + \frac{\pi}{6} + x = \pi \Rightarrow 6\pi + 5\pi + 30x = 30\pi$
   $\Rightarrow 30x = 19\pi \Rightarrow x = \frac{19}{30}\pi$

17. 

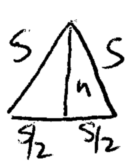
22. $\theta = 150° = \frac{5\pi}{6}$ $\frac{5\pi}{6} = s/1 \Rightarrow s = \frac{5\pi}{6}$ inches

24. a) $r = 4cm$ $\theta = \frac{\pi}{10}$ $A = \frac{1}{2}r^2\theta = \frac{1}{2}(4)^2(\frac{\pi}{10}) = \frac{4\pi}{5}cm^2 \approx 2.51cm^2$
   b) $r = 16m$ $\theta = 5°$ $A = (\frac{1}{2})(16^2)(\frac{5}{180}\pi) = \frac{32}{9}\pi m^2 \approx 11.17m^2$
   c) $r = 21ft$ $\theta = \frac{11\pi}{6}$ $A = \frac{1}{2}(21^2)(\frac{11\pi}{6}) = \frac{1617}{4}\pi f^2 \approx 1269.99 ft^2$
   d) $r = 4.2in$ $\theta = 170°$ $A = \frac{1}{2}(4.2^2)(\frac{170}{180}\pi) = 8.33\pi in^2 \approx 26.17 in^2$

28. $\theta = 135° = \frac{135}{180}\pi$ radians $r = 3m$ $S = \theta r = \frac{135}{180}\pi(3) = 2.25\pi$
perimeter $= 3 + 3 + 2.25\pi = 6 + 2.25\pi \approx 13.07m$
Area $= \frac{1}{2}(3^2)(\frac{135}{180}\pi) = \frac{27}{8}\pi \approx 10.60m^2$

36. diameter is 3ft x revolutions distance 22619 ft.
revolution is distance traveled $(2\pi)(\frac{3}{2}) = 3\pi$
$3\pi x = 22619 \Rightarrow x = \frac{22619}{3\pi} \approx 2400$ revolutions.

43. $\theta = 71°23' = 71\frac{23}{60}$ so $71\frac{23}{60} \cdot \frac{\pi}{180} = \frac{4283}{10800}\pi$
arc length is $s = r\theta = 3960$ miles $\cdot \frac{4283}{10800}\pi = \frac{47113\pi}{30} \approx 4930$ miles.

49. 

$h = \sqrt{s^2 - \frac{s^2}{4}} = \sqrt{\frac{4s^2 - s^2}{4}} = \frac{s}{2}\sqrt{3}$

$A = \frac{1}{2}(s)(\frac{s}{2}\sqrt{3}) = \frac{s^2}{4}\sqrt{3}$

51. each angle is $60° = \pi/3$ radians ; each arc length is
$r\theta = \frac{\pi}{3}s$

a) perimeter $\frac{\pi}{3}s + \frac{\pi}{3}s + s = \frac{2\pi}{3}s + s$

b) ABC area $= \frac{1}{2}r^2\theta = \frac{1}{2}s^2 \cdot \frac{\pi}{3} = \frac{\pi}{6}s^2$

area of Δ from #49 is $\frac{\sqrt{3}}{4}s^2$

Area of equilateral arch is

$A = A_s + A_s - A_t = \frac{\pi}{6}s^2 + \frac{\pi}{6}s^2 - \frac{\sqrt{3}}{4}s^2 = \left(\frac{\pi}{3} - \frac{\sqrt{3}}{4}\right)s^2$

55. a) $s = r\theta$ $\quad 12 = r + r + r\theta \Rightarrow r(2+\theta) = 12 \Rightarrow r = \frac{12}{2+\theta}$

so $r(\theta) = \frac{12}{2+\theta}$

b) $A = \frac{1}{2}r^2\theta = \frac{1}{2}\left(\frac{12}{2+\theta}\right)^2(\theta) = \frac{720}{(2+\theta)^2}$

$A(\theta) = \frac{720}{(2+\theta)^2}$ which is not a quadratic

c) $r = \frac{12}{2+\theta}$ solve for θ $\quad 2+\theta = \frac{12}{r} \Rightarrow \theta = \frac{12}{r} - 2$

$= \frac{12-2r}{r}$

$\theta(r) = \frac{12-2r}{r}$

d) $A = \frac{1}{2}r^2\theta = \frac{1}{2}r^2\left(\frac{12-2r}{r}\right) = 6r - r^2$ which is a quadratic

e) zeros at $r=0$ and $r=6$

So vertex is at $r=3$ (by symmetry)

$A = 6(3) - 3^2 = 9$

So max area is $9\,cm^2$ when $r = 3cm$

$\theta = \frac{12}{3} - 2 = 2$ radians