Section 6.3 Evaluating the Trigonometric Functions - Solutions.

2 a) $300°$ ref angle is $60°$ b) $1000°$ ref. angle is $80°$

c) -15 ref. angle is $15°$ d) $15°$ ref. angle is $15°$

4. a) $\frac{5\pi}{6}$ ref. angle is $\frac{\pi}{6}$ b) $-\frac{\pi}{3}$ ref. angle is $\pi/3$

c) $\frac{2\pi}{3}$ ref. angle is $\frac{\pi}{3}$ d) $\frac{5\pi}{4}$ ref. angle is $\frac{\pi}{4}$

5. a) $\cos\frac{\pi}{3} = \frac{1}{2}$ (B) b) $\sin(\frac{\pi}{3}) = \frac{\sqrt{3}}{2}$ (A) c) $\tan\frac{\pi}{3} = \sqrt{3}$ (D)

d) $\cos\frac{\pi}{6} = \sqrt{3}/2$ (A) e) $\sin\frac{\pi}{6} = \frac{1}{2}$ (B) f) $\tan\frac{\pi}{6} = \frac{\sqrt{3}}{3}$ (C)

8a) $\cos(150°)$ ref. angle is $30°$ $\cos(30°) = \frac{\sqrt{3}}{2}$ and since $150°$ is in quadrant II $\cos(150°) = -\sqrt{3}/2$

b) $\cos(-150°)$ ref. angle is $30°$ $\cos(30°) = \frac{\sqrt{3}}{2}$ and $-150°$ is in quadrant III $\cos(-150°) = -\sqrt{3}/2$

c) $\sin(150)$ ref angle $30°$ $150°$ in Quad II $\sin(30°) = \frac{1}{2} = \sin(150°)$

d) $\sin(-150)$ ref angle $30°$ $150°$ in Quad III $\sin(-150) = -1/2$

16a) $\cos(\frac{2\pi}{3})$ ref angle is $\frac{\pi}{3}$ $\frac{2\pi}{3}$ in Quad II $\cos(\frac{\pi}{3}) = \frac{1}{2}$ $\cos(\frac{2\pi}{3}) = -\frac{1}{2}$

b) $\cos(-\frac{2\pi}{3})$ ref. angle $\frac{\pi}{3}$ $-\frac{2\pi}{3}$ in Quad III $\cos(-\frac{2\pi}{3}) = -\frac{1}{2}$

c) $\sin(\frac{2\pi}{3})$ $\frac{2\pi}{3}$ in Quad II $\sin(\frac{2\pi}{3}) = \sin(\frac{\pi}{3}) = \frac{\sqrt{3}}{2}$

d) $\sin(-\frac{2\pi}{3})$ $-\frac{2\pi}{3}$ in Quad III $\sin(-\frac{2\pi}{3}) = -\frac{\sqrt{3}}{2}$

18 a) $\cos(\frac{17\pi}{6})$ ref angle $\frac{\pi}{6}$ $\frac{17\pi}{6}$ in Quad 2 $\cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$ $\cos(\frac{17\pi}{6}) = -\frac{\sqrt{3}}{2}$

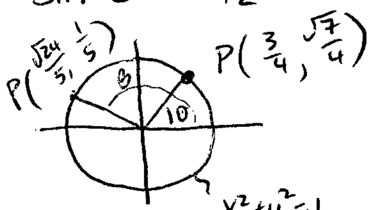
b) $\cos(-\frac{17\pi}{6})$ ref angle $\frac{\pi}{6}$ $-\frac{17\pi}{6}$ in Quad. 3 $\cos(-\frac{17\pi}{6}) = -\frac{\sqrt{3}}{2}$

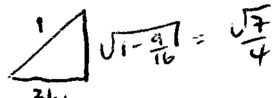
c) $\sin(\frac{17\pi}{6})$ $\sin\frac{\pi}{6} = \frac{1}{2} = \sin(\frac{17\pi}{6})$

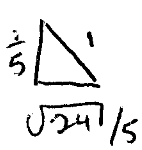
d) $\sin(-\frac{17\pi}{6}) = -\frac{1}{2}$

24. $\sin\theta = -1/2$ $\theta = \pi/6, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$, etc.

28.







a) $\sin\beta = 1/5$

b) $\cos\beta = \sqrt{24}/5$ $\beta = -\frac{\sqrt{24}}{5} \cdot 5 = -\sqrt{24}$

c) $\cos(\beta+\pi) = \sqrt{24}/5$ ↑ bigger

$\sin(\theta+\pi) = -\frac{\sqrt{7}}{4}$

29. $f(x) = \sin x \quad g(x) = \cos x \quad h(x) = \tan x \quad k(x) = 2x$

a) $f\left(\frac{5\pi}{6} + \frac{\pi}{6}\right) = f(\pi) = \sin \pi = 0$ b) $f\left(\frac{5\pi}{6}\right) + f\left(\frac{\pi}{6}\right) = \sin \frac{5\pi}{6} + \sin \frac{\pi}{6}$

$= \frac{1}{2} + \frac{1}{2} = 1$ c) $g\left(k\left(\frac{3\pi}{4}\right)\right) = g\left(2\left(\frac{3\pi}{4}\right)\right) = g\left(\frac{3\pi}{2}\right) = \cos\left(\frac{3\pi}{2}\right) = 0$

d) $k\left(g\left(\frac{3\pi}{4}\right)\right) = k\left(\cos \frac{3\pi}{4}\right) = k\left(-\frac{1}{\sqrt{2}}\right) = -\frac{2}{\sqrt{2}} = -\sqrt{2}$

e) $\frac{\Delta f}{\Delta x}\left[\frac{\pi}{4}, \frac{\pi}{2}\right]$ $f\left(\frac{\pi}{4}\right) = \sin \frac{\pi}{4} = \frac{1}{\sqrt{2}}$ $f\left(\frac{\pi}{2}\right) = \sin\left(\frac{\pi}{2}\right) = 1$

$$\frac{\Delta f}{\Delta x} = \frac{1 - \frac{1}{\sqrt{2}}}{\frac{\pi}{2} - \frac{\pi}{4}} = \frac{\frac{\sqrt{2}-1}{\sqrt{2}}}{\pi/4} = \frac{4(\sqrt{2}-1)}{\sqrt{2}\,\pi} = \frac{4 - 2\sqrt{2}}{\pi}$$

f) $\frac{\Delta g}{\Delta x}\left[\frac{\pi}{4}, \frac{\pi}{2}\right]$ $g\left(\frac{\pi}{4}\right) = \cos \frac{\pi}{4} = \frac{1}{\sqrt{2}}$ $g\left(\frac{\pi}{2}\right) = \cos \frac{\pi}{2} = 0$

$$\frac{\Delta g}{\Delta x} = \frac{0 - \frac{1}{\sqrt{2}}}{\frac{\pi}{2} - \pi/4} = \frac{-\frac{1}{\sqrt{2}}}{\pi/4} = \frac{-4}{\sqrt{2}\,\pi}$$

g). $\frac{\Delta f}{\Delta x}\left[\frac{5\pi}{6}, \frac{7\pi}{6}\right]$ $f\left(\frac{5\pi}{6}\right) = \frac{1}{2}$ $f\left(\frac{7\pi}{6}\right) = -\frac{1}{2}$

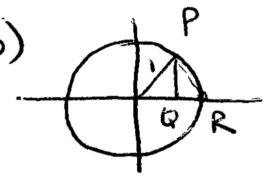
$$\frac{\Delta f}{\Delta x} = \frac{-\frac{1}{2} - \frac{1}{2}}{\frac{7\pi}{6} - \frac{5\pi}{6}} = \frac{-1}{\pi/3} = -3/\pi$$

h) $\frac{\Delta g}{\Delta x}\left[\frac{5\pi}{6}, \frac{7\pi}{6}\right]$ $g\left(\frac{5\pi}{6}\right) = -\frac{\sqrt{3}}{2}$ $g\left(\frac{7\pi}{6}\right) = -\frac{\sqrt{3}}{2}$

$$\frac{\Delta g}{\Delta x} = \frac{-\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2}}{\frac{7\pi}{6} - \frac{5\pi}{6}} = 0$$

31.a)

θ	$\sin\theta$	Bigger
0.1	0.09983	θ
0.2	0.19866	θ
0.3	0.2955	θ
0.4	0.3894	θ
0.5	0.479	θ

b) 

Show $PQ < PR < \theta$

In right triangle $PQ \leq$ hypotenuse $= 1$

In other right triangle $PQ \leq PR$

The arc length $PR = \theta$ and $\overline{PR}$ is $<$ the arc length so $\overline{PR} < \theta$

$PQ < PR < \theta$

c) if $0 < \theta < \frac{\pi}{2}$ then $\sin\theta < \theta$; since $\sin\theta = \overline{PQ}$ then $\sin\theta < \theta$

34. a) point P $(\cos\theta, \sin\theta)$

b) point Q $\left(\cos(\theta+\frac{\pi}{2}), \sin(\theta+\frac{\pi}{2})\right)$

if we rotate by P(a,b) by $\pi/2$ Q is (-b, a)

Therefore $\sin(\theta + \pi/2) = \cos\theta$

$\cos(\theta + \pi/2) = -\sin\theta$

c) i) $\cos(1+\frac{\pi}{2}) = -.84147$) same

$-\sin(1) = -.8414$

ii) $\sin(1+\frac{\pi}{2}) = .5403$) same

$\cos(1) = .5403$

iii) $\cos(17^\circ + 90^\circ) = -.2923$) same

$-\sin(17^\circ) = -.2923$

iv) $\sin(17^\circ + 90^\circ) = .9563$) same

$\cos(17^\circ) = .9563$