Section 3.3 Sketchs of Graph + Average Rate of Change

1. a) turning point is where the graph changes direction
   b) maximum value is the highest dependent value on the graph
   c) minimum value is the lowest dependent value on the graph
   d) If move from left to right on graph of f, the function is going up
   e) If move from left to right on graph of f, the function is going down

2. average rate of change of f on $[c,d]$ is $\frac{f(d)-f(c)}{d-c}$. This is the slope of the line through points $(c, f(c))$ and $(d, f(d))$

6. range: $-2 \le y \le 2$; max at $y=2$; min at $y=-2$; incr $0<x<1$ and $2<x<3$; decr $1<x<2$ and $3<x<4$

8. $f(x) = -x^4+3x+5$. max $(.91, 7.04)$ max value is 7.04
   incr $x<.91$ and decr $x>.91$

12. $g(x) = x^3-x$ on $[1,2]$ $g(1)=0$ $g(2)=7$ AROC $= \frac{7-0}{2-1} = 7$

14. $h(t) = 16-7t$ on $[-\sqrt{2}, 2\sqrt{2}]$ $h(-\sqrt{2}) = 16-7(-\sqrt{2}) = 16+7\sqrt{2}$ $h(2\sqrt{2}) = 16-14\sqrt{2}$

$$\frac{16-14\sqrt{2}-(16+7\sqrt{2})}{2\sqrt{2}+\sqrt{2}} = \frac{-21\sqrt{2}}{\sqrt{2}} = -21$$

16. a) $(0,8)$ $\frac{f(8)-f(0)}{8-0} = \frac{\frac{1}{2}-1}{8} = -\frac{1}{16}$ grams/day $(8,16)$ $\frac{f(16)-f(8)}{16-8} = \frac{\frac{1}{4}-\frac{1}{2}}{8} = -\frac{1}{32}$ grams/day

$(16,24)$ $\frac{f(24)-f(16)}{24-16} = \frac{\frac{1}{8}-\frac{1}{4}}{8} = -\frac{1}{64}$ gr/day $(24,32)$ $\frac{f(32)-f(24)}{32-24} = \frac{\frac{1}{16}-\frac{1}{8}}{8} = -\frac{1}{128}$ grams/day

b) $\left(-\frac{1}{128}-\frac{1}{64}-\frac{1}{32}-\frac{1}{16}\right) \div 4 = -\frac{15}{512}$ c) $\frac{f(32)-f(0)}{32-0} = \frac{\frac{1}{16}-1}{32} = \frac{-15}{512}$ b + c are the same

19a $\frac{f(1996)-f(1980)}{1996-1980} = \frac{20-18.5}{16} = \frac{1.5}{16} = \frac{3}{32} \approx .094$ millions of tons/year. of red meat

b) $\frac{20.32-18.68}{16} = .1025$ millions of tons of red meat / year.

22. $f(x) = 4x^2$ a) $\frac{f(x)-f(-2)}{x+2} = \frac{4x^2-4(-2)^2}{x+2} = \frac{4x^2-16}{x+2} = \frac{4(x^2-4)}{x+2} = \frac{4(x+2)(x-2)}{x+2}$
$= 4(x-2)$

b). $\frac{f(x)-f(a)}{x-a} = \frac{4x^2-4a^2}{x-a} = \frac{4(x-a)(x+a)}{x-a} = 4(x+a)$

28. $f(x) = 2x^2-x+1$ a) $\frac{f(x)-f(a)}{x-a} = \frac{2x^2-x+1-(2a^2-a+1)}{x-a} = \frac{2x^2-2a^2-x-a}{x-a}$

$= \frac{2(x-a)(x+a)+(x+a)}{x-a}$

$\frac{f(x+h)-f(x)}{h} = \frac{2(x+h)^2-(x+h)+1-(2x^2-x+1)}{h} = \frac{\cancel{2x^2}+4xh+2h^2\cancel{-x}-h\cancel{+1}\cancel{-2x^2}\cancel{+x}\cancel{-1}}{h}$

$= \frac{4xh+2h^2-h}{h} = \frac{\cancel{h}(4x+2h-1)}{\cancel{h}} = 4x+2h-1$

29. $f(x)=\frac{1}{x}$ $\quad \frac{f(x)-f(a)}{x-a} = \frac{\frac{1}{x}-\frac{1}{a}}{x-a} = \frac{\frac{a}{ax}-\frac{x}{ax}}{x-a} = \frac{\frac{a-x}{ax}}{x-a} = \frac{-(x-a)}{ax}\cdot\frac{1}{x-a}$
$= -\frac{1}{ax}$

$\frac{f(x+h)-f(x)}{h} = \frac{\frac{1}{x+h}-\frac{1}{x}}{h} = \frac{\frac{x}{x(x+h)}-\frac{x+h}{x(x+h)}}{h} = \frac{x-(x+h)}{x(x+h)}\cdot\frac{1}{h} = \frac{-h}{x(x+h)}\cdot\frac{1}{h} = \frac{-1}{x(x+h)}$

34. $s(t)=16t^2$ a) $[2,3]$ $\frac{s(3)-s(2)}{3-2} = \frac{144-64}{1} = 80$ ft/sec $\quad [3,4]$ $\frac{256-144}{1} = 112$ ft/sec

$[2,4]$ $\frac{256-64}{2} = 96$ ft/sec $\quad$ b) $\frac{80+112}{2} = 96$ ft/sec yes they are the same

38. $f(t)=t^3-6t^2+9t$ a) $\frac{f(t_2)-f(t_1)}{t_2-t_1}$ units are degrees / hour

b) $f(0)=0$ $\;f(.1)=.841$ $\;f(.01)=.089401$ $\;f(.001)=.008994$ $\;f(.0001)=.0008994$

$[0,.1] = \frac{.841-0}{.1} = 8.41$ $\quad [0,.01] = \frac{.89401-0}{.01} = 8.94$ $\quad [0,.001] = \frac{.008994-0}{.001} = 8.994$

$[0,.0001] = \frac{.0008994}{.0001} = 8.9994$ $\quad \frac{\Delta f}{\Delta t} \to 9$

39. $y=|x|$ D: $\mathbb{R}$ R: $y\geq 0$ TP $(0,0)$ max: none min: 0 inc $x>0$ dec $x<0$

$y=x^2$ D: $\mathbb{R}$ R $y\geq 0$ TP $(0,0)$ max: none min: 0 inc $x>0$ dec $x<0$

$y=x^3$ D: $\mathbb{R}$ R: $\mathbb{R}$ TP: none max: none min: none incr $(-\infty,\infty)$
decr: never.

40. $y=\frac{1}{x}$ D: $x\neq 0$ R: $y\neq 0$ TP: none max/min: none incr: never decr $x<0$, $x>0$

$y=\sqrt{x}$ D: $x\geq 0$ R: $y\geq 0$ TP: none max: none min: 0 inc: $x>0$ decr: never.

$y=\sqrt{1-x^2}$ D: $-1\leq x\leq 1$ R: $0\leq y\leq 1$ TP: $(0,1)$ max: 1 min: 0 incr. $-1<x<0$
decr $0<x<1$

41. $f(x)=\frac{1}{x}$ $[1,b]$ $\quad \frac{\frac{1}{b}-\frac{1}{1}}{b-1} = -\frac{1}{5} \Rightarrow \frac{1-b}{b(b-1)} = -\frac{1}{5} \Rightarrow -\frac{1}{b} = -\frac{1}{5} \Rightarrow b=5$

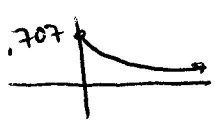
$(1-b) = -(b-1)$

43. $f(x)=ax^2+bx+c$ $\quad \frac{f(x+h)-f(x)}{h} = \frac{a(x+h)^2+b(x+h)+c-(ax^2+bx+c)}{h}$

$= \frac{\cancel{ax^2}+2axh+ah^2+\cancel{bx}+bh+\cancel{c}-\cancel{ax^2}-\cancel{bx}-\cancel{c}}{h} = \frac{2axh+ah^2+bh}{h} = \frac{\cancel{h}(2ax+ah+b)}{\cancel{h}}$

$= 2ax+ah+b$

47. $h(x)=\sqrt{x}$ $\quad f(x) = \frac{h(x)-h(2)}{x-2} = \frac{\sqrt{x}-\sqrt{2}}{x-2}$ $\quad g(x) = \frac{1}{h(x)+h(2)} = \frac{1}{\sqrt{x}+\sqrt{2}}$



$f(x)=g(x)$ $\quad g(x) = \frac{1}{\sqrt{x}+\sqrt{2}}\cdot\frac{\sqrt{x}-\sqrt{2}}{\sqrt{x}-\sqrt{2}} = \frac{\sqrt{x}-2}{x-2} = f(x)$

since $f(x)$ is undefined at $x=2$ $g(x)=f(x)$ except when $x=2$