Section 3.5 - Combining functions

2. a) $(fh)(x) = f(x)h(x) = (2x-1)(x^3) = 2x^4 - x^3$

b) $(h/f)(x) = \frac{h(x)}{f(x)} = \frac{x^3}{2x-1}$

c) $\left(\frac{f}{h}\right)(x) = \frac{f(x)}{h(x)} = \frac{2x-1}{x^3}$

7. a) $(f/m)(x) = (m/f)(x) = \frac{f(x)}{m(x)} - \frac{m(x)}{f(x)} = \frac{2x-1}{x^2-9} - \frac{x^2-9}{2x-1} =$

$\frac{2x-1}{x^2-9}\left(\frac{2x-1}{2x-1}\right) - \frac{x^2-9}{2x-1}\left(\frac{x^2-9}{x^2-9}\right) = \frac{4x^2-4x+1-(x^4-18x^2+81)}{(x^2-9)(2x-1)} = \frac{-x^4+22x^2-4x-80}{(x^2-9)(2x-1)}$

b) $\frac{f(0)}{m(0)} - \frac{m(0)}{f(0)} = \frac{-80}{(-9)(-1)} = -\frac{80}{9}$ (used final form from part a)

10. a) $(fog)(x) = f(g(x)) = f(x+1) = 1-2(x+1)^2 = 1-2(x^2+2x+1)$

$= -2x^2-4x-1 = -(2x^2+4x+1)$

b) $(fog)(-1) = f(g(-1)) = f(0) = 1$

c) $(gof)(x) = g(f(x)) = g(1-2x^2) = 1-2x^2+1 = -2x^2+2$

d) $(gof)(-1) = g(f(-1)) = g(-1) = 0$

e) $(fof)(x) = f(f(x)) = f(1-2x^2) = 1-2(1-2x^2)^2 = 1-2(1-4x^2+4x^4)$

$= -8x^4+8x^2-1$

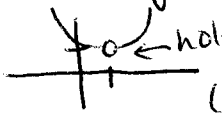
f) $(gog)(-1) = g(g(-1)) = g(0) = 1$

14. $f(x) = \frac{1}{x^2}+1 \quad g(x) = \frac{1}{x-1}$

a) $(fog)(x) = f(g(x)) = \frac{1}{\left(\frac{1}{x-1}\right)^2}+1 = (x+1)^2+1 = x^2+2x+1+1$

$= x^2+2x+2$

b) Domain: $x \neq 1$ because range of $g(x) \neq 0$ which is the domain of $f(x)$

c) 

$x^2-2x+2 = (x-1)^2+1$ found by completing square

17. from graph $g(3)=0 \quad f(3)=4 \quad h(3)=2 \quad f(0)=1 \quad g(4)=-3$

a) $f(g(3)) = f(0) = 1$ c) $f(h(3)) = f(2) = -1$ b) $g(f(3)) = g(4) = -3$

d) $(hog)(2) = h(g(2)) = h(1) = 2$ e) $h(f(g(3)) = h(f(0)) = h(1) = 2$

f) $g(f(h(f(2)) = g(f(h(-1)) = g(f(3)) = g(4) = 3$

20.

x	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$
$(g\circ f)(x)$	$\pi/2$	$\pi/3$	$\pi/4$	$\pi/6$	0

24. $F(x) = -x^2$ and $G(x) = \sqrt{x}$

F∘G domain $x \geq 0$ since range of G is that

G∘F domain $x = 0$ – this is the only value in range of F that is not negative.

25. $r = f(t) = 15 + t^{1.65}$

A. $A(t) = \pi(15 + t^{1.65})^2$

t	0	.5	1	1.5	2	2.5	3	3.5	4	4.5	5
$(A\circ f)$	707	737	804	903	1034	1199	1402	1648	1940	2284	2685

b) After one hour, area of spill is approx $800\ m^2$

c) When $t = 0$ spill was $707 m^2$, doubled in $\approx$ 3 hours.

d) $(0, 2.5)$ Avg ROC $= \frac{1199 - 707}{2.5 - 0} = 196.8\ m^2/\text{hour}$

$(2.5, 5)$ Avg ROC $= \frac{2685 - 1199}{5 - 2.5} = 594.4\ m^2/\text{hour}$

Area is increasing fastest between 2.5 and 5 hours.

27. $C(x) = 100 + 90x - x^2 \quad 0 \leq x \leq 40$

$x = f(t) = 5t \quad 0 \leq t \leq 8$

A. $(C\circ f)(t) = C(5t) = 100 + 90(5t) - (5t)^2 = 100 + 450t - 25t^2$

b) $(C\circ f)(3) = \$1225$

c) $(C\circ f)(6) = \$1900$ so the cost has not doubled if the time has doubled.

28. $N(T) = -2T^2 + 240T - 5400 \quad 40 \leq T \leq 90$

$T(t) = 10t + 40 \quad 0 \leq t \leq 5$

A) $N(T(t)) = N(10t + 40) = -2(10t + 40)^2 + 240(10t + 40) - 5400$

$= -2(100t^2 + 800t + 1600) + 2400t + 9600 - 5400$

$= -200t^2 + 800t + 1000 = -200(t^2 - 4t - 5)$

b) $N(T(0)) = -200(0^2 - 4(0) - 5) = 1000$ bacteria

$N(T(2)) = -200(2^2 - 4(2) - 5) = 1800$ bacteria

$N(T(5)) = -200(5^2 - 4(5) - 5) = 0$ bacteria

30. $a(x)=x^2$ $b(x)=|x|$ $c(x)=3x-1$

A) $f(x)=(3x-1)^2$ $f(x)=(a\circ c)(x)$

B) $g(x)=|3x-1|$ $g(x)=(b\circ c)(x)$

C) $h(x)=3x^2-1$ $h(x)=(c\circ a)(x)$

32. $a(x)=\frac{1}{x}$ $b(x)=\sqrt[3]{x}$ $c(x)=2x+1$ $d(x)=x^2$

$n(x)=x^{2/3}$ $(b\circ d)(x)$ OR $(d\circ b)(x)$

34. Average ROC $(A\circ f)(x)$ is $\frac{(A\circ f)(12)-(A\circ f)(6)}{12-6}$ between $t=6$ and $t=12$

$$= \frac{\frac{9\pi}{49}-\frac{9\pi}{64}}{6} = \frac{9\pi(64-49)}{(49)(64)(6)} = \frac{45\pi}{6272} \approx .0225 \frac{mi^2}{hour}$$

$$(A\circ f)(6) = \frac{\pi(6^2)}{4(6)^2+16(6)+16} = \frac{9\pi}{64}$$

$$(A\circ f)(12) = \frac{\pi(12^2)}{4(12^2)+16(12)+16} = \frac{144\pi}{784} = \frac{9\pi}{49}$$

$$\frac{\Delta(A\circ f)}{6-0} = \frac{3\pi}{128} \approx .07 \, mi^2/hour$$

So the area of the leak grew faster over 1st 6 hours.