Recent Work

• In high-speed transmission (at 40 Gb/s and above) the major nonlinear penalties come from intrachannel interactions: IFWM and IXPM.
• A common approach – a proper modulation format.
• Recent work

Novel modulation formats:
– Forzati, 2002, Alternate-phase RZ modulation format
– Liu, 2002, Inversion between adjacent marker blocks
– Xie, 2004, Alternate-polarization formats
– Djordjevic, 2004, Modified Duobinary RZ Modulation

Constrained (Modulation, Line) Coding:
Intra-Channel Nonlinear Effects

• A ghost pulse effect:
RZ, uncoded eye diagram, 80 spans

Time, t [ps]

Electrical filter output voltage, V [mV]
Pseudo-ternary code of rate 0.78, RZ, 60 spans

Optical Power, $P$ [mW]

Time, $t$ [ps]
Constrained Codes

- The key idea: forbid the bit patterns that cause a ghost pulse effect.
- Translation from unconstrained bit stream to constrained bit stream done in the transmitter by a modulation code.
- Translation back in the receiver.
Outline

- Constrained coding fundamentals.
- Codes to combat intra-channel nonlinearities.
Constraint Classification

- Time-domain constraints
  - Runlength constraints
  - Special forbidden pattern
    - distance enhancing codes
    - self synchronized codes

- Spectral constraints
  - dc free constraint
  - spectral null constraint
  - higher order spectral zeros
  - discrete spectral components

- Composite constraints
Examples of Constraints

Runlength constraints
(given by finite list $F$ of forbidden words)

Forbidden word $F = \{11\}$

Forbidden words $F = \{101, 010\}$

Spectral null constraints

Biphase

Even
Alternating Mark Inversion (AMI)

- A known code used in optical communications
A \((d,k)\) Runlength-Limited Constraint

- A \((d,k)\) runlength-limited sequence - binary sequence such that:
- The number of consecutive zeros between two ones is at least \(d\) and at most \(k\).
- For \(F=\{11\}\), \((d,k) = (1,\infty)\)

\[1 0 0 0 0 1 0 1 0 1 0 0 0 1 0 1 0 0 1 0 1 0 0 0 1 0\]
Encoder and Decoder

• Modulation encoder is a finite state machine with rate \( R = \frac{p}{q} \)

\[
\begin{align*}
\mathbf{a}^{(n)} &= f(s^{(n-1)}, \mathbf{b}^{(n)}) \\
s^{(n)} &= g(s^{(n-1)}, \mathbf{b}^{(n)}) \\
\{\mathbf{b}^{(n)}\} &= (b_1 \cdots b_p)^{(n)} \\
\mathbf{a}^{(n)} &= (a_1 \cdots a_q)^{(n)}
\end{align*}
\]

• Decoder should be state independent (preferably the sliding window type).

\[
\hat{\mathbf{b}}^{(n)} = \xi(\hat{\mathbf{a}}^{(n-\mu)} \cdots \hat{\mathbf{a}}^{(n)} \cdots \hat{\mathbf{a}}^{(n+\alpha)})
\]
Codes and Capacity

- What is the maximal code rate?
- Shannon defined the capacity of the constrained system:

\[ C = \lim_{n \to \infty} \frac{1}{n} \log N(n) \]

\(N(n)\) - the number of sequences of length \(n\) satisfying the constraint.

- \(N(n) \approx 2^{nc}\)
- Theorem [Shannon, 1948]: If there exists a constrained decodable code at rate \(R = \frac{p}{q}\), then \(R \leq C\).
- Theorem [Shannon, 1948]: For any rate \(R = \frac{p}{q} < C\) there exists a block constrained code with rate \(mp:mq\), for some integer \(m \geq 1\).
Counting Number of Sequences

- Transitions in one step determined by the graph adjacency matrix $A$.
- Transitions in $n$ steps determined by $A^n$.

Shannon showed that, for suitable representing graphs,

$$C = \log \rho(A)$$

$\rho(A)$ the spectral radius of the matrix $A$. 
Computing Capacity

- Example \((d,k)=(1,\infty)\)

\[
A = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}
\]

\[\lambda^2 - \lambda - 1 = 0\]

\[\lambda_0 = \frac{1 + \sqrt{5}}{2}\]

\[C = \log_2(\lambda_0) = 0.694...\]

\[R = \frac{p}{q} = \frac{2}{3} < C \Rightarrow p = 2, q = 3\]

- Can be done for an arbitrary constraint.
Constrained Decoding Theorems

• Theorem [Adler-Coppersmith-Hassner, 1983]
  Consider a finite-type constrained system. If $p/q \leq C$, then there exists a rate $p:q$ sliding-block decodable, finite-state encoder.
  (Proof is constructive: ACH or state-splitting algorithm.)

• Theorem [Karabed-Marcus, 1988]
  Similar statement for spectral null constraints.
Finite State Encoder

- Example \((d, k) = (1, \infty)\):
- \(R < 2/3, \; p=2, \; q=3\).

\[(b_1 \cdots b_p)^{(n)} \rightarrow \text{Finite State Machine} \rightarrow (a_1 \cdots a_q)^{(n)}\]

\[\begin{array}{c}
\text{a} \quad 0 \quad \text{b} \quad 0
\end{array}\]

\[G\]

\[\begin{array}{c}
\text{a} \quad 010 \quad \text{b} \quad 000
\end{array}\]

\[G^3\]
Code Construction Example: ACH Algorithm

Constraint Graph

Encoder

State Splitting

State Merging
Application to 40 Gb/s Transmission

- In high-speed transmission (40 Gb/s and above) the major impairments are due to intrachannel nonlinearities:
  - intrachannel four-wave mixing (IFWM) and
  - intrachannel cross-phase modulation (IXPM).
- The strongest interaction occurs in the regime where the pulses partially overlap
Intra-Channel Nonlinear Effects

\[
\frac{\partial A}{\partial z} = -\frac{\alpha}{2} A - \frac{j}{2} \beta_2 \frac{\partial^2 A}{\partial T^2} + \frac{1}{6} \beta_3 \frac{\partial^3 A}{\partial T^3} + j \gamma \left[ |A|^2 A - T_R A \frac{\partial |A|^2}{\partial T} + \frac{j}{\omega_0} \frac{\partial}{\partial T} \left( |A|^2 A \right) \right]
\]

\[
A = A(z, T) \quad T = t - z / v_g
\]

\[
\beta_2 = -\frac{\lambda^2}{2\pi c} D_c \quad \beta_3 = -\frac{\lambda^3}{(2\pi c)^2} \left( D_c + \lambda \frac{dD_c}{d\lambda} \right) \quad \gamma_0 = \frac{2\pi n_2}{\lambda A_{\text{eff}}} \quad \gamma = \frac{\gamma_0 A}{1 + b_s |A|^2} \approx \gamma_0 \left( 1 - b_s |A|^2 \right)
\]

\[
A = \sum_{l=1}^{L} A_l \Rightarrow \sum_{l=1}^{L} \left( \frac{\partial A_l}{\partial z} + \frac{\alpha}{2} A_l + \frac{i}{2} \beta_2 \frac{\partial^2 A_l}{\partial T^2} - \frac{\beta_3}{6} \frac{\partial^3 A_l}{\partial T^3} \right) = i \gamma \sum_{l,k,m=1}^{L} A_l A_k A_m^*
\]

- SPM: \( l=k=m \)
- IFWM: \( l\neq k\neq m \) or \( l=k\neq m \)
- IXPM: \( l=m\neq k \) or \( k=m\neq l \)
Intrachannel Cross-Phase Modulation

- In dispersion managed systems, when neighboring pulses overlap, the time derivative of the power of one pulse edge causes a shift in frequency of the other pulse:

\[
\frac{d\Delta f_{XPM}}{dz} \approx \pm 0.15 \frac{\gamma E_0}{T^2} F\left(\frac{\tau}{T}\right)
\]


IXPM cont.

- The frequency shift is negligibly small when $t < 0.4 \, T$ as the pulses do not overlap.
- The maximum distortion occurs when $\tau \approx T$, while for $\tau >> T$ the effect becomes negligible as the pulse power is low due to the large broadening.
Intrachannel Four-Wave Mixing

- Dispersed pulses experiencing the nonlinearity see a portion of their field shifted by a discrete frequency value due to FWM of spectral components within the same wavelength channel.

- At sufficiently high dispersion the frequency shift is translated in a discrete time shift located near the middle of a neighboring pulse.

\[ t_i + t_k - t_m \text{ (approximately)} \]
Constrained Codes

- The key idea: forbid the bit patterns that cause a ghost pulse effect.
- Possible Approaches:
  - the most troublesome sequences are identified and forbidden by constrained encoder,
  - the ‘zero’ symbol in so called resonant position is converted into ‘one’ symbol,
  - the constrained encoder is designed on such a way that different contributions to a ghost pulse creation cancel each other in resonant positions.
Binary Constrained Codes

\[ A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \]

\[ C = 0.8791 \]
• **Sliding window decoder:**

If $ABC = "011"$

$$I_1 = 0$$

$$I_2 = Y'$$

For all other patterns of $ABC$

$$I_2 = C' (B' E F' + B D' F) + E' (A' B' C F' (I' + G'))$$

$$+ D' (B C' H' (G I' + G' I) + A' B' C F))$$

and,

$$I_1 = I_2 (F' (E' H' I' + C (H' I + G)) + A B) + C' (A' B' D F'$$

$$+ D' (A (B' F' I2' + E F) + A' (I2' (B (I' + G') + E' F) + E F')))$$

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<th>Next State</th>
<th>Input Pattern</th>
<th>Branch Label</th>
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Pseudo-Ternary Codes

• The proper choice of initial pulse phases may result in complete cancellation of different ghost-pulse contributors at a zero-bit position in “resonance”.

\[ C = 0.78 \]

1+4-2: 0+0-\pi
2+5-4: \pi + \pi -0
A rate 4/5 non-linear pseudo-ternary block code

• Any sequence of length 5: \( c_i c_{i+1} c_{i+2} c_{i+3} c_{i+4} \), satisfies the following constraint: if for \( k, l, m \in [i, i+4] \) and \( k+l-m \in [i, i+4] \), \( k \) and \( l \) not necessarily distinct, \( c_k = c_1 = c_m = 1 \) or \( c_k = c_l = c_m = -1 \), then \( c_{k+l-m} \neq 0 \)

<table>
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<td>1110</td>
<td>10-110</td>
</tr>
<tr>
<td>1111</td>
<td>01-101</td>
</tr>
</tbody>
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Simulator features
- All major impairments in long-haul optical transmission:
  • ASE noise.
  • Pulse distortion due to fiber nonlinearities (Kerr nonlinearities, SRS, ...).
  • Chromatic dispersion (GVD, second-order GVD).
  • Crosstalk effects.
  • Intersymbol-interference (ISI), etc.
- Various modulation formats, detection and decoding schemes.
Eye Diagrams

Uncoded signal, RZ, 25 spans

Optical Power, P [mW]

Time, t [ps]

Uncoded signal, RZ, 60 spans

Optical Power, P [mW]

Time, t [ps]

Pseudo-ternary code of rate 0.78, RZ, 25 spans

Optical Power, P [mW]

Time, t [ps]

Pseudo-ternary code of rate 0.83, RZ, 25 spans

Optical Power, P [mW]

Time, t [ps]

Pseudo-ternary code of rate 0.78, RZ, 60 spans

Optical Power, P [mW]

Time, t [ps]

Pseudo-ternary code of rate 0.83, RZ, 60 spans

Optical Power, P [mW]

Time, t [ps]
Eye diagrams-cont.

**Left Diagram:**
- Pseudo-ternary code of rate 0.76, RZ, 25 spans
- Optical Power, $P$ [mW]
- Time, $t$ [ps]

**Right Diagram:**
- Pseudo-ternary code of rate 0.76, RZ, 60 spans
- Optical Power, $P$ [mW]
- Time, $t$ [ps]
Q-factor Improvement

- In the absence of ASE noise
- In the presence of ASE noise
Eye Opening Penalty Improvement

![Graph showing Eye Opening Penalty Improvement for different pseudo-ternary constrained codes of varying rates.](image-url)

- Pseudo-ternary constrained code of rate 0.76
- Pseudo-ternary constrained code of rate 0.78
- Pseudo-ternary constrained code of rate 0.80
- Pseudo-ternary constrained code of rate 0.83

The graph illustrates the improvement in eye opening penalty (ΔEOP) in dB as the number of spans (N) increases for different rates of pseudo-ternary constrained codes.
Combined Constrained and Error Control Coding

Uncoded signal bit-error rate, BER

Bit-error rate, BER

- LDPC(1057,813)
- Combined constrained end FEC coding
Constrained Codes of Rate 0.69

- RZ, 30 spans
- Power, $P$ [mW]
- Time, $t$ [ps]

Constrained code of rate 0.69, RZ, 30 spans

- RZ, 60 spans
- Power, $P$ [mW]
- Time, $t$ [ps]

Constrained code of rate 0.69, RZ, 60 spans
Conclusion

• The use of constrained codes to counter the effects of IFWM and IXPM is proposed.
• Significant Q-factor improvement up to 9.75 dB, and significant eye-opening penalty improvement of more than 10.24 dB, depending on code rate and number of spans, are demonstrated.
• At 40 Gb/s and above, constrained codes can significantly improve the transmission distance and system capacity.
• The constrained codes are capable of improving the FEC threshold in systems with severely degraded performance due to intrachannel nonlinearities.
• Notice that the complexity of the constrained encoder/decoder is significantly simpler than any state of the art FEC scheme employed in long-haul optical transmission.