Introduction to Statistics and Biostatistics: Practice Exercises for Sec. 8.1

Name: _____**Answer Key**_____

Instructions: Read each question carefully before determining the best answer. For numerical answers, report all final numerical values to a precision of 4 units past the decimal point. **SHOW ALL YOUR WORK**; do not rely on a computer output to satisfy the answer.

1. In each of the following circumstances with $X \sim B(n,p)$, state whether or not you may use the plus-four/Agresti-Coull confidence interval for estimating an unknown p.

Want $X \geq 10$ and $n-X \geq 10$:

a. $n = 70$, $X = 50$

   Yes   **X**   X = 50 ≥ 10 and n–X = 20 ≥ 10

   No   ☐

b. $n = 80$, $X = 15$

   Yes   **X**   X = 15 ≥ 10 and n–X = 65 ≥ 10

   No   ☐

c. $n = 10$, $X = 5$

   Yes   ☐

   No   **X**   X = 5 < 10

d. $n = 60$, $X = 50$

   Yes   **X**   X = 50 ≥ 10 and n–X = 10 ≥ 10

   No   ☐

e. $n = 20$, $X = 15$

   Yes   ☐

   No   **X**   n–X = 5 < 10

Introduction to Statistics and Biostatistics: Practice Exercises for Sec. 8.1

Name: _____**Answer Key**_____

2. Gambling is an issue of great concern to those involved in intercollegiate athletics. Because of this concern, the National Collegiate Athletic Association (NCAA) surveyed student-athletes concerning their gambling-related behaviors. They found that 1337 out of a total of 3381 female student-athletes reported participation in some gambling activity. Employ the plus-four/Agresti-Coull confidence interval to estimate the true proportion with a 95% confidence interval. (Use the space below to show your work.)

Lower limit: 0.3791

Upper limit   0.4120

We have X ~ B(3381,p) with X = 1337 observed.

First: check the plus-four condition (X ≥ 10 and n–X ≥ 10): X = 1337 ≥ 10
                                                                                       n–X = 2044 ≥ 10

Next, calculate $\tilde{p} = \dfrac{X + 2}{n + 4} = \dfrac{1339}{3385}$ = 0.3956, with standard error $SE_{\tilde{p}} = \sqrt{\dfrac{\tilde{p}(1-\tilde{p})}{n+4}} = \sqrt{\dfrac{(0.3956)(0.6044)}{3385}} = \sqrt{7.0635 \times 10^{-5}} = 0.0084.$

At C = 0.95, find $z_{.025}^{*}$ = 1.96. Then, the m.o.e. is m = (1.96)(0.0084) = 0.0165.

Thus the lower limit is 0.3956 – 0.0165 = 0.3791

and   the upper limit is 0.3956 + 0.0165 = 0.4121.

Introduction to Statistics and Biostatistics: Practice Exercises for Sec. 8.1

Name: _____**Answer Key**_____

3. A survey of 1300 student loan borrowers found that 444 had loans totaling more than $20,000 for their undergraduate education. Give a 90% plus-four/Agresti-Coull confidence interval for the proportion of all student loan borrowers who have loans of $20,000 or more for their undergraduate education. (Use the space below to show your work.)

Lower limit: _____0.3205_____

Upper limit _____0.3635_____

We have $X \sim B(1300, p)$ with $X = 444$ observed.

First: check the plus-four condition ($X \geq 10$ and $n{-}X \geq 10$): $X = 444 \geq 10$
$$n{-}X = 856 \geq 10$$

Next, calculate $\tilde{p} = \dfrac{X+2}{n+4} = \dfrac{446}{1304} = 0.3420$, with standard error $SE_{\tilde{p}} = \sqrt{\dfrac{\tilde{p}(1-\tilde{p})}{n+4}} =$ $\sqrt{\dfrac{(0.3420)(0.6580)}{1304}} = \sqrt{1.7258{\times}10^{-4}} = 0.0131$.

At C = 0.90, find $z_{.05}{}^* = 1.645$. Then, the m.o.e. is $m = (1.645)(0.0131) = 0.0215$.

Thus the lower limit is $0.3420 - 0.0215 = 0.3205$

and the upper limit is $0.3420 + 0.0215 = 0.3635$.

Introduction to Statistics and Biostatistics: Practice Exercises for Sec. 8.1

Name: _____**Answer Key**_____

4. In a study of the relationship between pet ownership and physical activity in older adults, 615 subjects reported that they owned a pet, while 1942 reported that they did not. Give a plus-four/Agresti-Coull 90% confidence interval for the proportion of older adults in this population who are pet owners. (Use the space below to show your work.)

Lower limit: _____0.2269_____

Upper limit _____0.2549_____

We have $X \sim B(2557, p)$ with $X = 615$ observed.

First: check the plus-four condition ($X \geq 10$ and $n–X \geq 10$): $X = 615 \geq 10$

$n–X = 1942 \geq 10$

Next, calculate $\tilde{p} = \dfrac{X + 2}{n + 4} = \dfrac{617}{2561} = 0.2409$, with standard error $SE_{\tilde{p}} = \sqrt{\dfrac{\tilde{p}(1–\tilde{p})}{n+4}} =$

$\sqrt{\dfrac{(0.2409)(0.7591)}{2561}} = \sqrt{7.1405 \times 10^{-5}} = 0.0085.$

At $C = 0.90$, find $z_{.05}{}^* = 1.645$. Then, the m.o.e. is $m = (1.645)(0.0085) = 0.0140.$

Thus the lower limit is $0.2409 – 0.0140 = 0.2269$

and   the upper limit is $0.2409 + 0.0140 = 0.2549.$

Introduction to Statistics and Biostatistics: Practice Exercises for Sec. 8.1

Name: _____ **Answer Key** _____

5. Many people die in bicycle accidents each year. One study examined the records of 1911 bicyclists aged 15 or older who were fatally injured in bicycle accidents in a five-year period, who also had their blood alcohol concentrations recorded. In this study 390 bicyclists had blood alcohol levels above 0.10%, a level defining legally drunk at the time. Give a plus-four/Agresti-Coull 99% confidence interval for the underlying true proportion who were legally drunk according to this criterion. (Use the space below to show your work.)

Lower limit: _____ 0.1810 _____

Upper limit _____ 0.2284 _____

We have $X \sim B(1911, p)$ with $X = 390$ observed.

First: check the plus-four condition ($X \geq 10$ and $n-X \geq 10$): $X = 390 \geq 10$

$n-X = 1521 \geq 10$

Next, calculate $\tilde{p} = \dfrac{X + 2}{n + 4} = \dfrac{392}{1915} = 0.2047$, with standard error $SE_{\tilde{p}} = \sqrt{\dfrac{\tilde{p}(1-\tilde{p})}{n+4}} =$

$\sqrt{\dfrac{(0.2047)(0.7953)}{1915}} = \sqrt{8.5012 \times 10^{-5}} = 0.0092$.

At $C = 0.99$, find $z_{.005}^* = 2.576$. Then, the m.o.e. is $m = (2.576)(0.0092) = 0.0237$.

Thus the lower limit is $0.2047 - 0.0237 = 0.1810$

and the upper limit is $0.2047 + 0.0237 = 0.2284$.