$$U = \frac{(n-2)MSE}{\sigma^2} \sim \chi^2(n-2)$$

indep of $Z = \dfrac{b_1 - \beta_1}{\sigma / \sqrt{\sum (X_i - \bar{X})^2}}$

$$T = \frac{Z}{\sqrt{U/(n-2)}} = \frac{b_1 - \beta_1}{\sigma / \sqrt{\sum (X_i - \bar{X})^2}} \Big/ \sqrt{\frac{\frac{(n-2)MSE}{\sigma^2}}{n-2}}$$

$$= \frac{b_1 - \beta_1}{\sigma / \sqrt{\sum (X_i - \bar{X})^2}} \sqrt{\frac{\sigma^2}{MSE}}$$

$$= \frac{b_1 - \beta_1}{\sigma / \sqrt{\sum (X_i - \bar{X})^2}} \frac{\sigma}{\sqrt{MSE}}$$

$$= \frac{b_1 - \beta_1}{\sqrt{MSE / \sum (X_i - \bar{X})^2}} = \frac{b_1 - \beta_1}{s\{b_1\}}$$

$$\sim t(n-2)$$