

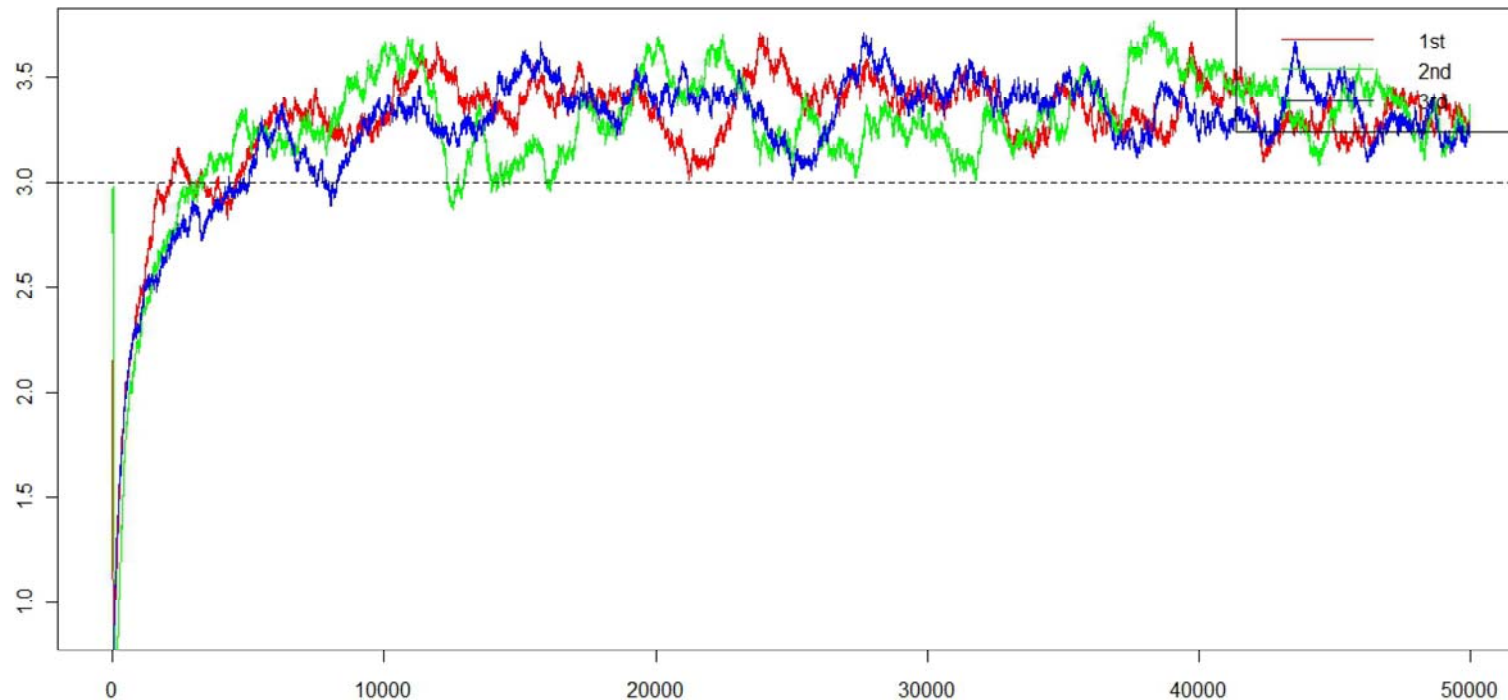
# A Brief Introduction to Geweke's Diagnostics

STAT 675

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# McMC and Burn-in

- McMC algorithms (Gibbs, M-H, hybrid methods...) generate Markov Chains which simulate the posterior distribution.
- Burn-in: early proportion of the chain that may not converge to target distribution yet. (See example below.)



# Geweke's Diagnostic

- Can we treat, say, the first 10,000 iterations as burn-in?
- In a long-enough chain whose trace plot suggests convergence to the target distribution, we assume the second half of the chain has converged to the target distribution, and we test if the first 10% can be treated as burn-in.
- Idea: mimic the simple two-sample test of means. If the mean of the first 10% is not significantly different from the last 50%, then we conclude the target distribution converged somewhere in the first 10% of the chain.

# Geweke's Diagnostic, continued

- Two-sample ( $X_1$  and  $X_2$ ) T test of mean (unequal variance)

$$T = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\frac{s_1^2}{n} + \frac{s_2^2}{m}}},$$

The D.F. for T are from Satterthwaite (=ugly) but can be treated roughly as  $\min(n-1, m-1)$ .

- When  $n, m \rightarrow \infty$ , T can be approximated using the standard normal Z.
- However, the sample variances,  $s_1^2$  and  $s_2^2$  need to be adjusted as the two 'samples' are not independent.
- Geweke's diagnostics use spectral densities to estimate the sample variances (Geweke, 1992).

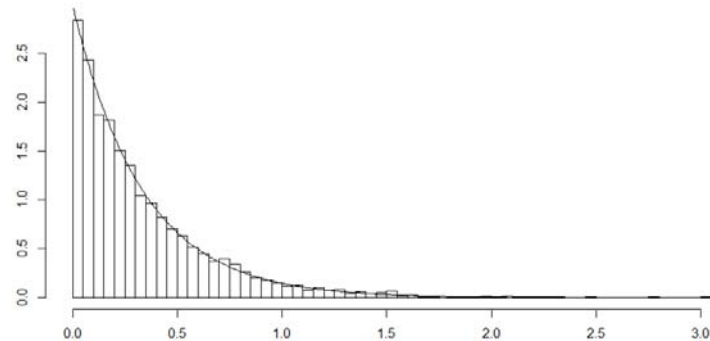
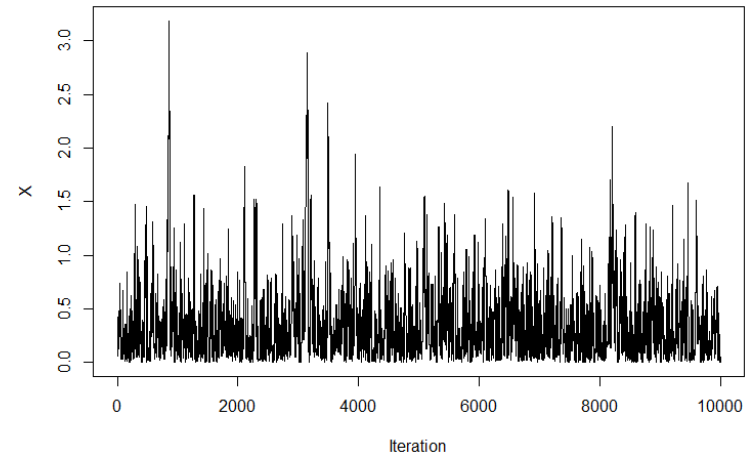
# Geweke's Diagnostic, continued

- Originally, Geweke's diagnostic was only designed for the univariate scenario. One way to expand it to a bivariate scenario is to also consider the sample covariance between the two variables  $X$  and  $Y$ .
- Suppose, a bivariate chain  $(X, Y)$  is considered.
- After performing Geweke's diagnostic for each chain, calculate  $\gamma_i = (x_i - \bar{X})(y_i - \bar{Y})$  for each draw of  $X$  and  $Y$ , and then perform Geweke's diagnostic for the sample of  $\gamma$ . Burn-in is chosen as the early proportion that passes all three diagnostics.

# Example 1. Metropolis Random Walk Sampler for Exp(3)

- R code

```
MH. Gamma =  
function(NN=10000, start=0.1, sd.norm=0.3)  
{  
  X = length(NN)  
  X[1] = start  
  for (i in 2:NN){  
    Xcandi = rnorm(1, mean=X[i -  
1], sd=sd.norm)  
    if ((dexp(Xcandi, 3)/dexp(X[i -  
1], 3))>=runif(1)){  
      X[i] = Xcandi  
    } else{  
      X[i] = X[i -1]  
    }  
  }  
}  
X  
}  
library(coda)  
set.seed(100)  
object = MH. Gamma()  
pnorm(abs(geweke.diag(mcmc(object))$z), l  
ower.tail=FALSE)*2
```



Z-statistics = 0.5700696. P-value for two-sided test is 0.5686305

# Example 2. Bivariate Normal Gibbs sampler

- Bivariate normal with

$$\mu_x = \mu_y = 0 \text{ and}$$
$$\sigma_x^2 = \sigma_y^2 = 1.$$

- Joint p.d.f of X and Y:

$$\frac{1}{2\pi\sqrt{1-\rho^2}} \exp \left[ -\frac{1}{2(1-\rho^2)} (x^2 + y^2 - 2\rho xy) \right].$$

- The conditional p.d.f.s are

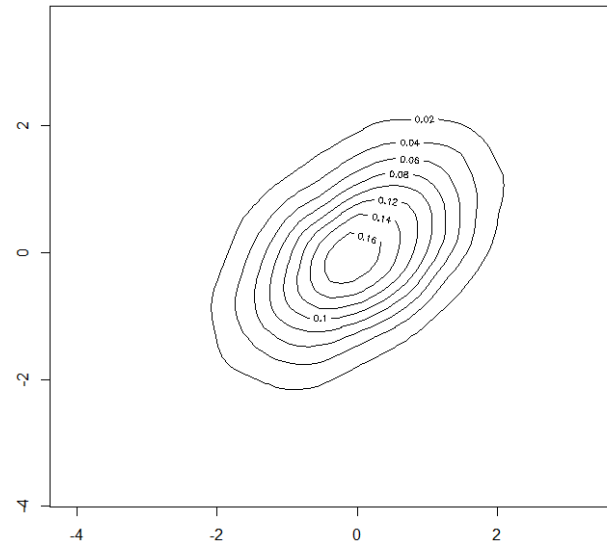
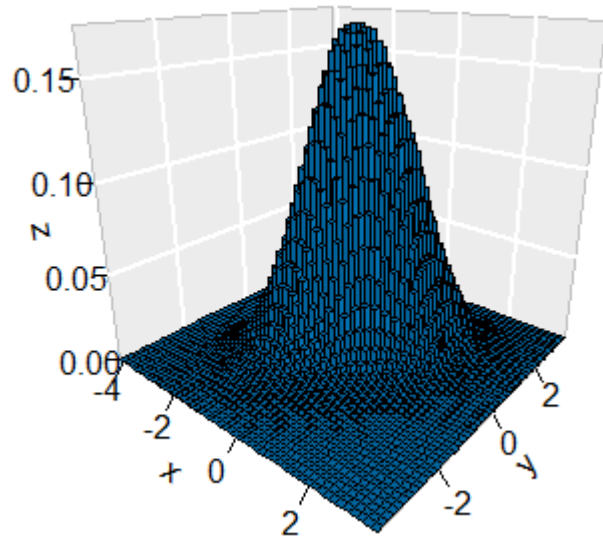
$$X|Y \sim N(\rho y, 1 - \rho^2)$$

$$Y|X \sim N(\rho x, 1 - \rho^2)$$

- R code

```
Gi bbs. bnorm =  
functi on(NN=10000, rho=0. 5){  
  X = Y = length(NN)  
  X[1]=Y[1]=0  
  for (i in 2: NN){  
    X[i] = rnorm(1, rho*Y[i -1], sqrt(1-  
rho^2))  
    Y[i] = rnorm(1, rho*X[i], sqrt(1-  
rho^2))  
  }  
  return(list(X=X, Y=Y))  
}  
set. seed(100)  
obj ect = Gi bbs. bnorm()  
pnorm(abs(geweke. di ag(mcmc(obj ect$X))$z)  
, lower. tai l=FALSE)*2  
pnorm(abs(geweke. di ag(mcmc(obj ect$Y))$z)  
, lower. tai l=FALSE)*2  
gamma=(obj ect$X-  
mean(obj ect$X))*(obj ect$Y-  
mean(obj ect$Y))  
pnorm(abs(geweke. di ag(mcmc(gamma))$z), lo  
wer. tai l=FALSE)*2
```

# Example 2, continued



- All the p-values from the Z tests are greater than 0.05 ( $p_x = 0.407$ ,  $p_y = 0.329$ ,  $p_\gamma = 0.198$ ), so we may treat the first 10% as burn-in.



# Conclusion

- Geweke's diagnostic can be used to determine the burn-in period. (The smallest early portion of the chain that passes the diagnostic.)
- It is similar to the two-sample test of mean.
- Easy to apply in R, (*coda* package, `geweke.diag` function).
- When expanding to multivariate situation, apply additional diagnostics on between-variable covariances.

# Reference

Geweke, J. (1992). Evaluating the accuracy of sampling-based approaches to the calculation of posterior moments. In *Bayesian Statistics 4*, Bernardo, J. M., Berger, J. O., Dawid, A. P. and Smith, A. F. M. (eds.), 169-193. Oxford: Oxford University Press.