STAT 675

Statistical Computing

Additional Homework Exercises – Chapters 13–14

In addition to Textbook Exercises 13.1, 13.4, 13.6:

14.A. The following samples of counts, Y, were reported from an ecological study of species abundance in song birds. Recorded were counts of species and ambient wind speed. Download the full data from http://math.arizona.edu/~piegorsch/675/birds.csv.

x = wind speed (m/sec)	1.1	0.5	···	2.3	0.4
Y = # species	17	45	···	15	27

For these data, build a *log-linear regression model* where $Y_i \sim \text{Poisson}(\mu_i)$, with $\mu_i = \exp\{\beta_0 + \beta_1 x_i\}$ and use maximum likelihood (ML) to estimate β_0 and β_1.

(i) Write the (Poisson) log-likelihood function here as a function of β_0 and β_1. You can ignore any constants that do not affect the maximization.

(ii) Calculate the ML estimates using the log-likelihood from part (i). Use **R**'s `optim()` function to maximize this log-likelihood with respect to β_0 and β_1, via the Nelder-Mead simplex method. Select $\beta_0 = 1$ and $\beta_1 = -1$ as initial values and report the consequent MLEs. Check if any issues occur with convergence.

14.B. Suppose data are observed as $X_i \sim$ i.i.d. *Exp*(θ) for $i = 1, ..., n$, and that an additional, independent observation, $X_{n+1} \sim Exp(\theta)$ is missing. Extend the EM approach from in-class Example 11.A to find the MLE of θ. Show both the E step and M step operations. Illustrate your EM equations by finding the MLE of θ if the observed data are the following n = 62 recorded waiting times between serious earthquakes over a 75-year period (the 'next' waiting time has yet to be observed, since the next serious earthquake has yet to happen):

840	157	145	44	33	121	150	280	434	736	584
887	263	1901	695	294	562	721	76	710	46	402
194	759	319	460	40	1336	335	1354	454	36	667
40	556	99	304	375	567	139	780	203	436	30
384	129	9	209	599	83	832	328	246	1617	638
937	735	38	365	92	82	220.				

Download the full data from http://math.arizona.edu/~piegorsch/675/earthq.csv.