Math120R: Precalculus Final Exam Review, Spring 2016

This study aid is intended to help you review for the final exam. Do not expect this review to be identical to the actual final exam. Refer to your course notes, in-class test reviews (posted on the precalculus homepage), homework, worksheets, and Webassign problems as well in preparing for the final exam.

1. The equation of the line passing through the ordered pair \((a, 0)\) and parallel to the line \(x + 4y = 1\) is:

   (A) \(y = \frac{1}{4}x - \frac{a}{4}\)

   (B) \(y = -\frac{1}{4}x + a\)

   (C) \(y = \frac{1}{4}x + \frac{a}{4}\)

   (D) \(y = -4x + 4a\)

   (E) \(y = -\frac{1}{4}x + \frac{a}{4}\)

2. The police can determine the speed, \(S\), that a car was traveling from the length of the skid mark, \(L\), that the car leaves. Assuming \(S\) varies directly with the square root of \(L\), express \(S\) as a function of \(L\).

   (A) \(S(L) = kL^2\)

   (B) \(S(L) = k\sqrt{L}\)

   (C) \(L(S) = kS^2\)

   (D) \(S(L) = k\sqrt{S}\)

   (E) \(L(S) = kS\)
3. The distance traveled by a falling object is directly proportional to the square of the time it takes to fall that far. If the object falls 100 feet in 2.5 seconds, how far does it fall in 5 seconds?
   (A) 800 feet    (B) 400 feet    (C) 325 feet
   (D) 250 feet    (E) 200 feet

4. Find the domain of the function \( g(t) = \frac{4t}{\sqrt{5-t}} \)
   (A) \((-\infty,5) \cup (5,\infty)\)    (B) \((-\infty,0) \cup (0,5)\)    (C) \((-\infty,\infty)\)
   (D) \((0,5) \cup (5,\infty)\)    (E) \((-\infty,0) \cup (0,5) \cup (5,\infty)\)

5. Let \( f(x) = \begin{cases} 4x+3 & x \geq 2 \\ x^2 - 4 & x < 2 \end{cases} \)

   Find the \( y \)-intercept of \( f(x) \).
   (A) \((0,3)\)    (B) \((0,-4)\)    (C) \(0, -\frac{3}{4}\)
   (D) \((0,2)\)    (E) None of these

6. Let \( f(x) = 4x^2 \). Find and simplify \( \frac{f(3+h)-f(3)}{h} \).
   (A) 1    (B) 4h    (C) \(h+9\)    (D) \(4h+24\)    (E) None of these
Problems 7 and 8 refer to the graph of the function $y = f(t)$ shown below.

7. Determine the interval(s) where $f(t) < 0$.

(A) $(-4, 0)$  
(B) $(-4, -2)$  
(C) $(-5, -2) \cup (4, 5)$  
(D) $(-5, -2) \cup (2, 5)$  
(E) $(-2, 0)$

8. Determine the average rate of change of $f(t)$ from $t = -3$ to $t = 5$.

(A) 8  
(B) -8  
(C) $-\frac{1}{8}$  
(D) $\frac{1}{8}$  
(E) 0

Use the table of values for a function $f(x)$ below to answer question 9.

<table>
<thead>
<tr>
<th>$x$</th>
<th>-5</th>
<th>-2</th>
<th>2</th>
<th>5</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f(x)$</td>
<td>1</td>
<td>6</td>
<td>-1</td>
<td>-3</td>
<td></td>
</tr>
</tbody>
</table>

9. If $f(x)$ is an odd function, what is the missing value in the table?

(A) 6  
(B) -6  
(C) $\frac{1}{6}$  
(D) $-\frac{1}{6}$  
(E) 0
10. Consider the functions \( f(x) \) and \( g(x) \) in the tables below.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
<th>( x )</th>
<th>( g(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>1</td>
<td>2</td>
<td>-1</td>
</tr>
<tr>
<td>6</td>
<td>4</td>
<td>3</td>
<td>-4</td>
</tr>
<tr>
<td>7</td>
<td>2</td>
<td>4</td>
<td>-2</td>
</tr>
<tr>
<td>8</td>
<td>3</td>
<td>5</td>
<td>-3</td>
</tr>
</tbody>
</table>

Which ONE of the following expresses the correct relationship between \( f(x) \) and \( g(x) \)?

(A) \( g(x) = -f(x) + 3 \)
(B) \( g(x) = f(-x + 3) \)
(C) \( g(x) = -f(x + 3) \)
(D) \( g(x) = f(-x) - 3 \)
(E) \( g(x) = -f(x - 3) \)

11. If the point \((8, -3)\) is on the graph of \( f(x) \), find the corresponding point on the graph of the transformation \( y = 2f(-x) \).

(A) \((16, 6)\)  (B) \((4, 3)\)  (C) \((-16, -3)\)
(D) \((-8, -6)\)  (E) \((-8, 6)\)

12. If the domain of \( f(x) \) is \([-5, 8]\), what is the domain of the function \( y = f(x + 2) - 1 \)?

(A) \([-7, 6]\)  (B) \([-3, 9]\)  (C) \([-4, 6]\)
(D) \([-7, 7]\)  (E) \([-3, 10]\)
13. Let \( f(x) = \frac{2}{x} \) and \( g(x) = \frac{x}{x-1} \). Find and simplify \( g(f(x)) \) completely.

(A) \( \frac{2}{x-1} \) \hspace{1cm} (B) \( \frac{2}{2x-1} \) \hspace{1cm} (C) \( \frac{2}{2-x} \)

(D) \( \frac{2(x-1)}{x} \) \hspace{1cm} (E) \( \frac{x}{2-x} \)

14. Let \( f(t) = \sqrt{3t-1} \). Find \( f^{-1}(4) \).

(A) \( \frac{17}{3} \) \hspace{1cm} (B) \( \sqrt{13} \) \hspace{1cm} (C) 5 \hspace{1cm} (D) \( \sqrt{11} \) \hspace{1cm} (E) 4

15. The function \( S = f(b) \) gives a student’s score on a standardized test as a function of the number of books \( b \) the student has read. If \( S \) is invertible, what is the meaning of \( f^{-1}(35) = 20 \)?

(A) The student’s score increases by \( \frac{35}{20} \) for every additional book the student reads.

(B) When the student’s score is 20, the student has read 35 books.

(C) The student’s score increases by \( \frac{20}{35} \) for every additional book the student reads.

(D) When the student’s score is 35, the student has read 20 books.

(E) There is not enough information to determine the meaning.
16. The hypotenuse of a right triangle is four times its base. If \( b \) is the base of the triangle, express the area \( A \) of the triangle as a function of \( b \).

(A) \( A = 2b^2 \)  
(B) \( A = \frac{\sqrt{3}b^2}{2} \)  
(C) \( A = \sqrt{3}b^2 \)

(D) \( A = 2\sqrt{15} b^2 \)  
(E) \( A = \frac{\sqrt{15}b^2}{2} \)

17. Which ONE of the following is true about the function \( f(x) = -3(x - p)^2 + q \), provided that \( p \neq q \)?

(A) \( q \) is the maximum value of \( f(x) \)

(B) \( p \) is the maximum value of \( f(x) \)

(C) \( q \) is the minimum value of \( f(x) \)

(D) \( p \) is the minimum value of \( f(x) \)

18. Determine whether \( f(x) = 2x^2 - 6x + k \) has a maximum or minimum value, and determine the maximum or minimum value.

(A) Maximum value: \( k \)

(B) Minimum value: \( k \)

(C) Maximum value: \( k - \frac{9}{2} \)

(D) Minimum value: \( k - \frac{9}{2} \)

(E) Minimum value: \( k + \frac{9}{2} \)
19. Which ONE of the following represents the complete factorization of 
\[2(3x+1)^7 - 16x(3x+1)^6\]?

(A) \(2(3x+1)^6(1+11x)\)
(B) \(2(3x+1)^6(1-5x)\)
(C) \(2(3x+1)^6(1-4x)\)
(D) \(2(3x+1)^6(1-11x)\)
(E) \(2(3x+1)(1-4x)\)

20. Suppose the graph of a polynomial function \(y = f(x)\) has the following end behavior:

\[y \to -\infty \text{ as } x \to \infty\]
\[y \to -\infty \text{ as } x \to -\infty\]

Which ONE of the following statements must be true?

(A) The degree of \(f(x)\) is an odd number.
(B) \(f(x)\) is an even function.
(C) \(f(x)\) has a minimum value.
(D) The range of \(f(x)\) is all real numbers.
(E) The leading coefficient of \(f(x)\) is a negative number.

21. Solve for \(x\):

\[\frac{x^2 - 6x}{x + 2} = 0\]

(A) \(x = 0, x = 6, \text{ or } x = -2 \text{ only}\)  
(B) \(x = 6 \text{ only}\)

(C) \(x = 0 \text{ or } x = 6 \text{ only}\)  
(D) \(x = -2 \text{ only}\)

(E) \(x = 6 \text{ or } x = -2 \text{ only}\)
22. Find the value of $A$ so that $y = -4$ is the horizontal asymptote of $g(x) = \frac{3x+7}{Ax-2}$.

(A) $A = -\frac{4}{3}$  
(B) $A = -\frac{3}{4}$  
(C) $A = -\frac{1}{2}$  
(D) $A = -2$  
(E) None of these

23. Suppose $x = 6$ is a vertical asymptote of a function $y = f(x)$. Which ONE of the following must be a vertical asymptote of $y = 2f(3x)$?

(A) $x = 2$  
(B) $x = 3$  
(C) $x = 12$  
(D) $x = 18$  
(E) $x = 6$

Questions 24 and 25 refer to the function $f(x) = \frac{3x^2-7x-20}{x+2}$.

24. Find the zero(s) of $f(x)$.

(A) $x = -\frac{5}{3}$ and $x = 4$ only  
(B) $x = -2$ only  
(C) $x = -\frac{5}{3}$, $x = -2$ and $x = 4$ only  
(D) $x = -20$ only  
(E) $x = -10$ only

25. Find the horizontal or slant asymptote of $f(x)$.

(A) $y = 3x-1$  
(B) $y = 3$  
(C) $y = 3x-10$  
(D) $y = 3x-13$  
(E) $y = 3x$
26. Find the value of $p$ so that the vertical asymptote of $f(x) = \frac{6px}{4x + p}$ is $x = 5$.

(A) $p = \frac{10}{3}$  
(B) $p = -20$  
(C) $p = -\frac{5}{4}$

(D) $p = 10$  
(E) None of these

27. The range of $f(t) = -3^t + 200$ is:

(A) $(-\infty, \infty)$  
(B) $(-\infty, 0)$  
(C) $(-\infty, 200)$

(D) $(200, \infty)$  
(E) $(0, \infty)$

28. Find the inverse function of $h(t) = 7^t + 19$.

(A) $h^{-1}(t) = \log_{19}(t - 7)$

(B) $h^{-1}(t) = \log_{7}(t - 19)$

(C) $h^{-1}(t) = -19 + 7^t$

(D) $h^{-1}(t) = -19 + \log_{7}(t)$

(E) $h^{-1}(t) = \frac{1}{7^t + 19}$
Use the following information to answer questions 29 and 30.

The velocity of a skydiver, in feet per second, \( t \) seconds after jumping out of an airplane, is modeled by the function \( v(t) = a(1 - e^{-bt}) \), where \( a \) and \( b \) are positive constants.

29. Based on this model, what happens to the skydiver’s velocity as \( t \to \infty \)? The skydiver’s velocity approaches:

(A) \( \infty \)  (B) \( a + b \)  (C) \( a - b \)  (D) \( a \)  (E) \( b \)

30. Assume that \( a = 100 \). If the skydiver’s velocity is 70 feet per second after 10 seconds, determine the exact value of \( b \).

(A) \( b = \frac{\ln(10)}{70} \)  (B) \( b = \frac{\ln(0.7)}{10} \)  (C) \( b = \frac{\ln(0.7)}{-10} \)

(D) \( b = \frac{\ln(0.3)}{10} \)  (E) \( b = \frac{\ln(0.3)}{-10} \)

Problems 31 and 32 refer to the function \( f(x) = \log_7 (11x + 3) \).

31. Find the domain of \( f(x) \).

(A) \( \left[ -\frac{3}{11}, \infty \right) \)  (B) \( \left( -\frac{11}{3}, \infty \right) \)  (C) \( (0, \infty) \)

(D) \( \left( -\frac{3}{11}, \infty \right) \)  (E) \( \left( \frac{3}{11}, \infty \right) \)

32. Find the exact zero of \( f(x) \).

(A) \( x = -\frac{2}{11} \)  (B) \( x = -\frac{4}{11} \)  (C) \( x = \frac{10}{11} \)

(D) \( x = \log_7 (3) \)  (E) None of thes
33. Simplify the expression completely: \( \ln(4e^x) \)

(A) \( 4x \) \qquad (B) \( \ln(4) + x \) \qquad (C) \( \ln(x) + 4x \)

(D) \( x\ln(4) + x \) \qquad (E) \( \ln(4) + e^x \)

34. Solve for \( x \): \( \log_4(2x+1) - \log_4(x-3) = 1 \)

(A) \( x = -2 \) only

(B) \( x = 4 \) or \( x = \frac{7}{2} \) only

(C) \( x = \frac{11}{2} \) only

(D) \( x = -4 \) only

(E) \( x = \frac{13}{2} \) only

35. Let \( \tan w = \frac{2}{5} \), where the terminal point of \( w \) is in Quadrant I. Find the exact value of \( \cos w \).

(A) \( \cos w = \frac{5}{\sqrt{29}} \) \qquad (B) \( \cos w = \frac{2}{\sqrt{21}} \)

(C) \( \cos w = -\frac{2}{\sqrt{29}} \) \qquad (D) \( \cos w = \frac{5}{\sqrt{21}} \)

(E) \( \cos w = 5 \)
Problems 36 and 37 refer to the following information.

Suppose a real number \( t \) determines the terminal point \((a, b)\) on the unit circle. See the graph below.

36. Find the terminal point determined by the real number \(-t\).

(A) \((b, a)\)  
(B) \((-a, b)\)  
(C) \((a, b)\)  
(D) \((-a, -b)\)  
(E) \((a, -b)\)

37. Find \(\sin(t + \pi)\).

(A) \(b\)  
(B) \(a\)  
(C) \(-b\)  
(D) \(-a\)  
(E) None of these

38. Find the terminal point \((x, y)\) on the unit circle determined by the real number \(t = \frac{3\pi}{4}\).

(A) \(\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)\)  
(B) \(\left(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)\)  
(C) \(\left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)\)  
(D) \(\left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)\)  
(E) \(\left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)\)
Problems 39 and 40 refer to the graph of \( f(t) = A\sin(Bt) + C \), shown below.

39. Determine the value of \( B \).
   (A) \( B = \frac{\pi}{3} \) \hspace{1cm} (B) \( B = \frac{\pi}{2} \) \hspace{1cm} (C) \( B = 6 \)
   (D) \( B = 3\pi \) \hspace{1cm} (E) \( B = 12 \)

40. Determine the value of \( C \).
   (A) \( C = 6 \) \hspace{1cm} (B) \( C = 0 \) \hspace{1cm} (C) \( C = -1 \)
   (D) \( C = 4 \) \hspace{1cm} (E) \( C = -5 \)

41. In a certain region of Australia, the population of a particular type of kangaroo is modeled by the function \( P(t) = 1300 - 140\sin(2t) \), where \( t \) is measured in years. According to the model, what is the maximum kangaroo population?
   (A) 1160 \hspace{1cm} (B) 1440 \hspace{1cm} (C) 1300
   (D) 1580 \hspace{1cm} (E) There is no largest number.
42. The minimum value of \( g(x) = -37 \cos(x - 3) + 21 \) is:

(A) \(-58\)  \quad (B) \(-16\)  \quad (C) \(37\)  \quad (D) \(21\)  \quad (E) \(16\)

43. Which ONE of the following is a vertical asymptote of the graph of \( f(x) = \tan \left( x + \frac{\pi}{3} \right) \)?

(A) \(x = -\frac{\pi}{3}\)  \quad (B) \(x = -\frac{\pi}{6}\)  \quad (C) \(x = 0\)

(D) \(x = \frac{\pi}{6}\)  \quad (E) \(x = \frac{2\pi}{3}\)

44. The domain of \( f(t) = \cos^{-1}(t) \) is:

(A) \([0, 2\pi]\)  \quad (B) \(\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]\)  \quad (C) \((-\infty, \infty)\)

(D) \([0, \pi]\)  \quad (E) \([-1,1]\)

45. Simplify the expression \( \tan \left( \sin^{-1} \left( \frac{x}{3} \right) \right) \). Assume \(0 < x < 3\).

(A) \(\frac{x}{3-x}\)  \quad (B) \(\frac{x}{3}\)  \quad (C) \(\frac{\sqrt{x^2-9}}{3}\)

(D) \(\frac{x}{\sqrt{9-x^2}}\)  \quad (E) \(\frac{\sqrt{9-x^2}}{x}\)
46. Suppose \( \cot \theta > 0 \) and \( \sec \theta < 0 \). In which quadrant could \( \theta \) terminate?

(A) Quadrant I  (B) Quadrant II  

(C) Quadrant III  (D) Quadrant IV

47. Find the length of an arc that subtends a central angle of 135° in a circle with radius 5.

The length of the arc is:

(A) \( 675\pi \)  (B) 675  (C) \( \frac{27\pi}{4} \)  (D) \( \frac{3\pi}{20} \)  (E) \( \frac{15\pi}{4} \)

48. The volume of a cone, shown below, is given by \( V = \frac{1}{3} \pi r^2 h \).

Express the volume as a function of \( \theta \).

(A) \( V = \frac{16}{3} \pi \sin^2 \theta \cos \theta \)

(B) \( V = \frac{64}{3} \pi \sin^2 \theta \cos \theta \)

(C) \( V = \frac{64}{3} \pi \cos^2 \theta \sin \theta \)

(D) \( V = \frac{16}{3} \pi \sin \theta \cos \theta \)

(E) \( V = \frac{16}{3} \pi \cos^2 \theta \sin \theta \)
49. Use the angle $32.8^\circ$ to determine the exact value of $x$ in the figure below.

\[
\begin{align*}
\text{(A)} \quad x &= 14 \sin(32.8^\circ) \\
\text{(B)} \quad x &= \frac{\cos(32.8^\circ)}{14} \\
\text{(C)} \quad x &= 14 \tan(32.8^\circ) \\
\text{(D)} \quad x &= \frac{14}{\sin(32.8^\circ)} \\
\text{(E)} \quad x &= 14 \cos(32.8^\circ)
\end{align*}
\]

50. Which ONE of the following angles is coterminal with $-245^\circ$?

\[
\begin{align*}
\text{(A)} \quad -115^\circ & \quad \text{(B)} \quad 25^\circ & \quad \text{(C)} \quad 65^\circ \\
\text{(D)} \quad 115^\circ & \quad \text{(E)} \quad 245^\circ
\end{align*}
\]

51. Let $\cos \phi = -0.4$. Determine the value of $\cos(-\phi)$.

\[
\begin{align*}
\text{(A)} \quad -0.4 & \quad \text{(B)} \quad \pi - 0.4 & \quad \text{(C)} \quad \pi + 0.4 \\
\text{(D)} \quad 2\pi - 0.4 & \quad \text{(E)} \quad 0.4
\end{align*}
\]
52. Simplify the expression \( \sin A (\csc A - \sin A) \).

(A) \( 1 - \cos A \)

(B) \( \sin^2 A \)

(C) \( 1 \)

(D) \( 1 - \sin A \)

(E) \( \cos^2 A \)

53. Let \( \sin A = \frac{1}{3} \) where \( A \) terminates in Quadrant I, and let \( \cos B = \frac{2}{3} \), where \( B \) terminates in Quadrant IV. Using the identity \( \cos(A - B) = \cos A \cos B + \sin A \sin B \), find \( \cos(A - B) \).

(A) \( \frac{\sqrt{8} - 2}{3} \)

(B) \( \frac{2\sqrt{8} - \sqrt{5}}{9} \)

(C) \( \frac{2\sqrt{8} + \sqrt{5}}{9} \)

(D) \( \frac{\sqrt{8} + 2}{3} \)

(E) \( 1 \)

54. Suppose \( \sin x = -\frac{5}{13} \), where \( x \) terminates in Quadrant III. Using the identity \( \sin(2x) = 2 \sin x \cos x \), find \( \sin(2x) \).

(A) \( \frac{120}{169} \)      (B) \( \frac{8}{13} \)      (C) \( -\frac{10}{13} \)

(D) \( -\frac{120}{169} \)   (E) \( -\frac{8}{13} \)
55. Solve for $t$: $2\cos t - 1 = 0$ on the interval $0 \leq t < 2\pi$.

(A) $t = 0$, $t = \pi$, $t = \frac{\pi}{3}$, or $t = \frac{5\pi}{3}$ only.

(B) $t = 0$ or $t = \pi$ only.

(C) $t = \frac{\pi}{3}$ or $t = \frac{5\pi}{3}$ only.

(D) $t = \frac{\pi}{6}$ or $t = \frac{11\pi}{6}$ only.

(E) $t = 0$, $t = \pi$, $t = \frac{\pi}{6}$, or $t = \frac{11\pi}{6}$ only.

56. Solve for $x$: $\frac{\sin(3x) - 2}{\cos 3x} = 0$ on the interval $0 \leq x < \frac{\pi}{2}$

(A) $x = \frac{\pi}{6}$ only

(B) $x = \frac{\tan^{-1}(2)}{3}$ only

(C) $x = 0$ only

(D) $x = \sin^{-1}\left(\frac{2}{3}\right)$ only

(E) No solution