Facilitating Whole-Class Discussion in Secondary Mathematics Classrooms

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Dedication

To my parents, Beverly and James McGraw

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Abstract

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This dissertation presents findings from a qualitative, case study of two teachers' efforts to engage high school students in whole-class discussions as part of the process of learning mathematics. Characteristics of the process of facilitating whole-class discussion are examined, as well as struggles and supports. Analysis of the setting suggests that features of the process of facilitating discussion include (a) developing opinions and motivation for discussion, (b) structuring the physical space of the classroom, (c) bringing students' ideas to the forefront of discussion, (d) creating reasons for listening, (e) encouraging students to question and respond to each other, and (f) pushing position-taking and creating a need for consensus. Analysis also suggests that teachers who attempt to utilize whole-class discussion as a tool for learning mathematics struggle with external time and curricular constraints, student expectations, and a high level of uncertainty with respect to their role during discussion. Further research is needed to determine the extent to which these findings translate to a range of educational settings.

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Chapter 1: Introduction

My dissertation tells the story of the efforts of two teachers, Kathryn Thomas and me, to use whole-class discussions as a tool for helping students learn mathematics. Kathryn and I planned, implemented, and reflected on instruction during the first semester of the 2000-2001 school year. We worked with two groups of ninth grade, Algebra 1 students, at a large high school in a school district adjacent to a major, Midwestern city. Throughout the semester, I gathered data to answer the following questions:

- What are the features of the process of facilitating whole-class discussions in secondary mathematics classrooms?
- What do teachers struggle with as they attempt to facilitate discussion?
- What supports teachers in their efforts to facilitate discussion?

This study of the process of facilitating whole-class discussion was situated within a particular way of thinking about the teaching and learning of mathematics. Therefore, I begin this manuscript with a description of the epistemological and pedagogical perspectives that informed my work.

Cobb and Bauersfeld (1995) identify two general theoretical positions regarding the relationship between individual and social processes and learning, both of which are significant for this study. The first position involves the treatment of learning as an individual constructive process. The second position involves consideration of learning as acculturation into social practices and traditions. I will outline each of these positions and then consider a third position that accommodates both the social and psychological processes of learning. Finally, I will describe a way of thinking about the process of teaching that builds upon the third perspective.

The Learning Process

Traditionally, doing math in school has meant following a set of rules as stated by the teacher and knowledge has been determined by one's ability to remember and apply these rules (Lampert, 1990). Teaching involved showing and telling, and mathematical content was broken up into smaller and smaller pieces until each piece could be shown to students during a typical class period. Underlying this way of thinking about mathematics teaching and learning is the view of knowledge development as the transmission of information from the teacher who knows to the students who don't. Freire (1970), calling this the "banking concept of education" (p. 58), described how teachers attempt to "deposit" bits of information into students for them to "receive" and "store". The banking, or transmission, model of teaching and learning rests on an understanding of knowledge as a set of ideas separate from human interpretation, and students as passive receptacles capable of being filled with these ideas.

Many educators have rejected this view of teaching and learning in favor of one based on student action and interpretation. Radical constructivists call into question the notion of a body of knowledge independent of knowers and capable of being transmitted to passive learners. They "give up the requirement that knowledge represents an independent world, and admit instead that knowledge represents something that is far more important to us, namely what we can *do* in our *experiential world*" (von Glasersfeld, 1995, p. 6-7, italics in original). Rooted in the work of Piaget, radical constructivism holds that knowledge is not passively received but rather actively constructed and interpreted by learners as they try to make sense of their experiences. “Learning is not a stimulus-response phenomenon. It requires self-regulation and the building of conceptual structures through reflection and abstraction. Problems are not solved by the retrieval of rote-learned ‘right’ answers.” (von Glasersfeld, 1995, p. 14).

For Piaget, the building of conceptual structures is an equilibrium process in which the individual incorporates new ideas into existing cognitive schemas through assimilation and accommodation. Assimilation involves the incorporation of new information into existing schemas, while accommodation involves the restructuring of schemas to make sense of new information. Assimilation and accommodation occur through reflection and abstraction, as a result of cognitive conflict generated during one’s interaction with the external environment. Radical constructivists believe that the individual’s environment, including social processes, plays an important role in learning; however, they view learning as primarily an individual cognitive activity. The sociological perspective, to which I now turn, focuses on learning as participation in social processes, with individual cognitive activities contributing to the development of the social.

Theorists operating from a social perspective, sometimes termed social constructivists or social constructionists, are similar to the radical constructivists in that they too reject the transmission or banking model of knowledge development. Theorists who view learning as primarily social, however, give priority to social and cultural processes in learning over individual cognitive activity. Those who follow the Vygotskian and activity theory tradition, for instance, consider learning to be “located in coparticipation in cultural practices” (Cobb & Bauersfeld, 1995, p. 4), rather than located primarily within the cognizing individual. For these theorists, “the central concern is to delineate the social and cultural basis of personal experience” (Cobb & Bauersfeld, 1995, p. 5) in an effort to explain “how participation in social interactions and culturally organized activities influences psychological development” (Cobb & Bauersfeld, 1995, p. 4). Some theorists (e.g., Solomon, 1989; Walkerdine, 1988) go so far as to reject consideration of psychological development altogether. For such theorists, knowledge development is evidenced not by the creation of increasingly complex cognitive schemas, but by students’ ability to engage in the pregiven discursive practices of the classroom.

Just as constructivist theories of learning have been criticized for treating knowledge as exclusively psychological, theorists who view learning as a social process have been criticized for describing knowledge in social terms and not attending to the psychological side of students’ constructive activities. The theoretical perspective on knowledge development that underlies my study represents an attempt to coordinate the psychological and social perspectives. This “emergent perspective” (Cobb, 1995) does not give preeminence to either perspective, but instead asserts the usefulness of each for understanding how students come to know.

The emergent perspective holds that individual constructive activities and classroom social processes are reflexive and mutually constraining.

> On the one hand, the emergent perspective goes beyond exclusively psychological approaches by viewing students’ mathematical activity as being necessarily socially situated. Therefore, the products of students’ mathematical development – increasingly sophisticated ways of reasoning – are seen to be related to their participation in particular communities of practice such as those constituted by the teacher and the students in the classroom. On the other hand, the emergent perspective questions the subordination of psychological processes to social processes and attributes a central role to analyses of individual students’ mathematical activity. (Cobb, 2000, p. 309)

In terms of research on students' mathematical development, proponents of the emergent perspective assert the usefulness of holding either the psychological or social as primary and then coordinating analysis between the two.

My study does not consider students' mathematical development directly. Rather, I consider teachers' efforts to engage students in certain kinds of classroom discourse. Although I focus on the classroom community as a whole and the efforts of the teachers to establish norms and practices at this level, I consider teachers' efforts to be informed and constrained by their perceptions of students' understandings. In addition, I view the teaching activity of facilitating discussion to be both a social classroom process and a cognitive activity on the part of individual teachers. Thus, the emergent perspective on learning that I have adopted applies equally to teachers, as they reflect on practice and build professional knowledge and expertise.

Because the emergent perspective suggests that research on knowledge development must include attention to both the psychological and the social, teachers, attempting to support and promote knowledge development, must also concern themselves with both of these aspects. Building upon the emergent perspective, Simon (1997) has developed a view of the teacher's role that includes both the psychological and the social, and it is upon his view of teaching that I base my analysis of facilitating discourse. His view is described in the next section.

The Teaching Process

In building his model of teaching, Simon (1997) attempted to answer the question "If we give up showing and telling as the teacher's principal means for promoting students' mathematical development, what do we have to replace them?" (p. 68). He suggested two activities in which teachers can engage: posing problems and facilitating discourse. When teachers choose problems to pose they may be thought of as taking a cognitive view of the learning process, and when they attempt to facilitate classroom discourse they may be thought of as taking a social view.

According to Simon (1997), choosing problems to pose can be thought of as an attempt to influence the growth of cognitive structures by intentionally promoting and supporting accommodation and assimilation.

> From the perspective of assimilation, the teacher focuses on what tasks might be undertaken successfully (but not trivially), given her model of the students' schemes. From the perspective of accommodation, the teacher focuses on what tasks might challenge the limitations of current schemes. . . . . The teacher's challenge is to identify student tasks that result in an appropriate balance between assimilation and accommodation. (p. 69)

In addition, teachers can promote the cognitive activities of reflection and abstraction by choosing tasks that encourage students to reflect on their ways of thinking and make generalizations. The teacher as problem poser also contributes to students' views about the nature of mathematical activity. For example, problems can be posed in ways that engage students in searching for patterns, developing hypotheses, and justifying conclusions

There are many ways to judge the quality of mathematical activities or problems. Kathryn and I were influenced in our choice of problems by the notion of "level of cognitive demand" (Stein & Smith, 1998). Problems with a low level of cognitive demand require only that students perform an algorithm by rote, whereas high-level tasks require pattern finding, generalizing, and making connections. The relationship between the use of high-level tasks and teachers' attempts to engage students in discussion is not entirely clear. However, if students are to engage in justifying their ideas and arguing among ideas, then it is reasonable to believe that high-level tasks have the potential to support discourse in ways that low-level tasks do not. Thus, there is an obvious relationship between teachers' efforts to pose problems that influence cognitive processes and teachers' efforts to influence social processes.

In terms of the social processes of knowledge development, the teacher contributes "to establishing a classroom community in which particular conceptions about mathematics are more likely to be developed and particular norms and practices established" (Simon, 1997, p. 72). Although the students and teachers interactively constitute what is appropriate activity, the teacher as facilitator of classroom discourse exerts a tremendous influence over what is viewed as legitimate mathematical activity.

> The teacher as facilitator of the classroom discourse has a major effect on what issues are explored, which significantly influences what is seen as of mathematical interest, which in turn influences views of what constitutes mathematics (e.g., what is central to the discipline). Further, the teacher's role in orchestrating discussion, as well as his or her initiation of particular conversations about students' mathematical activity contributes to establishing what it means to be an effective member of this mathematical community, including the valuing and appropriateness of certain types of reasoning and communication. (Simon, 1997, p. 72).

Chazan and Ball (1999) characterize teacher actions during discussions as "the product of subtle improvisation in response to the dynamics and substance of student discussion" (p. 7). These researchers suggest that orchestrating discussions requires a continuous analysis of (a) the value of the mathematics under discussion in relation to students, (b) the direction of the discussion and the degree of momentum, and (c) the social and emotional tone. Teachers' responses to each of these considerations influence students' views of mathematics and of the learning processes.

Simon's conception of teaching as posing problems and facilitating discourse was consistent with Kathryn and my desire to create classrooms similar to the "inquiry mathematics classrooms" described by Cobb, Wood, Yackel, and McNeal (1992). Inquiry classrooms are places where students explore problems, make and test conjectures, explain and justify their ideas, and attempt to make sense of and evaluate the ideas of others. Students spend time in small groups exploring and discussing non-routine problems, and in large groups contributing their ideas, and debating and building upon the ideas of other students and the teacher. Teachers engaged in inquiry-based instruction emphasize understanding and making connections among ideas. Teachers do not explain a set of rules to be followed, instead they engage students in developing methods for solving problems. In inquiry-based classrooms, mathematical knowledge is determined by the ability to engage with the problem solving process, explain and justify ideas, and discuss and evaluate the ideas of others. Mathematical truth is no longer the sole possession of the teacher, rather it is the joint construction of the teacher and the students (Cobb, Wood, Yackel, & McNeal, 1992).

With regard to classroom discourse, our views were consistent with the recommendations of the Indiana Professional Standards Board (IPSB) (1998) and the National Council of Teachers of Mathematics (NCTM) Standards documents (1991, 2000), both of which are consistent with Simon's conceptualization. Their recommendations, together with Simon's conceptualization, provided me with a vision of the classrooms we hoped to create and the nature of the teaching activities we could engage in. What remained was to find a way of thinking about the teaching process that included activities teachers engage in outside of class time. Fortunately, Simon (1995,

1997) suggested a way of thinking about the teaching process that proved useful in this respect.

The Mathematics Teaching Cycle (Simon, 1995) is a conceptual framework developed to describe the author's own teaching of pre-service elementary teachers. This cycle (Figure 1) describes the relationships among teacher knowledge, teacher planning, and teacher interaction with students. The types of problems posed by the teacher and the way the teacher facilitates classroom discourse contribute to the "interactive constitution of classroom practice." Teacher-student interaction, including the teacher's efforts to understand students' mathematics through observation and communication, influences teacher knowledge. Teacher knowledge takes a variety of forms, from knowledge of mathematics and students' ways of thinking to knowledge of teaching and learning. Teacher's knowledge and external factors become the basis for choosing goals for student learning. The learning activities chosen to help students reach these goals are based on the teacher's hypothesis of the process of learning.

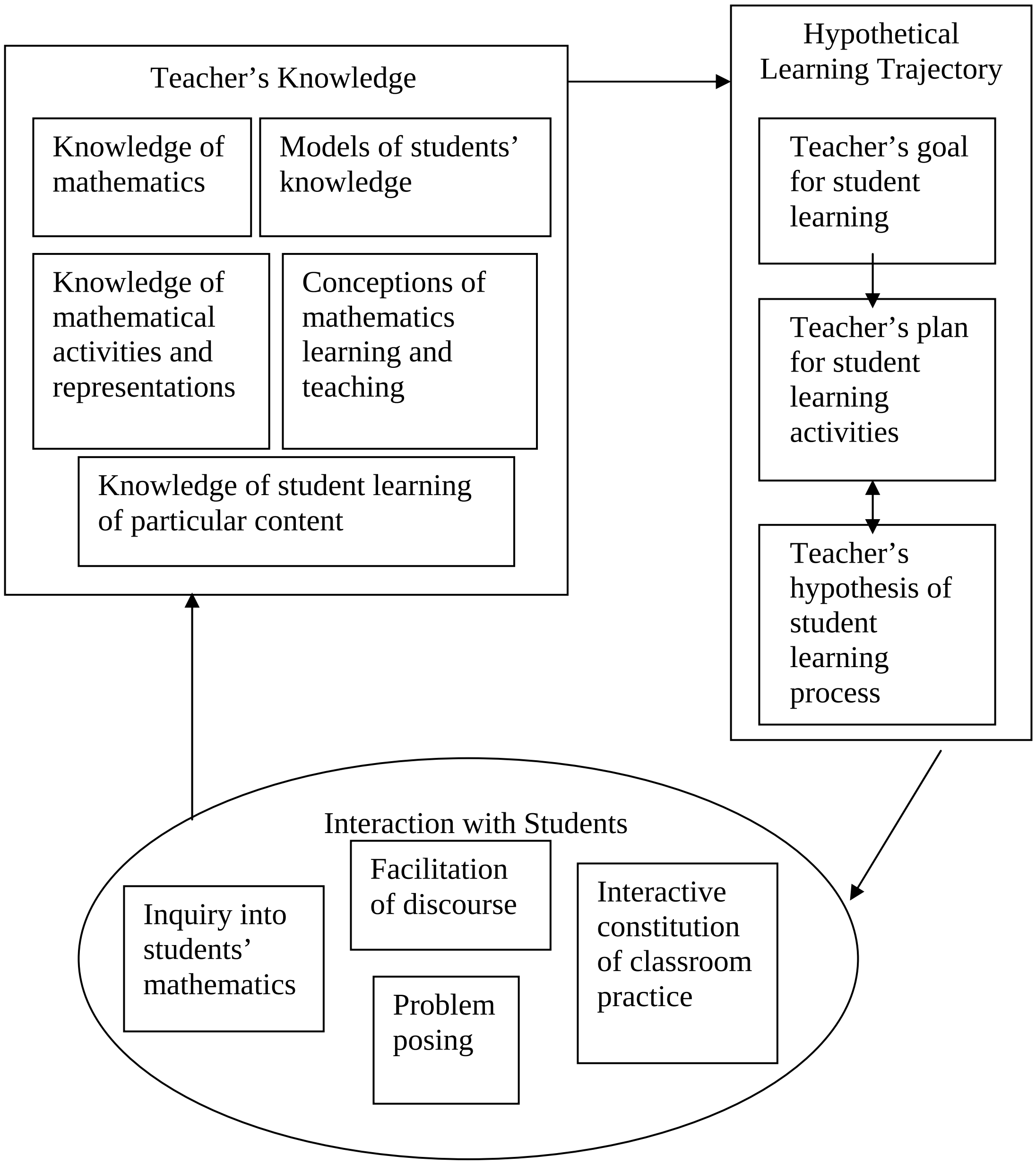


Figure 1. The Mathematics Teaching Cycle (adapted from Simon, 1995, 1997)

Teachers' learning goals for students may include the development of norms and practices consistent with a particular view of what it means to learn and do mathematics. Whole-class discussions may be viewed as learning activities with both content- and process-related goals. Teachers make decisions about the appropriate placement and content of discussion based on their hypotheses of the process of learning and in response to their interactions with students.

I situate my examination of the process of facilitating discussion within the larger process of teaching as described by this framework. It influenced my data collection and analysis in that I gathered data and then looked for evidence of our efforts to facilitate discussion during planning and reflection times, as well as during implementation. Of course, the actual discussions occurred within the "interaction with students" portion of the cycle, however, when we reflected on the discussions we were attempting to add to our "teacher knowledge" about discussion which then influenced us when we planned for discussion. The framework was useful to me in that it allowed me investigate our efforts and struggles not as isolated activities, but rather with respect to a specific way of thinking about teaching.

Having described the epistemological and pedagogical perspectives that undergirded and framed my research, I now turn to the literature on classroom discourse and describe, in the following chapter, how it informed Kathryn and my efforts to facilitate whole-class discussions and how it influenced my research. Then, in Chapter 3, I describe the methods and procedures I used to investigate the process of facilitating whole-class discussions. In Chapter 4, I present results of the study, including features of the process of facilitating discussion, and related struggles and supports. Lastly, in Chapter 5, I return to Simon's conceptualization of teaching and situate my findings within it, relate my findings to the literature on classroom discourse, and suggest areas for further study.

Chapter 2: Review of Selected Literature

The literature on classroom discourse informed my study at two levels; Kathryn and I drew upon this literature as we attempted to engage students in whole-class discussions, and this literature informed my examination of the process of facilitating discussion. I have divided the literature on classroom discourse discussed here into three parts: (a) purposes and patterns of classroom discourse, (b) strategies for altering patterns of discourse, and (c) methods for judging the quality of discussions. The ideas discussed in each part were useful to Kathryn and my efforts to facilitate whole-class discussion as well as to my analysis of our efforts.

Purposes and Patterns of Classroom Discourse

Discourse in the classroom serves a multitude of purposes including shaping social identities and relationships, creating a sense of what it means to learn in school, and communicating beliefs about the nature of particular subject matter. Patterns of discourse reflect and serve to reinforce teachers' and students' beliefs about the nature of teaching and learning in school. Knowledge of the purposes of classroom discourse and common and alternative patterns of discourse is important for teachers as they consider how to engage students in discussions about mathematics. In the following paragraphs I examine these purposes and patterns.

Power and Authority

Classroom talk is intimately tied to classroom power and authority relations because "social identities and social relationships are signalled and reproduced *in the act of speaking*." (Edwards, 1980, p. 239) Typically, the teacher, in a position of power over students, has the right "to 'invite stories,' to tell 'stories,' and to announce in so many words what these expositions have 'really' been about." (Edwards, 1980, p. 241) Students are obligated to wait to be nominated to speak, to respond when questioned, and to have their talk evaluated (often publicly) by the teacher. Although power relations are *interactively* constituted by teachers and students, and students have many ways of resisting and disrupting teacher's attempts to control and limit their actions (e.g., see Manke, 1997), unequal communicative rights characterize much of teacher-student talk. Classroom talk frequently serves the purpose of establishing the teacher as *the* knowledgeable and powerful person in the classroom.

If classroom discourse can be used for the purpose of establishing the teacher's power, then it can also be used for the purpose of empowering students. Kreisberg (1992), examining the characteristics of classrooms in which teachers and students share power, found that the ways teachers and students interacted provided opportunities for students and teachers to share decision-making and thus to share power. Robinson (1994) described modes of interaction that break down traditional barriers between teachers and students as "transformative" because they "distribute learner roles more evenly to all participants in the dialogue." (p.137) Kreisberg and Robinson did not conceive of power as a zero sum game. Rather, they viewed the sharing of ideas and responsibilities in the classroom as empowering to both teachers and students.

In addition to reflecting and reestablishing teacher-student power relationship, classroom discourse serves the purpose of communicating information about the nature of subject matter and of the learning process itself. The more the teacher talks, the fewer opportunities students have to share, and listen and respond to each other's ideas. When the teacher’s ways of thinking dominate classroom discourse, students may be less likely to reflect on their own thinking. When teaching involves primarily “showing and telling,” the view of learning communicated to students is that of the transmission or banking model discussed previously. Conversely, when students’ ways of thinking are brought to the fore and students are encouraged to listen and respond to each other, a very different view of learning and students’ role in the process emerges. Students can come to view themselves as powerful agents in their own learning.

When students use each other’s words as thinking devices their relationship to discourse can be termed “dialogic.” Building on the concept of “functional dualism” (Lotman, 1988) and on their own research on classroom discourse, Wertsch and Toma (1995) argued that discourse serves two primary functions: (a) to accurately convey information; and (b) to assist the speaker and listener in creating meaning. Wertsch and Toma used the terms “univocal” and dialogic” to refer to these functions of discourse, respectively, and associated the univocal function with the transmission model of teaching and learning. They argued that the relative dominance of either function has major implications for classroom discourse.

> It is reasonable to expect that when the dialogic function is dominant in classroom discourse, pupils will treat their utterances and those of others as thinking devices. Instead of accepting them as information to be received, encoded, and stored, they will take an active stance toward them by questioning and extending them, by incorporating them into their own external and internal utterances, and so forth. (Wertsch & Toma, 1995, p.171)

Wertsch and Toma called for increased opportunities for students to engage in discourse grounded in the dialogic function. Engagement in this type of discourse encourages students to view themselves as contributors to each other’s understandings with the power to evaluate correctness and make meaning no longer resting solely with the teacher.

Students' views about what constitutes legitimate mathematical activity are also influenced by patterns of classroom discourse. Through classroom discourse, teachers can communicate to students a vision of mathematics as involving mainly rule and procedure memorization, or a vision that involves investigation, reflection, and application. Teachers can use classroom discourse to highlight specific kinds of mathematical activities, such as understanding and evaluating the correctness of a variety of solution methods. Discourse can be used to help establish an environment in which the teacher is the sole voice of mathematical authority, or it can be used to encourage students to become authorities, too. In short, the relationship students develop with a given subject matter is largely a result of how they engage with the teacher in discourse *about* the subject matter.

Knowledge Development

Attempts to engage students more fully in classroom discourse stem from a belief that the sharing of ideas is a critical part of the learning process. The emergent perspective on knowledge development, discussed in Chapter 1, maintains the usefulness of thinking of learning in both psychological and social terms (Cobb, 2000). From a Piagetian, psychological perspective, classroom discourse can serve the purpose of stimulating disequilibrium and reflective abstraction. From a sociological perspective, knowledge is mainly a social construct, and student learning is evidenced by the ability to engage in appropriate discourse. In either case, knowledge is thought of as being actively constructed by the learner, rather than passively received, and the nature of classroom discourse affects the mathematical knowledge development of students.

Classroom discourse is the central aspect through which relationships among students, teachers, the learning process, and the subject matter are conceived and maintained. It can even be thought of as the place within which knowledge is located. A growing understanding of the centrality of talk to learning has led researchers to examine patterns of discourse and to begin to document alternative patterns and speculate on their potential for enhancing student learning. I turn now to the literature on patterns of discourse and describe a few ideas that may be particularly important for teachers attempting to facilitate discussion.

The IRE Pattern

One very common pattern of teacher-student interaction is the Initiate, Respond, Evaluate (IRE) sequence (Cazden, 1988). The IRE sequence consists of three parts; (a) the teacher called on a student to share, (b) the student responded to the teacher's query, and (c) the teacher commented on the students' response. This sequence allows the teacher to maintain tight control over the direction and momentum of talk, and provides the teacher with some information about students' knowledge. As Cazden noted, it may be argued that the IRE sequence is helpful to both students and teachers. When students have internalized the pattern and implicitly understand this process of classroom talk, they may be freer to attend to the content of the discussion. Teachers may find that the IRE pattern minimizes disruptions, thereby functioning as a controlling or management mechanism. Stodolsky, et al (1981) found this type of discourse may have a positive impact on students' learning of content that is algorithmic and factual.

When the IRE pattern is used frequently, teacher authority and student dependence are likely to characterize interactions. Teachers choose the question to be asked and decide how they will ask it, while students, if they want to please the teacher, must decide what answer the teacher is seeking and how the teacher expects it to be expressed. Once an answer is given, the teacher evaluates it and the process begins again. The IRE pattern thus promotes a view of the teacher as the center of knowledge and authority and a view of learning as the ability to respond appropriately to questions posed by the teacher.

Cazden does not argue that the IRE pattern, or any other specialized pattern of speech, has no intellectual value. However, the IRE pattern does not promote self or peer evaluation nor does it provide students with opportunities to influence directly the direction of the discussion. Other patterns of discourse are needed if teachers expect students to question each other and build upon each other's ideas.

Shifting away from the IRE sequence may be a difficult task for teachers. One critical problem derives from teachers' desire to engage students in inquiry for the purpose of developing predetermined knowledge. When students' investigations do not lead in the desired direction, teachers find themselves in a dilemma. On one hand, students' investigations and understandings should be the focal point of instruction; on the other hand, there is a body of predetermined knowledge that students must master. Edwards and Mercer (1987) describe the impact of this dilemma on classroom discourse.

Cued Elicitation

After students have engaged in an exploratory activity and small group discussion, the teacher may reconvene the class in the hope that students' work can provide the basis for generalization and the development of common knowledge. The teacher may have a predetermined idea about what students should have found out through their explorations and how this information will fit together into a generalizable whole. In an effort to orchestrate the discourse to achieve these predetermined ends without resorting to telling, the teacher may attempt to elicit the necessary information from students by providing them with verbal and/or nonverbal cues. This strategy results in a pattern of interaction that Edwards and Mercer (1987) termed “cued elicitation.” [Note: other researchers have referred to this phenomenon as an *illusion of understanding* (Yackel, 1995) or an *illusion of competence* (Gregg, 1992)].

The cued elicitation pattern consists of the teacher asking students a question and providing clues (through intonation, pauses, or gestures) to improve students’ chances of responding correctly. The teacher then uses the students’ input as shared knowledge with which to build the next point in the construction of the predetermined generalization. Cued elicitation allows teachers to communicate their ways of thinking to students while maintaining the appearance that knowledge is based in students’ understandings.

Edwards and Mercer are critical of the cued elicitation pattern because it masks gaps between teacher and student knowledge, leading teachers to believe that students’ thinking mirrors their own. Cueing students so that they are better able to respond in predetermined ways can lead to a sort of game or ritual in which students become proficient at “reading” the teacher’s cues and responding appropriately, without necessarily understanding or even attempting to understand the underlying (in our case mathematical) processes. If patterns of discourse similar to cued elicitation dominate classroom interaction, doing math may come to mean merely going through the motions of the explorations, and then “reading” the teachers’ cues and responding appropriately.

Cued elicitation requires student participation while leaving control over the direction and content of the discourse in the hands of the teacher. Maintaining a tight control over discourse is one way teachers can ensure that predetermined common knowledge becomes explicit, despite disparate outcomes of student explorations. The cued elicitation pattern was “pervasive” in the data collected by Edwards and Mercer. Patterns similar to cued elicitation have been described by mathematics education researchers including Wood (1998), Yackel (1995), and Williams and Baxter (1996). Although many of the teachers studied by these researchers did engage students in exploratory activities, the teachers were generally not able to carry the inquiry approach successfully into their whole class discussions.

Ritualized Discourse

Researchers have documented a second troubling outcome of teacher’s desire to combine student investigations and student-centered discourse with the development of common knowledge, the “ritualization of discourse.” (Williams & Baxter, 1996) Ritualization occurs when students focus on satisfying the teacher’s desire for participation, rather than on making sense of each other’s ideas as a way to deepen their own understanding. Williams and Baxter observed a middle school teacher who attempted to encourage discourse by setting up norms and expectations for classroom talk, modeling and explaining expected behaviors, and encouraging student explanations, questions, and conjectures. These researchers found that for some students discourse became an end unto itself, and for other students it became just another extraneous requirement. The discussion-oriented environment the teacher sought to create became "part of the meaningless ritual of classroom life, rather than a tool for learning" (p.36). In this classroom, students seemed to lack motivation for actively listening, making sense of, and building off each other's ideas. If discussion lacks real purpose in the minds of students, then perhaps it is not surprising that talk becomes part of "doing school" rather than part of learning mathematics.

Balancing the desire to engage students in inquiry-based learning with the need to develop common knowledge is a difficult task. Recognizing cued elicitation, ritualized discourse, or any pattern of activity or interaction that is inconsistent with the goals of inquiry-based instruction is valuable, but finding other, more productive ways of interacting is equally important. Researchers have begun only recently to describe alternative modes and patterns of discourse; the three described here, all from elementary level classrooms, had particular application to my examination of the process of facilitating discussion.

Reflective Discourse and Collective Reflection

Reflective discourse occurs when the results of mathematical activity become the explicit object of discussion. Consider the following description based on excerpts from research on the reflective discourse of first graders by Cobb, et al. (1997).

> A first grade teacher shows her students a picture of two trees, one larger than the other, and five monkeys. She then presents students with the following problem. "If they all want to play in the trees, just think of ways that we can see all five monkeys in the two trees – all five monkeys to be in two trees."(p.261) The students suggest various combinations of monkeys in trees and the teacher records their ideas in a two-column table like the one shown below.

5	0
2	3
3	2

0	5
4	1
1	4

> Next, the teacher asks the students whether there are any more ways the monkeys could be in the trees.
>
> Teacher: Are there more ways? Elizabeth?
> Elizabeth: I don't think there are more ways.
> Teacher: You don't think so? Why not?
> Elizabeth: Because [that's] all the ways that they can be. (p.263)
>
> At this point another child suggests a way, which, it turns out, is already represented in the table. The teacher encourages students to further reflect on the list by asking another question.
>
> Teacher: Is there a way that we could be sure and know that we've gotten all the ways? (p.263)
>
> Students respond to the teacher's question by attempting to explain how the number pairs (e.g. 0/5 and 5/0 or 2/3 and 3/2) related back to the picture of the monkeys and the trees and how they can tell that "there are no more ways"(p.263).

With the help of their teacher, these children have had the opportunity to solve the original problem and reflect on their solutions. After considering the initial question, "How could all five monkeys be in two trees?" the teacher encourages the children to reflect on the problem when she asks "Are there any more ways?" A further shift in discourse occurs when the teacher asks "Is there a way that we could be sure and know that we've gotten all the ways?" This shift allows the results of the original question, represented by the table of numbers, to become the objects of discussion. As students describe how they can tell there are no more ways to put the monkeys in the trees, their discourse become reflective. As a community, the students and teacher have shifted from collective problem solving to collective reflection. Cobb et al. (1997) conjectured that participation in this type of discourse may support learning of specific mathematical content as well as fostering a general orientation to mathematical activity that includes reflecting on the mathematical relationships underlying a specific problem context.

Focusing

Wood (1998) describes two patterns of discourse she found in second grade mathematics classrooms: funneling and focusing. Funneling bears a striking resemblance to the IRE pattern, while focusing seems to allow for some student-student talk. In the funnel approach, the teacher uses a series of explicit questions to “funnel” the student toward the correct answer.

> [The teacher asks Jim to give the answer to 9+7.]
>
> Jim: 14.
>
> Teacher: OK. 7 plus 7 equals 14. 8 plus 7 is just adding one more to 14 which makes ____ ?
>
> Jim: 15.
>
> Teacher: And 9 is just one more than 8. So 15 plus one more is _____ ?
>
> Jim: 16. (p.170-171)

This bit of interaction is characterized by teacher authority and student dependence. The teacher initiates the content under discussion and controls the process by which it is discussed. The student is dependent on the teacher to lead him through the problem; he merely fills in the blanks as the teacher suggests an appropriate line of reasoning.

The focusing approach involves greater student participation and creates opportunities for students to learn from each other.

> [The problem students are solving is 66 - 28 = __ . The teacher asks John to tell the class his solution to the problem.]
>
> John: (He is writing on the overhead projector.) We put the 28 under the 66. (As he talks, he writes 66 – 28 in a vertical format.) And we

took away . . . we . . . I took . . . the 6 and 8 off. And we said there was 60 and 20 there. (He puts his finger on the 60 and then on 40.) And if you take away 20 from 60, it's 40. (He holds up his fist.) And you still have to take away 8. So we took . . . there's 46 left over. If you take that 6 back, and take away that 6 (points to the 6 in 46) and that's . . . um . . . back to 40 and you still have to take away 2, so 39 (he holds up a finger) then 38. (He writes 38)

[The teacher recognizes that John has invented an effective solution and that his strategy may help others. Rather than stepping in and repeating his idea, she asks the class if anyone has a question. One boy says that he does not understand, and the teacher then asks John to re-explain.]

John: [reexplains his solution]

Elisabeth: But, but why did you take the 6 and the 8 off?

John: It was more easier. (p.173-174)

[At this point, the teacher encourages John to go through his explanation step-by-step, writing and explaining each step as he goes.]

During this episode, the teacher and John shared control over the content of the discourse. The teacher determined which problem would be solved, but the solution method was John's. By allowing students to question John directly, the teacher turns over some control for the process of learning as well. The teacher's expertise is an important component throughout this episode; however; the teacher understands the conceptual power of John's method and encourages other students to ask questions. The teacher also knows the value of the process of explaining step-by-step and encourages John to adopt this method. By sharing authority for the process and content of the discourse, the teacher is better able to understanding how students are thinking, thereby extending her ability to effectively support learning.

Argumentation

Wood (1999) examined discourse in a second grade classroom in which the teaching practices were "fundamentally different from conventional mathematics instruction found in most elementary schools" (p.188). Children in this classroom were expected to explain their reasoning to each other during whole-class discussions. The teacher promoted an active stance toward listening; the children were expected to "follow the thinking and reasoning of others to determine whether what was presented was logical and made sense . . . and voice their disagreement and provide reasons for disagreeing." (p.189). Wood found the following pattern of interaction in all of the classroom discussions that involved argument.

1. A child provided an explanation of . . . [a] solution to a problem.
2. A challenge was issued by a listener who . . . might or might not tell why he or she disagreed.
3. The explainer offered a justification for her or his explanation.
4. At this point, the challenger might accept the explanation or might continue to disagree by offering a further explanation or rationale for his or her position.
5. The explainer continued to offer further justification for her or his solution.
6. This process continued and other listeners sometimes contributed in an attempt to resolve the contradiction.
7. The exchange continued until the members of the class (including the teacher) were satisfied that the disagreement was resolved. (p.179)

Wood argued that the existence of this type of discussion pattern is dependent on teachers setting appropriate expectations for listening and participating during discussion and on students accepting and internalizing these expectations. "When these classroom routines became the tacit patterns of interaction, the children no longer found it necessary to direct their cognitive attention to making sense of their social setting and could direct their mental activity to making sense of their mathematical experiences" (p.189).

Taken together, reflective discourse and collection reflection, focusing, and argumentation suggest what alternative patterns of classroom discourse might look like. Although researchers found these alternative patterns in elementary classrooms, it seems reasonable to consider their usefulness at the secondary level as well. Regardless of grade level, teachers attempting to emulate these patterns must actively seek to alter traditional teacher and student roles during classroom discourse. Teachers must act in ways that encourage students to reflect on their solutions, share their ways of thinking, and push for understanding and consensus.

Teachers attempting to use whole-class discussion as a tool for learning mathematics will need to consider strategies for breaking free from traditional patterns of classroom discourse and encouraging students to adopt more active, critical roles. Understanding how teachers use various strategies is an important goal of research in this area. Therefore, I next describe briefly some of the strategies commonly suggested in research and practitioner literature.

Strategies for Altering Patterns of Discourse

Strategies for altering patterns of classroom discourse can be as complex as changing the relationships among teacher, student, and subject matter, or as simple as using wait time after students speak. I have divided the literature in this area into three overlapping categories; (a) sharing authority over knowledge with students, (b) developing norms and expectations, and (c) using discourse enhancing strategies. In each category are ideas that may be useful for teachers attempting to facilitate discussion, as well as researchers engaged in examining the process of facilitating discussion.

Sharing Authority over Knowledge with Students

Oyler (1996) analyzed a first grade teachers' interactions with her students and identified four key aspects of shared authority. First, the teacher actively sought to share her expertise with students. Sharing authority, however, did not mean abdicating it; the teacher's expertise was valuable in orchestrating both the content and process of classroom activities. At the same time, students brought their own knowledge, expertise, and ways of understanding to the classroom. During discussions, the teacher shared her expertise but also provided students with opportunities to share their knowledge and expertise.

Oyler identified student initiations in terms of both the content and process of learning as the second key aspect of shared authority. Students initiate for content when they bring up their ideas and their ways of thinking about problems or when they decide or suggest the content to be studied. Students initiate for process when they make decisions or suggestions about *how* learning will occur. In terms of facilitating discussion, teachers must be sure that students have opportunities to influence both the content and flow of talk.

The third aspect of shared authority is the co-development of a shared agenda. Student initiations must not be viewed as an end product; rather, teachers must take up these initiations so that classroom activities are formed, in part, according to the interests of the students. "By allowing and encouraging student initiations and expertise, [teachers] actually deepened and extended [their] authority"(p. 135); Oyler described here the fourth key aspect of shared authority. Teachers can use the information they gain by listening to students to become experts on their students' needs and thus better facilitate students' growth.

In terms of facilitating discussion, teachers can make use of several ideas related to sharing authority. Teachers can make room for student initiations and allow student-initiated ideas to be taken up by the group as a whole. Rather than admonishing students for talking, the teacher can encourage cross-talk, thereby allowing students to make decisions about what questions need to be asked. When students speak directly to each other, they have the opportunity to control the flow of talk, as well as the content.

Oyler identified characteristics related to teacher-student interaction in a classroom where the teacher was already sharing authority over knowledge with her students. Mathematics education researchers have begun examine the classroom interactions of teachers' who are actively seeking to share mathematical authority for mathematics knowledge with their students (e.g., Wilson & Lloyd, 1996). Wilson and Lloyd followed three high school mathematics teachers over the course of one year as the teachers implemented an inquiry-based curriculum. All three teachers indicated their desire to share mathematical authority with students, but only one was able to maintain some balance between small-group and whole-class instruction throughout the year. One teacher became more and more directive, effectively stripping students of the decision-making power the new curriculum was intended to support. This teacher cited her biggest challenge as the "fear that her students would not be able to make appropriate connections on their own"(p.16). Another teacher abandoned whole class discussion entirely and spent his time giving "mini-lectures" to small groups. I would suggest that both these teachers engaged unsuccessfully in the struggle to balance inquiry-based methods and the desire to ensure common knowledge.

The third teacher in Wilson & Lloyd's study was successful in fostering a greater level of student autonomy and authority during mathematical activities and discussions. This teacher's methods for encouraging student leadership and participation during whole class discussions included (a) preparing overhead transparencies of students' work so that their work could serve as a resource for discussion, (b) asking a group of students to demonstrate the mathematical connections they had made between graphs, tables, and equations, and (c) physically placing himself in the position of a participant (in the back of the room). These strategies seemed to give students more opportunities to share control over the flow and, to some extent, the content of the discussions.

Teachers attempting to facilitate discussion often do so as part of an effort to engage students in an inquiry-based mathematics learning environment. Shared authority over knowledge is an integral part of such an environment. Therefore, teachers need to be cognizant of problems such as those uncovered by Wilson and Lloyd. If they became too directive during whole-class instruction or if they engage small groups in "mini-lectures" then they may undermine the inquiry process. Research suggests that establishing appropriate norms and expectations about teachers' and students' roles in the classroom is critical for inquiry-based learning. Teachers engaged in facilitating discourse may be particularly concerned with establishing norms and expectations related to whole-class discussion and I now turn to research in this area.

Developing Norms and Expectations

The quality of mathematics classroom discussions is dependent, in part, upon the degree to which the teacher and students share common beliefs with regard to discussions. Quality is determined partially by the extent to which members view sharing, evaluating, and building on each other's ideas through discussion as a critical part of the process of doing mathematics and by the extent to which they believe that significant learning occurs through such interaction. If discussion is not viewed by participants as valuable for these purposes, then it is unlikely that discourse will further one's own and others' understandings of mathematics.

Students enter the classroom with knowledge, beliefs, and values based on their previous experiences both inside and outside of school. Unless students have had previous experiences with high quality mathematics discourse, it is unlikely that teachers who wish to build richer discourse communities in their classrooms will encounter students who share their vision. Thus, the first step in improving the quality of classroom discourse is likely to involve the re-negotiation among students and the teacher of classroom norms and expectations regarding both the purpose of discussion and the roles of the teacher and students during discussions.

Researchers have recently begun to examine the process of establishing norms and expectations related to classroom discourse. Sherin, Mendez and Louis (2000) make a number of suggestions for establishing norms and expectations for discourse based on Louis's experiences building mathematics discourse communities in his own middle school classroom. Their suggestions include (a) having students brainstorm about what makes a good listener, (b) reflecting with students about discussion behaviors, both prior to and after discussions, (c) pointing out when students are referring to each other's ideas, and (d) requiring that everyone be accountable for a mathematical opinion. These researchers concluded that significant effort is required at the beginning of the school year to establish norms and expectations for behavior because students are not used to engaging in these behaviors.

Manouchehri and Enderson (1999) suggested that teachers list norms as rules or principles to be followed and use incidences where students instantiate or transgress social norms as opportunities to discuss expectations. They also recommend that teachers insist that students (a) solve personally challenging problems, (b) explain personal solutions to their peers, (c) listen to and try to make sense of one another's explanations, (d) attempt to achieve consensus about an answer, and (e) resolve conflicting interpretations and solutions. These recommendations can be used to establish appropriate norms for both whole-class discussion and small group work.

In addition to establishing norms and expectations for whole-class discussions, teachers can use specific strategies during discussion to facilitate student listening, sharing, and questioning. Researchers have suggested a variety of strategies that teachers may use to enhance the quality of classroom discourse.

Using "Discourse-Enhancing" Strategies

Suggestions for improving or facilitating whole-class discussions include establishing participant frameworks, using strategies for engaging students in discussion, and providing students with more time to wrestle with a problem or idea. Knowledge of recommendations such as these is important both to discussion facilitators and to researchers examining and characterizing the process of facilitating discussion.

Establishing Participant Frameworks Through Revoicing

One method of incorporating students' ideas into whole class discussions is to reconstruct students' solution methods or paraphrase students' responses. Often termed "revoicing", this strategy can serve the potentially beneficial purpose of helping to establish participant frameworks (Forman, Larreamendy-Joerns, Stein, & Brown, 1998; O'Conner & Michaels, 1993).

By reconstructing or paraphrasing students' contributions to classroom discourse, teachers can frame discussions and position students' ideas within these frames. Consider the following fictitious interchange between a teacher and student during a whole class discussion.

> Teacher: Who can describe how they began to solve this problem?
>
> Sean: I took the first two numbers and timesed them.
>
> Teacher: So, Sean says to begin by multiplying five and three. Did anyone begin a different way?

The first word in the teacher's response, "so", is a marker for a warranted inference (Shiffrin, 1987). The word "so" indicates that the speaker (teacher) intends to make an inference that she or he believes is warranted based on the remarks of the previous speaker (student). The word "so" can indicate to students that the teacher is about to recast the previous speaker's words. However, if cued elicitation or traditional IRE patterns dominate classroom discourse, students may "read" the teacher's use of the word "so" as a cue that they are about to have the "real" meaning or idea explained to them.

Next, the teacher engages in what Goffman (1981) termed *animation*; that is, the teacher brings the student to life by using the words "Sean says". Goffman called "says" a laminator verb because it is used to link the subject of the sentence (Sean) with some proposition (a method for beginning to solve the problem in question). Animating Sean in this way gives him a particular status or position within the discussion – Sean's personal method for beginning the problem becomes a suggestion for the entire class. Notice that the teacher has not merely restated Sean's idea, instead his idea has been recast using the more mathematically correct term "multiply", and specificity has been added by explicitly stating the numbers to be multiplied (five and three). In addition to animating and positioning students within a discussion, paraphrasing students' words is one way teachers can model the use of more formal mathematical language.

The teacher ends by asking if "anyone began a different way?" This move creates space in the discussion for alternative solution methods. As students offer their methods, the teacher can animate and position students and ideas, helping to create what Goodwin (1990) termed a "participant framework". Participant frameworks include how participants are aligned with each other, as well as how they are positioned relative to the ideas under discussion. These frameworks are co-constructed by teachers and students as they animate and position themselves and each other. Teachers exert influence over the structure of participant frameworks both by recasting students' speech and by posing questions such as "did anyone begin a different way?"

O'Conner and Michaels (1993) argued that revoicing may be useful for (a) positioning students and their propositions within a participant framework, (b) reformulating students' ideas in more official language while still crediting them verbally, and (c) strengthening a weak voice that might otherwise be overlooked. These potential benefits must balance against at least two dangers. First, students may learn that they need not listen to each other, since the teacher will likely restate any important ideas or suggestions. Second, in recasting students' comments, the teacher may help to create an illusion of understanding; the teacher may recast ideas so as to align them with predetermined lesson goals, thereby masking students' true understandings.

Establishing participant frameworks can be a powerful tool for engaging students in the examination of each other's ways of thinking. Students need encouragement, however, to explain their ideas and listen to and respond to each other. In addition to establishing appropriate norms and expectations related to classroom discourse, teachers can engage with students during discussions in ways that encourage these behaviors.

Engaging Students in Discussion

Sherin et al. (2000) created a model of classroom discourse based on research in Louis's middle school classroom. Central to their model are discourse strategies for students. The strategies, which Louis taught his students to use, are explain, build, and go beyond. The explain strategy involves giving reasons for ideas or answers, build refers to building on other students' ideas, and to go beyond is to generalize from particular examples to larger contexts. Louis explicitly taught students the strategies, modeled how to use them, and reflected with students on their use after discussions. These researchers found that discourse strategies for students, coupled with establishing norms and expectations for behavior, were useful for altering patterns of discourse during whole-class discussions.

Teachers can encourage students to explain, build, and go beyond by asking questions such as "Can you explain why?", "So, what do people think about that?", or "Do you agree or disagree with _______s idea?" (Sherin, et al., 2000). Teacher questions can encourage students to respond to and question each other's ideas. Suggestions in this vein include "Who can respond to _____s suggestion?" or "That is an excellent question. Anybody else want to take a shot at it?" (Manouchehri & Enderson, (1999).

Reinhart (2000) described strategies teachers can use to encourage students to participate in discussions, many of which do not involve asking questions. Teachers can (a) identify students with good ideas to share and give them a "heads up" prior to discussion, (b) use wait time before and after students speak, (c) try not to comment after each students' response, instead simply pause and wait for someone else to talk, (d) avoid answering your own questions or accepting only one response to a question, and (e) do not allow students to blurt out answers because this cheats others out of their right to think. Copes (2000) suggested that stopping when a student gives the "right" answer violates the spirit of inquiry. He also noted that "it doesn't do much good to answer a question that students haven't asked" (p 297).

Facilitating discussion is a complex and multi-faceted task. Strategies such as those discussed here can only be used successfully when teachers are sensitive to situational variables such as teacher-student relationships and students' level of interest and comfort in speaking. Teachers must also consider students' knowledge and the mathematical value of the particular topic under discussion when making decisions about when to use various strategies. Sometimes stopping whole-class discussion can be an appropriate method of facilitating discussion.

Providing Think Time

Sending students back to small-group or individual work when an interesting idea arises can be an important discourse-enhancing strategy (Sherin, et al., 2000; Reinhart, 2000). Students may need time to wrestle with a new idea or revisit an old one before they are prepared to share their ideas publicly. Teachers can use small group or individual work time to listen to how students are thinking. Understanding students' thinking gives teachers an opportunity to better facilitate discussion when the whole-class reconvenes. If students are not given time to wrestle with ideas that arise in discussion, teachers may be tempted to revert to more directive methods such as funneling or cued elicitation.

When discussions are viewed as opportunities to share ideas and move mathematical thinking forward, rather than times for wrapping up lessons or explaining to students the meanings behind their investigations, teachers may be more comfortable with not correcting faulty thinking. Although teachers can push students to attempt to reach consensus, consensus may not be attainable at a given point in time. By sending students back to their small groups for further investigation, teachers can help students move toward consensus without short-circuiting their inquiries.

Summary

The literature related to altering patterns of classroom discourse provides researchers and teachers with some important insights into the process of facilitating discussion. Students come to the classroom with deeply held beliefs about the nature of school mathematics. Teachers will likely need to re-negotiate classroom norms and expectations with students in order to create classroom environments in which discussion is viewed as a legitimate part of the learning process. Finding ways to share authority for the content and process of learning may become part of teacher's efforts. Awareness of various discourse-enhancing strategies and the interplay between individual work, small group work, and whole-class discussion is important, as is an understanding of the complex and multi-faceted nature of the process. Facilitating discussion involves attention to the needs of particular students at particular moments in time, in addition to concern for the development of particular mathematical understandings.

In addition to knowledge of various strategies for altering patterns of classroom discourse and facilitating discussion, teachers and researchers need methods for reflecting on whole-class discussions and evaluating their quality and usefulness. The methods considered here are those that played a particular role in this study.

Methods for Judging the Quality of Discussions

The research in this area is in its infancy. Researchers can point to the quality of particular interchanges between a teacher and one or more students (e.g. Wood, 1998), and a few have attempted to analyze the quality of entire discussions (e.g. Forman, et al., 1998). One potentially useful framework for analyzing mathematics discussions was recently proposed by Mendez (1998). Mendez used six dimensions to evaluate both the level at which the mathematics was discussed and the level of student involvement in the discussion (Figure 2).

Levels of Justification	Proof - logical argument or counterexample is given
	Explain - student explains why an answer holds and/or how an answer was found
	None - no justification given, but description of what answer is may be given
Levels of Representation	Unpacked - more than three representations, possibly applications or extensions
	Amplified - topic is amplified with two or three representations
	Single - compressed with only one representations
Levels of Generalization	Generalization - generalization beyond particular examples or categorization or recognition of a pattern
	Concrete - limited to one particular situation
Levels of Intensity	Volunteer - students voluntarily join the discussion without teacher mediation
	Elicited - teacher nominates student speakers or asks questions of students
	None - off topic remarks or no students enter discussion
Level of Engagement	Count the number of student speakers as a proxy for number engaged
Levels of Building	Build - responses build on earlier comments with new ideas and are integrated into the discussion
	Neutral - first responses or repetition of ideas

Figure 2. Summary of Mendez's (1998) categories for use in discussion analysis.

This framework is useful for helping teachers consider (a) to what extent did students examine and critique each other's reasoning? (b) did students attend to and build upon each other's ideas or do they merely restate their own positions? (c) did students justify their ideas? and (d) did students move beyond the specific situation, that is, were they able to generalize beyond the work at hand? Mendez's framework does not attend to the importance of the mathematical content under discussion. Questions such as “Does conversation focus on procedural details or mathematical relationships?” and “Is the mathematics of significant importance to merit discussion?” must also be considered when judging the quality of mathematics discussions.

In addition to analyzing the ways in which students engage in talking about mathematics, teachers can ask students directly about the usefulness of discussions. When students write about the ideas they shared, the questions they asked or responded to, and the understandings they developed, they consider both how they contributed to and how they benefited from the discussion. In addition, students’ writing provides teachers with evidence of the level of involvement of students, regardless of whether or not they spoke during the discussion.

Thinking about the teaching process as represented in Simon’s Mathematics Teaching Cycle, reflecting on discussion feeds into teacher knowledge about facilitating discussion which feeds into the Hypothetical Learning Trajectory and then into interactions with students. Analyzing the content and process of teachers' reflective activities became part of my effort to examine the process of facilitating discussion.

Conclusion

Using whole-class discussions as a tool for learning mathematics requires significant time and effort on the part of teachers. Established patterns of interaction must be altered and strategies for encouraging students to listen and respond to each other must be learned. Decisions about direction, tone, and mathematical value must be made during discussions and some method must be used to judge the quality of discussions. Although professional organizations, such as NCTM, provide teachers with a vision of the role of discourse in the learning of mathematics (NCTM, 1991, 2000), mathematics educators do not yet understand very well how to help teachers achieve this vision in their classrooms. Careful examination of teachers engaged in the process of orchestrating discussions is needed if mathematics teacher educators are to support teachers in this important endeavor.

Chapter 3: Methods and Procedures

My study began with a problem defined from practice, specifically, the problem of engaging mathematics students in the types of discussions advocated by the National Council of Teachers of Mathematics (1989, 1991, 2000) and others. I became interested in this topic through my own experiences in the classroom and through working with teachers who expressed concern about their ability to facilitate discussion. My desire for an in-depth, detailed examination of the process of facilitating discussion led me to adopt a qualitative case study methodology (Merriam, 1988). According to Merriam, a qualitative case study is an "intensive, holistic description and analysis of a single instance, phenomenon, or social unit" (p. 21). The phenomenon I investigated was the facilitating of whole class discussion as it occurred through Kathryn and my efforts. My use of one particular case to gain insight into the process of facilitating discussion was consistent with an important purpose of case study research, "to reveal the properties of the class to which the instance being studied belongs" (Guba & Lincoln, 1981, p. 371).

My position with respect to the phenomenon I studied was that of an insider (Ball, 2000). When I facilitated discussion in Kathryn's classroom, I was engaged in creating the object of my own research. Insider research blends the construction of practice with its analysis. The researcher has the opportunity to participate in the design of the phenomena under investigation with the possibility of insights perhaps impossible, or at least very difficult, for an outsider to obtain. However, insider research also requires disciplined methods for the examination of data and the ability to connect the particulars of one's own experience to the broader subject under study. Managing the insider position was an important part of my work and I describe my efforts to do so in the data analysis section of this chapter.

The remainder of this chapter is divided into three sections. In the first section, I describe the context of the study, including descriptions of the schools, districts, study participants, and our collaboration. In the second section, I describe the data I collected and my collection methods and in the third section, I describe my data analysis process. I also address the credibility or trustworthiness of my study in this chapter, as part of my discussion of data analysis.

Context of the Study

The Genesis and Growth of Our Collaboration

Shortly after choosing the process of facilitating whole class discussions as my dissertation topic, I began working as a research assistant on a professional development project aimed at helping experienced secondary mathematics teachers become mentors for beginning teachers. Through this project, I was fortunate enough to meet two teachers with perspectives on teaching and interests in classroom discourse similar to my own. Kathryn Thomas was a high school mathematics teacher and department chairperson at a large urban high school in Indiana. Janet Raymond was a middle school mathematics teacher in a midsize suburban school district, also in Indiana (I provide further information about Kathryn and Janet in the "Study Participants" subsection of this chapter). As participants in the aforementioned professional development project, Kathryn and Janet videotaped their own classrooms and used the video segments to reflect on the quality of their teaching. Both Kathryn and Janet expressed dissatisfaction with the level of mathematical discourse they saw in their classroom videotapes and a desire to improve their teaching in this area. When I approached them about participating in a study about teachers' attempts to facilitate whole class discussion, they agreed to become my collaborators.

Kathryn, Janet and I shared a common goal: each of us wanted to align our teaching more closely with that recommended by the Indiana Professional Standards Board (IPSB) (1998) and the National Council of Teachers of Mathematics (NCTM) (1989, 1991, 2000). Our visions of ideal mathematics classrooms were roughly consistent with what Cobb, Wood, Yackel, and McNeal (1992) have termed "inquiry mathematics classrooms." As I described in Chapter 1, inquiry classrooms are places where students explore problems, make and test conjectures, explain and justify their ideas, and attempt to make sense of and evaluate the ideas of others. Teachers do not explain a set of rules to be followed, but instead engage students in developing methods for solving problems, and mathematical knowledge is jointly constructed by the teacher and students (Cobb, et al., 1992).

With regard to facilitating discussion, our vision was consistent with the IPSB recommendations which are based in large part on the NCTM Standards documents (1989, 1991). The IPSB recommendations with regard to the teacher's role in fostering discourse include: posing questions and tasks that elicit, engage, and challenge each student's thinking; asking students to clarify and justify their ideas orally and in writing; deciding when and how to attach mathematical notation and language to students' ideas; deciding when to provide information, when to clarify an issue, when to model, when to lead, and when to let a student struggle with a difficulty; monitoring students' participation in discussions and deciding when and how to encourage each student to participate. (IPSB, 1998, p. 9) The IPSB recommendations with regard to the students' role include: listen to, respond to, and questions the teacher and one another; initiate problems and questions; make conjectures and present solutions; and try to convince themselves and one another of the validity of particular representations, solutions, conjectures, and answers. (IPSB, 1998, p. 9)

With these recommendations in mind, Kathryn, Janet and I agreed to meet in July and August of 2000 to share resources and plan for the fall semester. Kathryn would be teaching first-year algebra to ninth graders using the textbook Heath Algebra 1: An Integrated Approach (Larson, Kanold, & Stiff, 1998). Janet would also be teaching Algebra 1, but to 8th graders in middle school. Janet would be responsible for teaching the material in the first half of the Prentice-Hall Algebra (Bellman, et al., 1998) textbook only, spread across the entire school year. In the fall semester, Kathryn would be responsible for the material in the first six chapters of her textbook, whereas Janet would be responsible for the material in the first three chapters of her textbook. I would assist Kathryn and Janet in examining their curricula for the fall semester and review with them strategies suggested in the literature for facilitating discussion. Once school began, I would observe and perhaps assist Kathryn and Janet in the classroom and help them to reflect individually and jointly on their efforts to facilitate discussion.

Initially, I did not plan to engage in insider research. However, almost immediately I realized that I would have to put myself on the line, so to speak, and join Kathryn in facilitating discussion in the classroom. Kathryn and I both felt awkward because I was fully engaged in planning and reflecting, but only she was experiencing the struggles of implementation. This inconsistency in my position was not helping either of us experience the process of facilitating discussion, nor was it helping me to make sense of the process. So my status quickly shifted from participant observer to insider. I became a “second teacher” in the classroom three days a week: Tuesday, Thursday, and Friday (commitments at my University prevented me from being in the classroom on Mondays and Wednesdays). I led discussions, introduced activities, and worked with individuals and small groups, and Kathryn and I planned for, implemented, and reflected on instruction together.

Janet’s position in our collaboration also shifted somewhat. Initially, I planned to divide my time between Kathryn's and Janet’s classrooms, acting as a support person in each classroom and gathering data to document each of their efforts to facilitate discussion. However, district preparation requirements for the state-mandated eighth grade test, and other school and district level requirements restricted Janet’s efforts to facilitate discussion. Rather than incorporating whole-class discussion into regular classroom activities, Janet used it only occasionally when she engaged students in developing their own solution methods to problems. Instead of participating in or observing Janet’s classroom activities on a regular basis, I visited only when she planned to facilitate discussion. Janet did meet with Kathryn and me periodically during the semester to share materials and reflect on our efforts to facilitate discussion, and in this way, our three-way collaboration continued throughout the semester.

The changes in Janet's situation and my role in Kathryn's classroom led me to focus my attention on the process of facilitating discussion as we (Kathryn and I) experienced it. Part of this process was our meetings with Janet (six over the course of the semester). None of my individual work with Janet, which was very limited in comparison to my work with Kathryn, was included in this study.

Having described the development of our collaboration and its relationships to my study of the process of facilitating discussion, I now provide further information about the schools, districts, and study participants. As an insider in my research, I include a description of myself in this subsection.

Districts, Schools, and Study Participants

Districts and Schools

Kathryn's school, South Jackson, is the only high school in a metropolitan school district serving a 39 square mile area adjacent to a large midwestern city. At the time of the study (Fall, 2000), the total enrollment at South Jackson High School was 3118 students (grades 9-12) with a full-time faculty equivalent of 156.5. The student population was 59% Caucasian, 34% African-American, 3% Hispanic, 2% Asian, and 2% multiracial. The female to male student ratio was nearly 1:1, and approximately 18% of the students were eligible for free or reduced lunch.

Janet's school, Valley Spring, is the sole middle school in a suburban school district serving a 49 square mile area. At the time of the study, the total enrollment at Janet's school was 1066 (grades 7-8), with a full-time faculty equivalent of 50.5. The student population was 96% Caucasian, 1% Hispanic, 2 Asian, and 1% multiracial. The female to male student ratio was nearly 1:1, and approximately 8% of the students were eligible for free or reduced lunch.

Study Participants

Kathryn. At the time of the study, Kathryn was already committed to and somewhat experienced in and knowledgeable about engaging students in investigating non-routine problems. Her view of teaching was consistent with that proposed by Simon: posing tasks and facilitating discourse. Further, she had been dissatisfied with her attempts to engage students in whole class discussions and was eager to examine this aspect of her role as teacher more closely. Kathryn was an experienced teacher and mathematics department chair at her high school. Her previous teaching experiences included 16 years at two public high schools, and two years at a public middle school. She also had had five years of experience as a mathematics department chair (two years at the middle school level and three years at South Jackson High School). During our collaboration, she was the teacher of record for three 9th grade Algebra 1 classes. In addition to her teaching and department chair responsibilities, Kathryn had been active for many years in local and state level professional development activities, as both a participant and facilitator.

Janet. Janet's beliefs about teaching and her frustration with facilitating discussion were similar to Kathryn and my own. Her 25 years of experience in education involved mostly middle school mathematics teaching with some time spent as technology and computer curriculum coordinator and one semester as an assistant principal. During our collaboration, she taught four eighth grade Algebra 1 classes and one eighth grade Honors Algebra class. My occasional visits to her classroom were to two of her regular Algebra 1 classes. Although my study focuses on Kathryn and my experiences facilitating discussion, I include Janet as a participant because part of our effort to facilitate discussion involved reflecting on what was happening in the classroom and Janet engaged with us in this process.

Rebecca. At the time of my study, I was beginning my fourth year in the mathematics education doctoral program at Indiana University. Before that, I taught mathematics for four years at a local area high school. As a doctoral student working on various professional development projects, I had facilitated pre-service and in-service teacher education at the elementary and secondary level. I developed an interest in classroom discourse in general, and whole-class discussions in particular, as a result of both my teaching experiences and graduate studies. Readers may refer to Chapter 1 for a description of the orientation to teaching and learning that Kathryn and I shared and which influenced my approach to the study of facilitating discussion.

Overview of Fall, 2000

Kathryn and my efforts to facilitate discussion were situated within the larger activity of teaching first semester Algebra 1 to two classes of ninth graders (Kathryn was responsible for three Algebra 1 classes in addition to her department chair responsibilities; I studied and worked with two of these classes). Both classes, Period 3 and Period 4, met daily for 57 minutes. There were 29 students in Period 3 and 30 students in Period 4. All of the students were ninth graders at the time of the study. The high school draws students from several public and private middle schools and most of the students knew some of their classmates from previous years.

Students at South Jackson High School were divided into several different mathematics levels, or tracks, at the time of the study. Students in Kathryn's classes were in a middle-level track; they were in neither the accelerated nor decelerated algebra sequences. Each mathematics track had its own first and second semester exam; previous years' teachers had created the Algebra 1 exams (see Appendix A for sample problems from the first semester exam). Each Algebra 1 teacher was responsible for preparing students for the semester exam by teaching the material in the first six chapters of the Heath Algebra 1 textbook (Larson, et al., 1998). Methods for teaching this content were not specified to teachers and teachers were allowed to supplement the content as long as they did not remove anything.

Teaching the Algebra 1 content as it was laid out in the textbook would have involved explaining procedures for finding solutions to problems to students and then assigning problems that would allow students to practice the procedures. Kathryn and I, agreeing with the vision of teaching and learning mathematics set forth by NCTM (1991, 2000) and IPSB (1998), sought something very different for her students. Therefore, we began the semester by analyzing the textbook and semester exam and determining how to reorganize the first semester Algebra 1 content in ways that would allow us to focus students' attention on developing understanding of mathematical concepts and relationships. We identified several concepts, relationships, and processes around which to organize instruction including (a) using graphs to represent relationships among quantities, (b) writing equations to represent relationships among quantities, (c) relationships among equations and graphs, (d) solving linear equations, and (e) rate of change. As a part of helping students develop their understandings in these areas, we knew we would need to ensure that they had opportunities to develop proficiency in the procedures that would be on their first semester exam. As the school year progressed, we periodically re-examined the textbook and semester exam to ensure that our instruction included this material.

Before the start of the school year and during our semester together, Kathryn and I found or created a variety of activities to help students learn algebra. A few of these were the bottle filling and calculator-based laboratory (CBL) experiments and the staircase, pool tiling, and rollerblading problems (see Appendix B for descriptions of the tasks that we used). Some of these tasks had been used by one of us in previous years and some were new to or invented by us. In addition to gathering and choosing tasks, planning involved sequencing activities and determining how to orchestrate discourse. Would students work individually, in small groups, or as a whole class? What kinds of communication should we require or encourage and when? When and what might students learn from trying to evaluate each other's ideas and reach consensus? We developed and continually revised our hypotheses about the paths of students' learning (what Simon (1995) would call "hypothetical learning trajectories") and altered the sequence and structure of classroom activities accordingly.

Kathryn and I set aside time after school (ten 1-2 hour sessions over the course of the semester) to reflect on our efforts to facilitate discussions and to continue to share ideas from the literature and our own practice. Janet joined us at six of these meetings. At each of the first nine meetings, we analyzed a transcript of a whole-class discussion that one of us had recently led. We watched videotape of the discussion and talked about various issues including patterns of interaction, questioning strategies, and students' level of interest in and engagement in discussion. At the last meeting, we reflected on the semester and had a more general discussion of issues related to facilitating discussion.

Information about these meetings can be found in Table 1; transcripts of the discussions that were the focus of these meetings can be found in Appendix C).

Table 1. Summary of Facilitating Discussion Focused Meetings

Date	Focus Discussion	Members
8/29/00	8/21/00 Period 4 Bottle Filling -discussion led by Kathryn centering on whether or not to connect the points	Kathryn Rebecca Janet
9/15/00	9/1/00 Period 4 Mystery Graphs -discussion led by Kathryn centering on which situation matches which graph	Kathryn Rebecca Janet
9/22/00	9/1/00 Period 3 Mystery Graphs -discussion led by Rebecca centering on which situation matches which graph	Kathryn Rebecca
9/29/00	8/29/00 Janet leads her 7th grade students in discussing their methods of solving the following problem: "Two T-shirts and two Cokes cost 44 dollars. One T-shirt and three Cokes cost 30 dollars. How much does 1 T-shirt cost? One Coke?"	Kathryn Rebecca Janet
10/6/00	9/29/00 Period 3 Classifying Equations -discussion led by Rebecca in which students use the classification system they've created to classify a given equation and then describe which characteristic of the equation is most important for determining the appearance of the graph	Kathryn Rebecca
10/17/00	9/29/00 Period 4 Classifying Equations -discussion led by Rebecca in which students use the classification system they've created to classify a given equation and then describe which characteristic of the equation is most important for determining the appearance of the graph	Kathryn Rebecca Janet
10/20/00	9/29/00 Period 4 Mystery Equations -discussion led by Rebecca centering on which pattern matches which equation	Kathryn Rebecca
11/7/00	10/31/00 Period 3 and 4 Writing Equations - discussions led by Kathryn (Period 3) and Rebecca (Period 4) in which the equations students' developed for a given set of data are analyzed and evaluated by the class	Kathryn Rebecca
11/10/00	11/2/00 Period 3 and 4 Writing Equations - discussions led by Kathryn (Period 3) and Rebecca (Period 4) in which students discuss their the equations they have written and their methods for finding equations for given data	Kathryn Rebecca Janet
12/1/00	Discussed general issues related to facilitating discussion and reflected on the semester as a whole. Reviewed and discussed suggestions from the literature.	Kathryn Rebecca Janet

Reflecting on practice was an important part of the process of teaching for us. As our chosen focus for the semester was facilitating discussion, we frequently reflected on this aspect of our teaching. Meeting with other teachers with similar interests to share ideas and reflect on practice was also a natural activity for us. For these reasons, I do not view the reflective sessions or Janet's participation as extraneous activities that potentially distort my effort to study the process of facilitating discussion, but rather as the type of reflective, professional development activities in which teachers who are attempting to improve their practice regularly engage.

Understanding the context of a qualitative study such as this one is essential to interpreting and making judgments about the usefulness of the results. The ways in which Kathryn and I engaged in teaching, including activities such as reflecting on and analyzing our own discussions, the students we taught, and the external constraints, such as semester exams, are important aspects of the context of this study. Our efforts to facilitate discussion occurred as part of teaching algebra, so the tasks we chose and the ways we sequenced and implemented them become an important aspect of the context as well. Having provided a synopsis of the context of my study in the preceding section of this chapter, and more detailed materials in the Appendices, I will now describe my methods for collecting data.

Data Collection

My goal was to trace Kathryn and my efforts to facilitate discussion through all three phases of instruction: planning, implementation, and reflection. I gathered both video and audio tape of classroom activities and audio tape of both planning and reflection. In addition, I kept field notes during implementation when I was not engaged in working with students or leading discussions. I audio taped my individual reflections and Kathryn kept a journal for reflections. I have summarized the data I collected in

Table 2 (see p. 54). Kathryn and I met to plan for and reflect on instruction approximately twice per week throughout the fall semester. All of our meetings were audio taped. We used video and audio tape to record each day's classroom activities, except on quiz or test days and the week of December 4th when students were engaged in reviewing for the semester exam. All ten of the discussion-focused meetings (Table 1) were audio taped.

When I was not engaged in teaching, for example when Kathryn was leading a discussion, I kept fieldnotes in which I recorded events as they occurred and the thoughts these events brought to mind. As I was unable to keep fieldnotes when I led discussion, I recorded my observations/reflections immediately after class on audio tape. Kathryn intended to keep a written journal for individual reflections, but only wrote in it twice.

In addition to audio and video tapes and fieldnotes, I collected classroom artifacts including lesson plans, worksheets and other materials distributed to students, and a selection of student work. I also collected copies of discussion transcripts that Kathryn and Janet had analyzed during our meetings and articles related to improving classroom discourse (see Appendix D) that we shared with each other prior to and during the semester.

I interviewed Kathryn three times during the course of the semester. Before school began, at the end of the Develop Graph Sense unit (September 12th, 2000) and at the end of the semester. These interviews helped me document Kathryn's impressions about the process of facilitating discussion in a way that separated them from my own impressions and from the ideas jointly constructed by us when we planned for and

Table 2. Summary of Data Collected (Fall, 2000)

August	7	18	21	22	24	25	28	29	30	31
Planning/Reflecting Kathryn/Rebecca	*	*		*		*		*	*	
Implementation Period 3	*		**	**	**	**	**	**	**	**
Implementation Period 4	*	**	**	**	**	**	**	**	**	**
Reflecting on Whole Class Discussions Kathryn/Rebecca										
Reflecting on Whole Class Discussions Kathryn/Rebecca/Janet								*		
Individual Reflections Rebecca	*	*		*		*		*		*
Individual Reflections Kathryn									*	

September	1	5	6	8	11	12	13	14	15	19	20	21	22	26	27	28	29
Planning/Reflecting Kathryn/Rebecca	*			*	*	*		*	*	*			*	*			*
Implementation Period 3	**	**	**	**	**	**	**	**	**	**	**	**	**	**	**	**	**
Implementation Period 4	**	**	**	**	**	**	**	**	**	**	**	**	**	**	**	**	**
Reflecting on Whole Class Discussions Kathryn/Rebecca													*				
Reflecting on Whole Class Discussions Kathryn/Rebecca/Janet									*								*
Individual Reflections Rebecca		*	*			*						*					
Individual Reflections Kathryn							*										

October	2	3	5	6	10	11	12	13	16	17	19	20	21	25	29	30	31
Planning/Reflecting Kathryn/Rebecca		*		*	*		*	*			*	*	*		*		*
Implementation Period 3	**	**	**	**	**	**	**	**	**			**		**		**	**
Implementation Period 4	**	**	**	**	**	**	**	**	**	**	**	**		**		**	**
Reflecting on Whole Class Discussions Kathryn/Rebecca				*								*					
Reflecting on Whole Class Discussions Kathryn/Rebecca/Janet										*							
Individual Reflections Rebecca					*		*			*							
Individual Reflections Kathryn																	

November	1	2	3	7	8	9	10	14	15	16	17	20	21	22	27	28	29	30
Planning/Reflecting Kathryn/Rebecca		*	*	*		*	*	*		*			*			*		*
Implementation Period 3	**	**	**	**	**		**	**	**	**		**	**	**	**	**	**	**
Implementation Period 4	**	**	**	**	**	**	**	**	**	**		**	**	**	**	**		**
Reflecting on Whole Class Discussions Kathryn/Rebecca				*														
Reflecting on Whole Class Discussions Kathryn/Rebecca/Janet							*											
Individual Reflections Rebecca		*		*							*							
Individual Reflections Kathryn																		

Dec	1	8	12
Planning/Reflecting Kathryn/Rebecca	*	*	
Implementation Period 3	**	**	
Implementation Period 4	**	**	
Reflecting on Whole Class Discussions Kathryn/Rebecca			
Reflecting on Whole Class Discussions Kathryn/Rebecca/Janet	*		
Individual Reflections Rebecca			*
Individual Reflections Kathryn			

* = audio taped

** = audio and video taped

reflected on instruction. Protocols for these interviews can be found in Appendix E. Prior to interviewing Kathryn, I recorded my own responses to the same set of questions.

Collecting data from multiple sources helped me develop a more complete picture of the process of facilitating discussion. Kathryn and I each had individual understandings of the process and, through planning and reflection, developed joint understandings as well. Interviews and individual written and audio taped reflections provided me with data on our individual perspectives. Observations, fieldnotes, and video and audio tape of whole-class discussions provided further evidence of the nature of each of our efforts. Audio tapes of dialogue from planning and reflecting sessions recorded how we made sense of our work and how our efforts to facilitate discussion wove their way into planning for instruction more generally. Compiling this vast quantity of data was the first step in the rather lengthy analysis process to which I now turn.

Data Analysis

> [Researchers on the inside] must be able to view the teaching, the students, and the learning in the context of, but also apart from, their efforts and desires. . . . [T]he researcher-teacher must cultivate a stance of inquiry and curiosity. . . . At the same time, to deny the personal is to undo the very project of first-person research, shutting out part of what is experienced on the inside. . . . [Insider] research requires both an unusual concentration on, and use of, self, combined with an almost unnatural suspension of the personal. (Ball, 2000, pp. 392-393)

Managing the insider position was a critical aspect of all phases of my research. With respect to data analysis, I began by deciding that my insider position could help me create a rich data set for analysis, particularly if I did the transcribing myself.

Creating the Data Set

Working chronologically, I transcribed all of Kathryn and my joint planning/reflecting sessions (38 meetings ranging from 10 minutes to 4 hours in length, see Table 2 for dates) as well as all ten discussion-focused meetings (ranging from 1 to 3 hours in length). I had transcribed 18 whole-class discussions during the fall semester (many of which had been the subject of discussion-focused meetings for Kathryn and me, or Kathryn, Janet, and me, see Table1 for details). I inserted these discussion transcripts into the data set, as well as Kathryn and my individual reflections and interviews. Transcribing the data myself was a lengthy and time-consuming process; however, as an insider in my research, I had knowledge of meanings and subtleties in our conversations that made it essential that I do the transcribing myself. This knowledge also allowed me to combine the talk I was transcribing with pieces of discussion, classroom artifacts, and individual reflections, thus creating a richer representation of our efforts to facilitate discussion.

Transcribing helped me remember all that had happened in the previous months and, in a way, I relived the experience. Moving through the semester's worth of data, I was able to literally hear us at work. My subsequent analysis benefited because when I looked at the written data set, I was better able to put myself back in the situation and utilize my insider position. Interestingly, transcribing myself also helped me establish a "stance of inquiry and curiosity" with respect to my data (Ball, 2000). As I transcribed, I reflected on our experience; its many possible meanings and interpretations. Following the advice of Merriam (1988) and Goetz and LeCompte (1984), I kept a log of my thoughts, many of which linked our work to ideas and theories on teaching and learning about which I was familiar. After coding (which I describe in the next subsection), I returned to my log to consider how the sense I was making of the data fit with the impressions I recorded as I transcribed.

Coding, Categorizing, and Representing the Data

Not wanting to pigeonhole prematurely our efforts to facilitate discussion, I had collected data on all aspects of planning, implementing, and reflecting on instruction; however, as I considered how best to begin coding my data, I was immediately faced with the need to reduce the data set. I began by separating out data that were "discussion related." My conceptualization of teaching (based on Simon, 1997) as posing tasks and facilitating discourse and facilitating whole-class discussions as an aspect of facilitating discourse, influenced my coding decisions. For example, prior to the beginning of school, Kathryn and I considered how best to structure the classroom space (e.g., the placement of desks). I view this activity as part of our effort to facilitate discourse, as opposed to thinking about what tasks to pose and how to pose them. Had I adopted a different conceptualization of teaching, for example posing tasks, facilitating discourse, *and* establishing the learning environment, this data would likely have been coded differently.

The close relationship between concepts and code in qualitative research in general, and in this study in particular, should be apparent. My broad conceptualization of what might be a part of the process of facilitating discussion had influenced the data I collected and what I coded as "discussion related."   I do not find this to be particularly problematic, because I also allowed my readings of the data to influence my perceptions about the boundaries of the concept of "facilitating discussion." Coffey and Atkinson (1996) described the relationship between theory/concept and code as follows:

> There is no doubt that theoretical ideas can, and should, inform the coding of data . . . . Likewise, it is easy to endorse the view that a careful examination of codes can help to generate theoretical ideas. . . . It should certainly not be assumed that theory can be "built" by the aggregation and ordering of codes. . . . One must always be prepared to engage in creative intellectual work, to speculate about the data in order to have ideas, to try out a number of different ideas, to link one's ideas with those of others, and so to move conceptually from one's own research setting to a more general, even abstract, level of analytic thought. (pp. 142-143)

Sensitivity to this interplay between the development of codes and concepts was a critical component of my research. As I coded segments of data and attempted to create a structure that would represent and encompass the data, I continually asked myself "What were we talking about and thinking about at that moment on that day? What were we trying to do or figure out? What was the context from which this piece of data was taken?" I alternated these questions with questions such as "What is this piece of data an example of? "To whose theory or concept of classroom discourse does this issue belong? How does this piece of data fit or not fit with the structure I am beginning to form?

After separating out "discussion related" data from the initial data set, I began to work, chronologically, through the reduced, but still large, data set. One reason that the data set was still substantial was that it included whole-class discussion transcripts and classroom artifacts related to whole-class discussions and transcripts of Kathryn and me planning for and reflecting on discussion. My initial coding suggested that in addition to our efforts during class time to facilitate whole-class discussion, Kathryn and I had spent a significant portion of our time outside of class planning for and reflecting on whole-class discussion.

I began the process of coding the “discussion-related” data by revisiting my research questions: “What are the elements of the process of facilitating whole-class discussion? With what do teachers struggle and what supports them as the attempt to facilitate discussion?” as these questions needed to be foremost in my mind as I examined the data. I then read through the data set, line by line, several times, looking for regularities and repeated themes in which to sort the data initially (as suggested by Merriam, 1988). The themes I identified at this stage were:

> Trying to get students to share their ideas with the class
> Trying to get students to listen to each other
> Students’ level of interest in discussion
> Trying to decide what to say and do during discussion (as the leader of discussion)
> Tasks and their relationships to discussion
> Trying to encourage students to “discuss” their ideas
> Time constraints (wrestling with them)
> Students’ expectations (wrestling with them)
> Discussion/reflection on our collaboration

An important part of this initial coding process was “unitizing” the data, that is dividing the data up into small segments (from a few lines to a page or two) that reveal information, stimulate thinking and, while small, are interpretable on their own (Lincoln & Guba, 1985). It was from these units that the initial themes were developed. I identified a new unit and then compared it to the emerging categories developed with respect to the previously defined units, searching throughout the process for anomalies and other, more representative, or useful, themes. My goal at this stage was to begin to fit the data together according to the content or focus of Kathryn and my work in the classroom, and our work in planning and reflecting on discussion.

At this point, I noticed that I had coded some units of data into more than one category. For example, I noticed that, in almost every instance, when Kathryn and I talked about students' level of interest in discussion, we also talked about ways in which students were or were not engaged in the tasks surrounding the discussion. The opposite, however, was not true. That is, data coded as "tasks and their relationship to discussion" included, for example, our decision-making in terms of where to place discussion within or among tasks. Therefore, I eliminated "students' level of interest" as a top-level category and included it instead as a subdivision of "tasks and their relationship to discussion."

At this stage, I also considered dividing the categories into two groups, one focusing on issues that emerged for us during discussion and one focusing on issues that arose at other times. This structure looked like this:

<u>During Discussion</u>

Trying to get students to share their ideas with the class
Trying to get students to listen to each other
Trying to decide what to say and do during discussion (as the leader of discussion)
Trying to encourage students to "discuss" their ideas

<u>Prior To and After Discussion</u>

Tasks and their relationships to discussion
Time constraints (wrestling with them)
Students' expectations (wrestling with them)
Discussion/reflection on our collaboration

Examining the data, I became convinced that this structure was not representative of our experiences facilitating discussion. Although "tasks and their relationships to discussion" and "discussion/reflection on our collaboration" did occur apart from whole-class discussions, we felt and responded to time constraints and students' expectations during discussions. In addition, a considerable amount of the data I had coded into the "during discussion" categories involved planning for and reflecting on discussion. I had coded our efforts to analyze our own discussion behaviors (with and without the aid of discussion transcripts) as “trying to decide what to say and do during discussion.” I had coded our efforts to reorganize the physical space in the classroom as “trying to get students to listen to each other” because as we moved desks around we focused on whether or not students would be able to see whomever was speaking. Although it made some sense to me, in the abstract, to separate the process of facilitating discussion into what teachers do during discussion and what they do outside discussion, this way of thinking about discussion did not seem to represent the intermingling of the two that I found in my data.

I next considered the following division, which proved more fruitful.

<u>Components of the Process of Facilitating Discussion</u>

Tasks and their relationships to discussion
Trying to get students to share their ideas with the class
Trying to get students to listen to each other
Trying to encourage students to “discuss” their ideas

<u>Struggles and Supports in the Process of Facilitating Discussion</u>

Trying to decide what to say and do during discussion (as the leader of discussion)
Time constraints (wrestling with them)
Students’ expectations (wrestling with them)
Discussion/reflection on our collaboration (support)

In addition to their congruence with my research questions, these groupings were satisfying in several respects. First, as I reread the data, I found that the first four categories, “tasks, sharing, listening, and discussing” were imbedded, from the beginning of the school year, into planning, implementing, and reflecting on instruction. How would discussion fit with tasks and how would we encourage students to share, listen, and discuss their ideas? This question was the focus of our work throughout the semester. Time constraints and students' expectations, on the other hand, felt more like obstacles than part of the process. I could feel a certain level of frustration and unease expressed by us in the data units in these categories, and I saw our concern for each of these issues growing across the semester. Of course, these issues were intimately tied to the process of facilitating discussion, as was trying to decided what to say and do during discussion. I thought seriously about including "trying to decide what to say and do during discussion" in the "process of facilitating discussion" category, however the data carried the feeling of a struggle. I found in that data our expression of the tension between pushing students to engage while trying to turn over responsibility for pushing for understanding to them (more on this in Chapter 5). I also found uncertainty for us in terms of choosing the appropriate action to take at any given moment during discussion. Both the content and tone of this data led me to include it as a "struggle" rather than as a "component" of facilitating discussion.

Satisfied for the present with my categorization scheme, I began to flesh out each category. I entered all the data (except for classroom artifacts) into QSR Nudist 4 (Qualitative Solutions and Research Pty Ltd., 1997) software and coded it using my current categories. I then printed out and analyzed the data in each category. At this point, I was looking for emergent themes. The process here was similar to the one I engaged in initially; that is, I unitized the data and compared successive units to previous ones until I had developed subcategories that exhausted each data set. As before, I sought anomalies and alternate categorizations throughout the process. In addition to developing subcategories, I made three changes to the existing scheme. First, the large amount and variety of data coded as "trying to encourage students to discuss their ideas" led me, as I was unitizing this data, to divide it into two categories, “trying to get students to question and respond” and “trying to get students to agree and disagree.” Second, I found that the “trying to get students to share” and “listen” categories, if put together, would be parallel, both in substance and syntax, to “questioning and responding” and “agreeing and disagreeing.” Finally, I took the “trying to get students to” out of the category titles, replacing it with “engaging students in” or “engaging students with,” and I retitled the struggles and supports, leaving me with the following categorization scheme.

<u>Elements of the Process of Facilitating Whole-Class Discussion</u>

Engaging Students with Tasks

- Deciding when and how to incorporate discussion
- Engaging students with tasks in ways that would help them develop ideas and opinions
- Motivating in students a need for discussion
- Managing differences in students’ understandings

Engaging Students in Sharing and Listening

- Structuring space
- Encouraging talk and presenting students’ ideas
- Motivating listening

Engaging Students in Questioning and Responding

- Encouraging students to question and build upon each other’s ideas
- Sharing the responsibility for questioning and responding with students

Engaging Students in Agreeing and Disagreeing

- Positioning Students’ Ideas in Opposition to One Another
- Inserting a Problem or Question
- Pushing Position-Taking

<u>Struggles and Supports in Facilitating Whole-Class Discussion</u>

- *Managing Internal Dialogue*
- *Managing Time/Curricular Constraints*
- *Managing Students’ Expectations*
- *Engaging in Collaborations*

Moving between a well-categorized set of data and a narrative description of the content of each category and subcategory represented another level of data reduction. To begin, I transferred the subcategory codes into QSR Nudist 4 (Qualitative Solutions and Research Pty Ltd., 1997) and printed out the data set by category and subcategory. This allowed me to examine the data for segments (of whole-class discussions, dialogue of planning and reflecting on instruction, and individual reflections) that might be useful for representing and illuminating the essence of that subcategory. Once I had chosen segments and written surrounding narrative, I went back through the data for each category and compared it to the narrative to further ensure agreement between the two.

Throughout the data collection, analysis, and writing process, my concern was to represent our experience as closely as possible and to create a structure that would convey it in a meaningful and useful way. My descriptions of data collection and analysis methods provide evidence of my efforts in represent fairly the object of study. I conclude this chapter by describing other methods I used for ensuring the trustworthiness of my results.

Trustworthiness

The trustworthiness of my findings rests on long-term and intensive participation at the research site, the use of multiple data sources, my data analysis methodology, coding and member checks, and clarification of biases (all of these strategies are suggested by Merriam, 1988). Researchers' biases include assumptions, world-view, and theoretical orientation; I provide information about these in Chapter 1 and in the Study Context section of this chapter. I described data sources and analysis methods in the previous sections of this chapter, and will describe coding and member checks in the proceeding paragraphs.

My insider position allowed me unique opportunities to make sense of the process of facilitating whole-class discussions; however, my proximity to the data and my role in developing its representation made it impossible for me to engage in a member check of my work. Therefore, I depended on my collaborator, Kathryn, to assist me in determining whether I had fairly represented the data. Kathryn and I reviewed each of the features of the process of facilitating discussion that I had identified, as well as related struggles and supports. As I described the results, Kathryn recalled her impressions of the process of facilitating discussion. Kathryn's comments resonated with my interpretation of our experiences. For example, Kathryn noted that "students need something to talk about, such as different answers," and "us not talking encourages talk." Both of these ideas are aspects of the process of facilitating discussion that I described in Chapter 4. In summary, Kathryn said that I did "a good job of capturing our experience" and she found nothing that she disagreed with in my findings.

In addition to reviewing the study results with Kathryn, I engaged an experienced coder to examine approximately 10 pages of data using the coding scheme I had developed. The pages chosen were representative of the entire data set and covered a range of coding categories. Initial agreement between our coding for the chosen data was over 90 percent. Discussion of code and data interpretation led to coder agreement of over 97 percent.

Summary

In this study, I used a qualitative case-study methodology to examine the process of facilitating discussion in secondary mathematics classrooms. Positioned as an insider, I engaged in creating the object of my own research. This position allowed me unique insight into the process of facilitating discussion, but required careful, disciplined methods of data gathering and analysis. My analysis depended on emergent themes; however, the literature on classroom discourse, as well as the emergent perspective on learning and Simon’s conceptualization of the teaching process, influenced the data I collected and the sense I made of it. To ensure trustworthiness of my results, I used multiple data sources and a thoughtful, disciplined analysis methodology. I also discussed my findings with Kathryn and checked a portion of my coding against that of a colleague. Having described the methods and procedures upon which my study rests, I turn now to the Results chapter and discuss both the features of the process of facilitating discussion and related struggles and supports.

Chapter 4: Results

In this study, I examined teachers' efforts to facilitate whole-class discussion in mathematics classrooms. Specifically, I consider the efforts of Kathryn (a high school teacher) and myself to facilitate discussion in two of her Algebra 1 classes. I collected before, during, and after instruction to answer the following questions:

- What are the features of the process of facilitating whole-class discussions in mathematics classrooms?
- What do teachers struggle with as they attempt to facilitate discussion?
- What supports teachers in their efforts to facilitate discussion?

I considered facilitating discussion to be part of a model of teaching mathematics suggested by Simon (1997, 1995). This way of thinking about teaching (for a description of Simon's ideas, see Chapter 1) underlies the results I present here. Specifically, I assume that teachers actively seek to plan for discussions (as part of the development of hypothetical learning trajectories) and reflect on them after they occur (adding to teacher knowledge), as both Kathryn and I did. I collected and analyzed data from planning and reflection times, as well as from classroom implementation, and consider the extent to which facilitating discussion is an activity that occurs throughout the mathematics teaching cycle.

I divide this chapter into three main sections, each section addressing one of my research questions. In the first section, I describe features of the process of facilitating whole-class discussion as experienced by Kathryn and me. I devote the second section of this chapter to the struggles we experienced as we attempted to facilitate discussions. In the third and final section, I focus on aspects of our work that supported us, and may be useful to other teachers attempting to facilitate whole-class discussions in mathematics classrooms.

Features of the Process of Facilitating Whole-Class Discussion

For Kathryn and me, the process of facilitating discussion was cyclic. Planning guided implementation, implementation was the basis for reflection, and reflection influenced planning. In terms of the Mathematics Teaching Cycle (Simon, 1995), we can think of this as developing hypothetical learning trajectories, interacting with students and reflecting on these interactions (causing changes in teacher knowledge), and revising hypothetical learning trajectories. Analysis of our experience planning, implementing, and reflecting on discussions suggests our work revolved around four major concerns or themes: (a) engaging students with tasks, (b) engaging students in sharing and listening, (c) engaging students in questioning and clarifying, and (d) engaging students in agreeing and disagreeing.

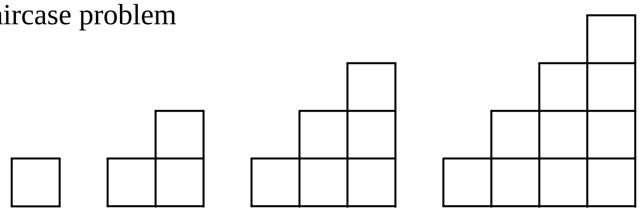
Engaging students with tasks refers to our efforts to engage students in thinking about particular mathematical problems or tasks, which influenced the types of whole-class discussions we were or were not able to have. Sharing and listening, questioning and clarifying, and agreeing and disagreeing refer to issues that arose for us during the discussions themselves, as well as during planning and reflecting. The four features of the process are highly interdependent and concern with one aspect generally involved concern with the other three. For example, concern with whether or not students' engaged in agreeing or disagreeing with each other was related to concerns over students' willingness to listen to each other and concern for students' behaviors during whole-class discussions was related to concerns about engagement with tasks prior to discussion.

In each of the first four of the following subsections, I describe aspects of one of the features of the model and provide several examples of the feature from the data. In the fifth and final subsection, I summarize my findings.

Engaging Students with Tasks

When Kathryn and I engaged in planning, we frequently reflected on students' understandings and our goals for student learning. We then chose a task or a series of tasks to implement in the classroom. Next, we considered how best to utilize discourse, both oral and written, to further students' mathematical development. Making decisions about when and how to incorporate whole-class discussions was, of course, an important part of this process. Consider the following example.

Task: The Staircase problem



The drawing shows staircases 1, 2, 3, and 4 steps high. How many blocks would be needed for a staircase 100 steps high? For n steps high?

Note: Our prior experiences with this task suggested that students often first notice that they can find the number of blocks for a staircase with n steps by adding n to the number of blocks in the preceding staircase. Developing a formula that is not recursive (called a "general" formula in the dialogue below) seems to occur subsequently and requires further thinking on the part of students.

Rebecca: I was thinking, could we have them in their groups or in pairs, but uh, and also we could notice who's working on similar ideas – we could put them together. But give them some butcher paper and have them make a big picture and big writing and have them share.

Kathryn: Just on the recursive idea or-

Rebecca: That's why I'm not sure whether or not we want to do that tomorrow, have them write it up on the butcher paper and then we can see what we've got and then they can share on Monday or whether we want to push them through general formulas and then have them, maybe that makes more sense, then they'll keep thinking about the problem. Maybe sharing recursive [formulas] would put a stamp on the problem that we don't want.

Kathryn: It might. Yeah.

Rebecca: But anyway what do you think about doing something where they kind of present their ideas to each other? . . . I guess I just felt like when we did mystery graphs, they almost, explanations were listened to, and there was something to look at, and there was some sense of I'm really supposed to be

Kathryn: This is important. Yeah, I agree. And I think you've commented on that before, making something that you know is going to go up for the whole class, you're going to take a little more care in putting the thing together.

Rebecca: And I think we can underscore that we want each of them to understand another way to look at the problem. (9/14/00 joint planning/reflecting time)

Our goals for student learning included developing recursive and general nonlinear formulas to describe a geometric pattern and analyzing others' ways of thinking. Thinking about how best to sequence classroom activities to attain these goals, we used our prior knowledge about the task (recursive formulas are often developed first) to make a decision about when to have students share solutions. How students would share was influenced by our goal for discussion (analyzing each other's solutions) and by our previous experiences with student engagement during discussions.

Considering the relationship between whole-class discussion and mathematical task was an important initial piece of the process of facilitating discussion for Kathryn and me. We also found ourselves concerned with (a) providing students with sufficient opportunities to develop opinions about the mathematics at hand prior to discussion and (b) engaging students with tasks in such a way that they would feel some need for discussion. Reflecting on discussions after they occurred (as in the following excerpt) helped us identify issues such as these that impacted the quality of our whole-class discussions.

Task: Students are given a set of 64 linear and nonlinear equations. Students are asked to think about what the graph of each equation would look like and then put equations that they expect to have similar graphs into groups. Next students graph the equations and reanalyze their groupings. (see Appendix B: Classifying Equations Project for detailed description of this task)

After students had grouped their equations (prior to beginning graphing), Kathryn and I asked students to share with the class how they had created their groups. (see Appendix C, 9/29/00 Period 4, for discussion transcript)

Kathryn: We talked before about that discussions don't really happen unless there's something to argue about, unless there's a conflict of some kind. And at this point, this is a question we want them to start thinking about, but they really, and I think you said it in one of the two classes too, that they were really still kind of guessing at this point, because they haven't had enough experience with looking at graphs and comparing them to equations to make a decision

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Rebecca: Generally, the arguments in period 4/5 were around order of operations and-

Kathryn: Yeah, the things that became real discussion questions, they weren't really what this question was about, they were offshoots of that and they were things they'd had some experience with and they felt like they had an opinion. "I remember order of operations and this is the way it works" and someone else was "Oh, yeah, I wasn't sure I needed to use it in this case" which I thought was really interesting. What would make you think that you wouldn't use it? But I think that's really good, it's really getting at some misconceptions they have about order of operations, they've learned it but they haven't figured out that they're supposed to use it all the time, sometimes they use it and sometimes they don't.

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Kathryn: But like I said, that was the thought I had about second period is they still didn't know enough to feel strongly enough about something to argue for their case. It was like "this is what I think, but it's just my opinion" and somebody else 'well this is what I think but I don't' really know, I'm just guessing' so I wondered if that's why that didn't go anywhere.

Rebecca: It's kind of like where Jackie said basically I'll share with you what I think but I can't tell you a reason for this. (10/6/00 joint planning/reflecting time)

In this instance, it seemed to us that students had not had sufficient time to develop strong opinions about the question at hand, nor was there a perceived need or motivation for discussion. Students' ideas were tentative and, although several students were willing to share their thoughts, they did not engage in justifying their own ideas or evaluating the ideas of others.

Motivating a need for discussion was something Kathryn and I considered as we planned for instruction. The structures and goals of some tasks seemed to motivate discussion, as in the "mystery graphs" activity described below. As we reflected on discussions after they occurred, Kathryn and I frequently considered how tasks, such as mystery graphs, might be modified to better motivate engagement in discussion. In the following excerpt, we discussed the level of student engagement and proposed modifications that may enhance discussion.

> Task: Small groups of students are each given a different real-world situation, such as "The number of people in (fill in the name of a nearby city) watching television." or "The number of people at the McDonald's drive through window." Each group of students is to create a graph that represents the given situation. Students are told that the graph must show a 24-hour period (midnight to midnight) on the x-axis and number of people on the y-axis (so that the graph shows number of people as a function of time). Students draw their untitled "mystery" graphs on butcher paper and post them around the room. Next, students are given a list of all the given situations and attempt, in their groups, to match the situations and graphs. Discussion about which situation matches which graph and why concludes the activity.
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> Kathryn: I was noticing, uh Jackie is not normally one who gets very involved, but because she worked at McDonald's she got really involved in that discussion and then she seemed to stay in the discussion even when they weren't taking about McDonald's, and uh, I remember fourth period, Amanda really getting into the discussion about the Children's Museum, [she said] "Well, I know because I worked at the Children's Museum.", so I guess the thought occurred to me instead of giving them the situations maybe even having them come up with a situation-
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> Rebecca: From their own experience, they'd make better graphs.
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> Kathryn: Yeah, that's what I thought. Because Jackie was like "I know that McDonald's goes up and down and it's going to be high here." and same thing with Amanda "I know that the people leave at a certain time." And it didn't really occur to me until I was watching Jackie stay involved all the way through and I don't know whether that was a product of the fact that at some point she did get super engaged because "I know this from my personal experience" and then she didn't lose that momentum. Now she was of course arguing against someone's graph based on her experience, I don't know if she had made her

own McDonald’s graph if that would have occurred or not. But maybe that would be another way to approach it. Like with this they came up with their own patterns. And you can set the guidelines, you know we want this to be a graph that involves a number of people.

Rebecca: Well you could have each group come up with three or four different situations that they could graph and then they turn them in and then look at them and decide. Because you need to have a certain similarity and variety in the product to have a good discussion so you could pick the one you want them to do off their list.

Kathryn: Yeah, because the ones you had, you had some that were high at night and low at night and you want that variety. (9/22/00 joint planning/reflecting time)

The Mystery Graphs activity provided students with opportunities to create graphs that reflected their understandings of the situations they were given and of graphical representations in general. Discussion was motivated as students attempted to decide which situation a mystery graph represented. We attempted to motivate a need for discussion even further by suggesting to students that we would accept more than one possible answer if students were able to argue for it to the satisfaction of their classmates and ourselves. Reflecting on discussion after it occurred, Kathryn and I noticed high levels of engagement from some students and considered how to further modify the activity to use students’ prior experiences to increase engagement in discussion.

Providing students with opportunities to develop ideas prior to discussion and engaging students in ways that would motivate discussion was helpful only if students actually engaged in the activities. We had particular difficulty, for example, with engaging students with discussion based on homework assignments. Students who did not do the homework, or who stopped when they reached a problem they were uncertain about, had difficulty participating fully in discussion. We eventually decided not to attempt to engage students in whole-class discussions based primarily on homework assignments.

Differences in levels of understanding students brought to discussion were concerns for us even when discussion was based on in-class activities. Variations in students' previous experiences and in their thinking about a given task were an important resource upon which to draw, but also, at times, an obstacle. The following excerpt illustrates how variations in students' experience can be a hindrance.

> Rebecca: Today I felt like some kids didn't speak because they didn't know what was going on, so they couldn't really say (i.e., speak), especially compared to some kids who really could, these kids who already understand slope and y intercept are telling the rest of the class what to look for. . . . they're so different in how much they understand – coming into Algebra 1 and at this point and there's no way you're going to have a discussion and get a variety of involvement if the situation is such that your six or eight top kids, they can have a nice discussion and other people can listen and learn things but
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> Kathryn: Yeah, it's not getting those other kids really involved.
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> Kathryn: They're just sitting back and somebody's going to say it. And in a way I almost think maybe more of it is slipping by them because they have been trained to only value what the teacher has said and so when the student says it they don't really place value on it and they're not busy writing down the things the kids are saying. They may not know whether what the kids are saying is even right or not whereas if it's coming from the authority in the classroom, well I'm pretty sure this is right and important and I'll write it down, but if this kid over here is saying it then I don't know whether this is something that's important to pay attention to or not.
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> Rebecca: And it's hard because we don't want to further, like promote that, we want to break the kids of that habit but at the same time we have to attend to the fact that they have it, so they're being hurt. (10/20/00 joint planning/reflecting time)

In this case, our concern was that students' differential experiences in previous courses and with the current task had negatively impacted both the level and type of student involvement during discussion, including students' tendencies, based on their prior experiences, to "only value what the teacher has said." For Kathryn and me, part of facilitating discussion was considering whether or not all students could engage in and profit from discussing a specific topic on a given day. Sometimes these considerations led us to decide against discussion, as the following excerpt demonstrates.

> Note: For the past few days, students had been working on problems in which they were given a few collinear data points in a simplified context and asked to find an equation that described the relationship among the quantities. For example: two days ago the temperature was -6 degrees, yesterday it was -5 degrees, and today it is -4 degrees. Assume the trend continues and find an equation that describes the relationship between the day and temperature.
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> Rebecca: If we say now "How did people find their equations?" I think we're going to get the same stuff we've been getting, just maybe a little more from more kids hopefully, but the same things. . . . And other people would hear it one more time but I'm not sure that would [help]
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> Kathryn: I don't know that it would help or not. I was remembering . . . at the end of the period Chrissy basically said "Well I know that the number you add on has to be the y-intercept and I know the number you put in front of x has to be [the slope]" and I don't remember if she said slope, . . . but she pretty much just said it, but does that mean that every kid got it?
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> Rebecca: No. . . . I don't know whether it makes sense to have discussion or just give them more time to finish up what they need to finish up and just let that be that.
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> Kathryn: Probably just let it be it. . . .(11/3/00 joint planning/reflecting time)

Kathryn and I did not simply give up on the students who had not made connections yet between the characteristics of equations and their graphs, but rather decided that student learning would not benefit from whole-class discussion at that time.

Facilitating classroom discourse in general, and whole-class discussion in particular, was a central component of teaching mathematics for us. As we planned for and reflected on instruction, we focused repeatedly on students' engagement with tasks and how this engagement related to discussion. We kept four considerations in mind: (a) when and how to best incorporate whole-class discussions to further students' mathematical development, (b) how to engage students with tasks in ways that would help them develop ideas and opinions, (c) how to motivate in students a need for discussion, and (d) how best to manage differences in students' understandings so that all students might benefit from whole-class discussions. I do not suggest that we found answers to these issues, but rather that the process of facilitating discussion involved consideration of them.

I turn now to the first of the three features of the process of facilitating discussion that focuses on student behavior *during* discussion: sharing and listening. Sharing and listening are, of course, essential features of a successful discussion. Unfortunately, they did not simply occur for us, but rather had to be carefully planned for and nurtured, as I will attempt to show in the following section.

Engaging Students in Sharing and Listening

Kathryn and I put a considerable amount of effort into trying to bring students' ideas and ways of thinking to the forefront of discussion and encouraging students to listen to each other. Our work in this area falls into three categories: (a) structuring space, (b) encourage talk and presenting students' ideas, and (c) motivating listening. The first category relates to both sharing and listening, the second primarily to sharing and the third to listening.

Structuring Space

The day before school started, we decided to rearrange the students' desks into a U-shape with the overhead projector at the top of the U. We reasoned that if students were to talk to and listen to each other, then they should be able to see each other as well. Toward the end of the semester, behavior and other management issues caused us to move students back into rows temporarily. It was after switching back into rows that the importance of the U-shape became apparent.

> Kathryn: When they were back in rows [listening to each other] really completely deteriorated and that to me points out the importance of having them in a seating arrangement where they can see each other and hear each other a little

> bit better, because if there's somebody in the front and they speak softly there is no way anyone in the back can hear. If they're [facing each other] and they speak softly maybe there's at least a chance. But so when they were sitting [in rows], I had really just pretty much given up, . . . I had pretty much given up on if they responded trying to make sure that everyone in the class was hearing what was going on. . . . (12/15 interview with Kathryn)

In addition to considering the position and orientation of students' desks, Kathryn and I thought about where we stood or sat during whole-class discussions, and whether or not we moved around the room. We thought that students would be more apt to listen to each other if they were looking at each other, but if we stood at the front of the room, then they would likely look at us. We set up stools in the back corners of the room so that we could experiment with sitting in different places during discussions. Kathryn found moving away from the front of the classroom to "feel strange" and neither of us found that it took the attention away from us to the extent we had hoped. In the end, both she and I spent the majority of our time during discussions in the front of the classroom. We did begin, however, to sit rather than stand beside the overhead, putting us nearer the level of the students. In the end, neither of us came to a definite position on the placement of the teacher during whole-class discussion, but we agreed that experimenting with one's own placement in the classroom should be part of the process of facilitating discussion.

Kathryn and I noticed during the mystery graphs activity (described previously in this chapter) that having reference objects (in this case graphs drawn on butcher paper) posted around the room seemed to shift attention away from the teacher and toward whoever was speaking and the reference object. We experimented with this strategy a few times during the semester with similar results and would suggest it to others. Placing three-dimensional reference objects, geometric solids for example, on a table in the center of the U, rather than at the top where the teacher is, might be another way to achieve a similar outcome.

Alterations to the physical structure of the classroom and the positions of its occupants might or might not have a significant impact on whole-class discussions in a given classroom. Kathryn and I made a few changes with mixed results. We found that the U-shape of students' desks and the placement of reference objects seemed to have a positive impact on sharing and listening. Where we positioned ourselves did not seem to have much of an impact, perhaps because we did not move away from the front of the room for a long enough period of time. Regardless of the particular outcomes in our case, an important part of the process of facilitating discussion was examining and experimenting with the structure of space in the classroom.

Encouraging Talk and Presenting Students' Ideas

Kathryn and I spent the months before school began reading the suggestions of researchers and practitioners about how to get students to talk. The importance of establishing norms and expectations for discussion became apparent to us, as did the necessity of changing our own behaviors by using wait time and other discourse-enhancing strategies (see Chapter 2 for descriptions of specific strategies). We began the school year with an activity designed to engage students in thinking about their own expectations for mathematics classrooms (see "Typical/Ideal Mathematics Classroom Activity in Appendix B). We used the activity as a springboard from which to share with students our view of the role of discussion in learning. We carefully planned for discussion and used it as a tool for learning on a regular basis. We tried to be consistent in asking students to share their ways of thinking and encouraging more students to share by using wait time and noticing when quieter students had their hands raised. Despite these efforts, Kathryn and I continued to struggle to find ways to engage all students during whole-class discussions.

Within a few weeks of beginning the school year, we could identify approximately 10 students in each of the two classes who were "talkers." They frequently raised their hands and seemed comfortable with sharing their thoughts with the rest of the group. Other students talked either infrequently or not at all. Our goal, however, was to facilitate discussion in such a way that a wide variety of students would talk and learn from participating in dialogue about mathematics. We knew that some students would likely be uncomfortable speaking in front of the class despite our efforts to create a supportive, non-threatening environment. We also realized that students would not share unless they believed they had something to contribute. And, of course, with approximately 30 students in the classroom, not everyone could contribute during a 15 or 20 minute long discussion. Further, a student who talked was not necessarily learning more from the discussion than a student who did not talk. Nevertheless, Kathryn and I were concerned when we noticed that fewer than half of the students in each class were doing the majority of the talking during whole-class discussions and we struggled to understand why.

> Rebecca: It's a struggle to get other kids involved, but I noticed Arial was talking [in this transcript]. And the other day when [the usual] people weren't talking, Robert and Arial were talking. . . . [T]here are the people who always are good for a comment, like Heather and whoever, and then there are people who some days, like Lisa, some days Lisa is [obviously] doing other things and she's not here and some days she's here. And then with some of the others, like with Robert . . . that aren't so obvious about whether or not they're paying attention, I don't know why they're quiet some days and why they're opinionated some days.

Kathryn: Or why certain people, like I was thinking the other day about Sarah, in third period, when she sat over here she used to talk a lot, and now she doesn't say much at all. And I wonder why, did something happen one day in class, did she feel like one of her ideas got ignored or put down, and maybe it's nothing.

Rebecca: Or did she get into arguments with ideas that she had while groups were working when she was sitting over with Cassie and Brad and now that she isn't sitting with them she doesn't get.

Kathryn: And she's with Melissa now, who's a weaker student. (10/20/00 joint planning/reflecting time)

In addition to trying to understand the experiences of individual students in the two classes, Kathryn and I talked about strategies with potential for drawing out ideas from a wider variety of students. We considered ideas from our previous years of teaching, such as assigning problems to small groups to present to the class, and experimented with strategies we had just heard about, such as announcing a certain number of hands that you want to see before you call on someone. We also looked at transcripts of our own whole-class discussions and identified questioning techniques that might help.

Rebecca: One of the things from the transcript we were looking at today is when one student, when Anna said "it was the fractions, you divide," rather than me saying to Anna "Can you say some more about that?" asking the whole class "Who can add to what Anna said?" It kind of keeps everyone involved. I mean you do want students to go deeper on their own ideas, you do want them to say more, and I know I've done that before, too, but I just think maybe you keep the whole class engaged.

Kathryn: Yeah. In fact I don't know that I wrote it down, but that was a thought that came to me too when I looked at that, that, oh, you hadn't asked her to expand more on her own, but the whole class. (10/17/00 joint reflecting time with Janet)

Assessing the potential effects of our own questioning methods on student participation in discussion provided us with some ideas about what to do in the future. We also considered the potential impact of students' words on other students' participation.

Kathryn: And then Joe, "Well, I thing she's wrong," and I don't remember how he said it, but I just noted to maybe encourage students instead of saying "I think

> they're wrong" to say "I disagree with what so and so said. And I didn't catch that when he said it and I don't know whether I would have even said anything. . .
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> Rebecca: And Joe, he's gruff . . . [but] not threatening, at least I see him that way.
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> Janet: And so she would not have taken offense at that remark.
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> Rebecca: Yeah, more than likely she wouldn't have felt like she couldn't speak up again, but you never know when somebody might [be intimidated] (9/15/00 joint reflecting time with Janet)

Attention to the tone of students' exchanges during discussion is as important as analysis of the words themselves. In this instance, we were uncertain about how other students perceived Joe's comment but we still felt that encouraging use of terms such as "agree" or "disagree" would be helpful.

Before long, Kathryn and I realized that some students' ideas would probably not become part of the discussion if we depended solely upon talk to bring them out. We began to experiment with other methods for bringing students' ideas to the fore. We experimented with having students write solutions on overhead sheets, listing students' ideas on the chalkboard or overhead as they spoke and then adding other ideas we had noted during group work, having students write ideas or solutions on butcher paper and posting them on the wall, and collecting written work and presenting a selection of ideas to the class. In these ways, students who did not want to talk might still have their ideas examined during whole-class discussion.

For Kathryn and me, the process of facilitating discussion involved both encouraging talk and developing strategies that would bring forward the ideas of students who were reluctant to speak. A crucial part of this process was reflective; it involved careful examination of our own whole-class discussions in order to better understand how patterns of interaction, questioning techniques, and word choices might be impacting the level of participation in discussions. We also thought about how the broader experiences of individual students in our class might be influencing their levels of participation. We chose not to force talk, but rather found ways to bring up or bring out students' ideas by presenting them to the class on overhead sheets, on butcher paper, or in some other fashion. Although these strategies were helpful, we continued to struggle with the varying levels of engagement we noticed throughout the semester, and we suspect that this struggle will be part of the process of facilitating discussion for other teachers as well.

Motivating Listening

For Kathryn and me, motivating students to listen to each other's ideas was a critical and difficult component of the process of facilitating discussion. We understood that our students beliefs about the nature of mathematics and the nature of schooling based on their prior experiences could lead them not to value whole-class discussion in general and listening to each other in particular. We attempted to change students' beliefs and motivate listening by making discussion an integral and necessary part of the learning process and leading discussions in ways that would encourage students to listen to each other. We also structured the physical space of the classroom in ways that seemed to support listening (see the "Structuring Space" section of this chapter for a description of this aspect of facilitating discussion).

Kathryn and I attempted to use discussion as a means of furthering students' thinking about mathematics. We asked students to use what they had heard, shared, and thought about during discussion to complete current tasks and inform future ones. When planning for instruction, we thought carefully about when and how to incorporate discussion (see the "Engaging with Tasks" section of this chapter for a discussion of this aspect of facilitating discussion). When thinking about possible methods for encouraging sharing, we were quick to consider the extent to which listening would also be encouraged. Below, Kathryn reflects on a way of structuring discussion that she had used in previous years.

> Kathryn: [Groups of students] had to draw a number and that was their assigned problem to present and so then I had them write up that specific one for me like on a little card, this was with pre-algebra first semester, they wrote it up on a little card and then I collected the card for that one problem that they were supposed to do and I looked at it to see if they had a reasonable answer and an explanation and made comments on it and then I gave it back to them the next day to make corrections, additions to, whatever . . . and then after they had their cards all fixed up, then they did their presentation and when they did their presentation then they were allowed to bring that card up with them. . . . But see the problem through all of that is again getting them to care about listening to [each other].
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> Rebecca: Right, it's the discussion aspect of it.
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> Kathryn: Because my group is up and I present my answer and then I go back to my seat and then I'm done and who in the class listened or cared?
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> Rebecca: Or learned from being a part of the explaining activity?
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> Kathryn: Right. (8/25/00 during joint planning/reflecting time)

Having students responsible for presenting their ideas to the class would certainly have created a wider variety of speakers, however neither one of us was convinced that this method would likely increase student listening.

Kathryn and I also thought about specific methods for holding students accountable for listening during and learning from discussion. For example, we considered concluding discussions by having students write about how the discussion impacted their thinking. Reflecting on the process of facilitating discussion at the end of the semester, both of us felt we had not done enough to hold students accountable for learning from discussion. We struggled to find time for classroom activities such as writing about discussion and we also struggled with how best to assess the impact of discussion on students' mathematical understandings. We felt that when low levels of student listening occurred, part of the reason was likely that we did not engage students more frequently in reflecting on discussions after they occurred. For us, a critical part of facilitating discussion was motivating listening and a critical part of motivating listening was finding ways to make discussion a valued part of the learning process

Leading discussions in ways that motivated listening was a surprisingly difficult part of the process of facilitating discussion for us. We realized that certain teacher actions, such as repeating students' comments, answering one's own questions, and responding differently to correct and incorrect statements would likely relieve students of the need to listen to and evaluate each other's ideas. A primary strategy we employed to shift responsibility for listening onto students was to try not to talk ourselves. We told students that we were going to try not to repeat what they said. Kathryn demonstrated to students how repeating their words would take away the need for them to listen to each other. We tried to resist the urge to fill silences and instead let students' words hang in the air. When we analyzed transcripts of our own whole-class discussions, however, we found that we had been successful only partially in terms of not repeating or using wait time after students spoke.

> Rebecca: As soon as I saw some math that I wanted everyone to see, I regressed in my speech patterns back to repeating and then telling, in the form of a question, but telling the thing I wanted people to think about. . . .when something interesting came up I started to want to push that with the group . . .

> Kathryn: Yeah, and I'm trying to think when this was, and I almost think it was Wednesday, and I don't' remember who it was now, but somebody was saying something and it was something that I thought was an important idea and I just had this sense that nobody was really listening or hearing and I did

say that ‘oh I do want everyone to listen to this because this is something that’s important’ but even as I was saying that I felt ‘oh should I really be saying this’ –

Rebecca: Then they’ll think that the only things they have to listen to are the ones that I stop and say listen to this.

Kathryn: And then that’s acknowledging, that’s basically like saying “This is a correct statement.” and if you’re trying to avoid making value judgements or saying “Yes, this is right.” or “This is wrong.” then I had just big time said “This is right, this is really important.” (9/22/00 joint planning/reflecting time)

Reflecting on discussions after they occurred, Kathryn and I noticed that when students did not seem to be listening to each other or attending to an idea we felt was important, we responded by repeating or otherwise regressing into speech patterns that served to focus students’ attention for the moment, but did not support the more general goal of developing in them the habit of listening to one another. Examining our own behaviors during whole-class discussions in order to uncover inconsistencies such as these was an important part of the process of facilitating discussion for us. Once we understood our own actions, we could think about how to respond to student behaviors, such as not listening, in ways that would be consistent with both our short and long term goals. The following is an example (excerpt of the discussion itself and our reflection on it) that may give the reader a sense of how Kathryn and I engaged in reflecting on our own behaviors during whole-class discussions.

Task: Students are given a set of 64 linear and nonlinear equations. Students are asked to think about what the graph of each equation would look like and then put equations that they expect to have similar graphs into groups. Next students graph the equations and reanalyze their groupings.

Note: The discussion excerpted below occurred after students had put their equations into groups but before they had begun graphing.

<u>Discussion Excerpt</u>

Note: An unedited version of the entire discussion can be found in Appendix C.

Rebecca: What do you all think? I'm going to give you an equation that's not on your list but I want you to compare it to groups that you have

[Rebecca writes on the overhead $y = \dfrac{3x^2 - 9}{3}$]

Rebecca: Can you all see that? Uh, what I want you to do is look at your sheet, your pair's sheet, and figure out where you would put this equation. What category would you put that one in?

[Rebecca walks around while students look at their papers and talk]

Rebecca: Ok. Alright, let's see. Ok. What wants to start us off? Who knows where they'd put it? Who's decided where they'd put it? Raise your hand if you've decided where you put it. Ok. Who do we have in the back? Deanna?

Deanna: Uh, we had like

Rebecca: Shh. Listen to her please. A little louder.

Deanna: Uh, we had for like our description for our category, each of these equations are formed in some kind of fraction and the denominator is always a number. So that's what we would put it in.

[Rebecca writes – fractions w/ denominator a #]

Rebecca: Does the fraction bar have to go all the way across these or could it be like 1/2 x + 2?

[Rebecca writes 1/2 x + 2]

Deanna: No we have it all the way across.

Rebecca: <u>ALL</u> the way across.

[Rebecca crosses out the 1/2 x + 2]

Rebecca: Another category. So that's one place this could go. Yeah?

Katie: The category we used were more than 2 operations.

Rebecca: Oh. Who sees what Katie's saying? Who can explain what Katie means by more than 2 operations in this equation? . . . (9/29/00 period 3)

<u>Discussion Reflection</u>

Rebecca: I think that when I say "Can you all see that?" I don't think I say it because I'm worried about whether they can see it or not, I think I'm worried about whether or not they're looking at it or not. So I wrote down "I do this to get their attention, to focus them on whatever I've just written." I was reading

> that and thinking that, you know, I know that I've always done that a lot, ask for their attention without asking for it. I wrote right after the "can you all see that?" I wrote "trying to get them involved or focused by giving them something to do rather than a question to think about." Like I said "I want you to look at your sheet." . . . I think this whole thing was trying to get the flow started. "What category would you put this equation in?" Then I asked people to be quiet. . . .
>
> Kathryn: I kind of thought that about [your "<u>ALL</u> the way across" comment], since you wrote it in caps and underlined it that you must have said it for emphasis.
>
> Rebecca: Yeah, <u>ALL</u> the way across. I suspect that I was feeling tense because everyone wasn't paying attention as much as I'd like.
>
> Kathryn: Uh huh. And down at the bottom, the very last part, I noted, what you were probably trying to do was check and see if they were listening. . . . "Who sees what Katie is saying?" . . . they were still kind of unfocused at that point. And I'm sure you were doing that just to kind of focus them, [as in] "Has anyone listened enough to Katie to tell me what she said?"
>
> Rebecca: Which would probably have been a better thing to say. I don't know. It would have been ok if I'd stopped at "Who can explain what Katie means?" [rather than repeating what Katie had said] (10/6/00 joint planning/reflecting time)

Becoming aware of our own tendencies, such as using repeating to "ask for [students'] attention without asking for it," was an important part of the process of facilitating discussion for Kathryn and me because it helped us identify inconsistencies among our goals and behaviors. We found both written transcripts and videotape to be useful for this purpose; written transcripts allowed us to focus on sequences of talk and videotape provided us with a sense of the physical environment and the level of involvement of the class as a whole. Unfortunately, Kathryn and I found it easier to identify problems in our behaviors than to find solutions. We did try, however, to use prompts (such as "Who can explain what Katie means?") to encourage and assess listening, as well as avoiding methods, such as repeating, that served to undermine our goals for student listening. We also made conscious decisions about how to ask students to listen. In the transcript of class discussion above, I first say "shh," which is basically a synonym for “be quiet.” It is not only “being quiet” that I am after, however, but also listening, which is an action. I ask students to “listen to her, please” and ask the student to speak “a little louder” in an effort to motivate listening, not just quiet.

To summarize, motivating listening was difficult for Kathryn and me. We did not do a very good job of holding students accountable for listening, and we were not always satisfied with the ways we responded when students did not listen to each other. In the end, we wondered if further integrating discussion into the learning process and thinking about ways to structure activities so that students would want to or need to listen to one another might not be the best ways to motivate listening. It seemed clear, however, that thinking about both how best to motivate listening and how our own actions during discussion might be supporting or discouraging listening were important parts of the process of facilitating discussion for us.

Engaging students in sharing and listening was essential if Kathryn and I were to be successful in using whole-class discussion to further students’ mathematical understandings. We began the school year by altering the physical space of the classroom in ways that would support sharing and listening. We explicitly discussed with students our expectations for sharing and listening during whole-class discussions. We also devoted a portion of our time together before and after class considering how best to motivate sharing and listening. Our main concerns with respect to sharing were how to encourage more students to talk and how to bring out the ideas of students who were reluctant to talk. Facilitating discussion involved developing and testing various methods for addressing these concerns. With respect to motivating listening, Kathryn and I endeavored to make whole-class discussions an integral part of the learning process. We also spent time examining our own efforts to lead discussion in order to understand how patterns of interaction and word choices might encourage or discourage listening. Wrestling with how best to motivate listening was an important component of the process of facilitating discussion for us.

Sharing ideas and listening to one another were essential parts of the discussion process. Whole-class discussions often began with the sharing of solutions or responses to a question. As the semester progressed, however, Kathryn and I began to differentiate between "sharing" and "discussing." Early in the semester, we had examined discussion transcripts for examples of *explaining*, *building*, and *going beyond* (Sherin, et al., 2000; see Chapter 2, p. 33 for definitions of these terms). Looking for examples of building, in particular, led us to notice that students frequently shared ideas without trying to integrate them with previous students' comments. Rather than reacting to and building upon each other's words, they often followed the separate tangents of their individual thoughts. Kathryn and I call this "sharing" and began to contrast it with "discussing," as in the following excerpt.

> Rebecca: Sometimes it seems like there's a conversation going on, it's not just "now they share, now I'll raise my hand, now I'll share," sometimes it's like someone's listening to what someone else says and that MAKES them say something and then whatever happens next MAKES them say something else. For example, with sharing you could almost mix up the order of who spoke, besides the teacher, you could just mix the people up and it would make just as much sense. With conversation you can't do that. (8/29/00)

Discussing occurred when students engaged in questioning each other, seeking additional information or clarification, and debating the merit of each other's thoughts. Over the course of the semester, we became increasingly focused on trying to motivate students to move past sharing to discussing. In the next two sections, "Questioning and Clarifying" and “Agreeing and Disagreeing,” I describe our efforts to engage students in “discussing” mathematics.

Engaging Students in Questioning and Clarifying

> It feels like discussion is a top or something, something you have to get spinning, something you have to do something to, to get it moving And once it’s actually moving it’s OK. Then you just have to keep [it going]. (Rebecca 10/6/00)

For Kathryn and me, the discussion top began to spin when one student’s idea caused another student to speak. Discussion involved not simply putting forth one’s own ideas, but making sense of other’s ideas through a process of questioning and clarifying. The critical issue for us was how to make this happen. Helping students develop their ideas prior to discussion and motivating a need for discussion were important (see the “Engaging with Tasks” section of this chapter for a discussion of our efforts in this area), as was developing norms and expectations for sharing and listening (see the “Encouraging Talk and Presenting Students’ Ideas and the “Motivating Listening” subsections of the “Sharing and Listening” section of this chapter). In addition, we used strategies during discussions that we hoped would encourage students to pursue understanding.

Techniques for encouraging students to question and build upon each other’s ideas suggested in the literature include asking questions such as “Who can add to what ____ said?” “Does anyone have a question for _______ ?” and using wait time after students speaks (Sherin, et al., 2000; Manouchehri & Enderson, 1999). Both asking for questions and using waiting encourage students to become pursuers of understanding. Part of the process of facilitating discussion involved simply remembering to follow suggestions such as these. Consider the following excerpts from two different discussions.

Discussion Excerpt 1

Task: Two days ago the temperature was -6 degrees, yesterday it was -5 degrees, and today it is -4 degrees. Assume the trend continues and find an equation that describes the relationship between the day and temperature.

Note: An unedited version of the entire discussion can be found in Appendix C.

Rebecca: I think that almost everybody in here has figured out that y= x-4 is the equation that works for this data. . . . The question is, uh, what did you figure out or learn or have you learned from this that can help you, how can we help each other learn to find equations?

Chrissy: Since it goes up it rises a degree every day, there is going to have to be a 1 in there somewhere and then there, but there isn't a 1 in that one [y=x-4], whatever, then I knew there had to be the number of days and that going to be x.

Rebecca: Ok, hold on one second. I want to write down Chrissy's idea so that kind of everybody can refer back to it . . .

Alyssa: Ok, and then since the first day was, or today is negative four, there has to be a negative four in there and since x is the number of days, x is going to have to be in there too . . . what I did was I took x as the number of days times 1 . . . then you have to add in there the first day and then the temperature was negative four . . .

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Rebecca: Now Chrissy looked at that table and noticed that the temperature was going up by one each time. Does anybody notice anything else from looking at that table? [Rebecca nominates Jessica]

Jessica: You need to add negative four.

Rebecca: Where do I do that?

Jessica: Anywhere. You just add negative four.

Rebecca: Brittany were you going to say something to expand on that idea?

Brittany: Uh, I just said, uh, the x column. Cause like uh you add the negative four to the days from now column, to the x column (11/2/00 period 4/5)

Discussion Excerpt 2

Note: An unedited version of the entire discussion can be found in Appendix C.

Task: Students are given a set of 64 linear and nonlinear equations. Students are asked to think about what the graph of each equation would look like and then put equations that they expect to have similar graphs into groups. Next students graph the equations and reanalyze their groupings.

Note: The discussion excerpted below occurred after students had put their equations into groups but before they had begun graphing.

Rebecca: So the question I want to ask everyone is this. What do you think is the most important characteristic of this equation [see below]? And what I mean by important is if you imagined graphing this equation, 'cause that's where we're going next, which of these things, the number of operations, the fractions, the exponent, whether the division bar goes all the way across or not, what do you think might be the most important thing in terms of what the graph is going to look like if you were to graph that equation? [15 sec wait time]

[Rebecca writes on the overhead $y = \dfrac{3(x)^2 - 9}{3}$]

Rebecca: Ok. Anybody have a vote for something else? [noisy] Whose got a vote for something besides division? Jeff?

Jeff: Variable

Rebecca: What do you mean?

Jeff: 'Cause you can't really do anything until you know what the variable is.

Rebecca: So, I'll add that up here.

[Rebecca writes on the overhead: What the variable is]

[Katie says something inaudible]

Rebecca: I think Katie needs you to say more about what you mean by that, Jeff.

Jeff: You can't do any of the other operations until you find out what the variable is because it comes first in the order of operations because it's in the parentheses

[17 sec wait time]

Tom: Does the variable matter? What it is? Because it can be whatever the equation asks for, like it doesn't matter what number it is, its whatever it says . . . so it shouldn't have anything to do with how the equation goes. (9/29/00 period 3)

In the first excerpt, I asked "How can we help each other learn to find equations?" hoping that this question would help focus students' attention on learning from each other through discussion. Next, I said that I wanted to "write down Chrissy's idea so that kind of everybody can refer back to it," indicating to students that referring back to ideas was something I expected to occur. After Jessica said "you need to add negative four" to create the equation, I might have waited or asked the class to expand on Jessica's words. Instead, I assumed the role of the questioner by asking, "Where do I do that?" I did better when I asked Brittany "Were you going to expand on that idea?" encouraging her to build on, or add to, Jessica's idea. In the second excerpt, I first asked Jeff, "What do you mean?" but then restrain myself and simply added his comment, "What the variable is," to the list. My silence seemed to prompt Katie to comment. I tried to encourage students to question and respond to each other when I said, "I think Katie needs you to say more about what you mean by that, Jeff." Finally, I used a lengthy wait time after Jeff described his reasoning, hoping that other students would question him. This move gave students time to think about Jeff's statement and time to formulate their own questions or responses.

As discussion leader, my goal was to act in ways that would encourage students to question and build upon each other's ideas. In the previous examples, however, I found myself fighting the urge to ask clarifying questions. Kathryn and I both found it difficult to manage this aspect of facilitating discussion, particularly when students said things we did not understand. The following excerpt of discussion and accompanying reflection illustrates this problem.

> Task: Small groups of students are each given a different real-world situation, such as "The number of people in [fill in the name of a nearby city] watching television," or "The number of people at the McDonald's drive through window." Each group of students creates a graph that represents the given situation. Students are told that the graph must show a 24-hour period (midnight to midnight) on the x-axis and the number of people on the y-axis (so that the graph shows the number of people as a function of time). Students draw their untitled "mystery" graphs on butcher paper and post them around the room. Next, students are given a list of all the given situations and attempt, in their groups, to match the situations and graphs. Discussion about which situation matches which graph and why concludes the activity.

Note: The focus of the following excerpt is on how to distinguish between the graphs of two situations: the number of people in the school hallways on a weekday, and the number of people in the school classrooms on a weekday.

Discussion Excerpt

Note: An unedited version of the entire discussion can be found in Appendix C.

Jon: Did anybody talk about how the, there's slants in the, like between the different times or whatever. . . . Because like up here there's no, like all the people leave [the classrooms], and so it's just a straight line, nobody's in there. And then that's like to represent the time that it takes for the passing period.

Kathryn: Ok, so you're talking about down here in this, actually up here and then down here in this little valley there's kind of a flat piece and in this graph it just comes to a point? [Kathryn draws on the overhead]

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Kathryn: Which, you know what, can you, go point to where you're talking about because I'm not sure exactly where you're-

Jon: [walks over to graph] Like right here. So this is later than down here. And over there, there's barely a-

Anna: So why does that one go so far over and ours goes straight up?

Jon: Yeah. This is like the time that, that's how to distinguish between the two. They're in the hallway or in the classroom.

Kathryn: Ok. So you're saying because it, are you saying because it's steeper-

Jon: Yeah.

Kathryn: Or because, actually that one's not as steep as this one, this one goes up a little more steeply? Ok. Alright. And that helped you decide-

Jon: Which was which.

Kathryn: Which was which? So your group thought this one was what? In the hallways?

Jon: Yeah.

Kathryn: Ok. Alright. And why did you think that in the hallways needed to be less steep than this one did?

Jon: Because, uh, it takes seven minutes, don't they have seven minute passing periods? It only takes like ten seconds to get out of here.

[7 second pause]

Kathryn: Ok. So this one took less time and that one would represent the passing periods you're saying?

Jon: No. Uh, the people in the classroom. It takes less time to leave the classroom.

Kathryn: Ok. Anyone else like to, like to comment on what Jon's saying? [seven sec wait time] Alright. Group D, can you tell us a little bit about your graph and why you designed it the way that you did? And this one I guess is the number of the students in the hallways. (9/1/00 period 4/5)

<u>Discussion Reflection</u>

Kathryn: And my question is how to handle this? And this situation has happened a couple times where a student is trying to explain and I don't understand what they're talking about. And I'm trying to ask questions to clarify my own thinking and I'm still not getting it. And I just thought this makes me feel so uncomfortable. How do you do this? And, like I said, I'm reading it now and I still don't know what he was talking about.

Rebecca: But I think it's great that he got up and went over [to the graph] and explained. And I'm not sure that because [we] didn't understand that other kids didn't understand . . .

Kathryn: I don't know. I guess, just the fact that he got up there, he pointed to the graph, he had such conviction about what he's saying, I'm thinking he must be saying something that makes some sense. So why I can't figure this out? And it happened with Robert that one day too. I'm just not sure. And I think at the end I said would anyone like to comment on what Jon's saying hoping maybe somebody would see my struggle with trying to understand it and that they would try to maybe explain it another way, but [that didn't happen] (9/15/00 joint reflecting time with Janet)

Rather than attempting to engage the class in questioning or adding to Jon's initial comments, Kathryn pursued understanding herself. Analysis of video tape of this discussion, and Anna's interjection in the excerpt above, indicate that other students were attending to the exchange, but Kathryn did not attempt immediately to engage them in questioning Jon. Reflecting on the discussion a couple of weeks later, Kathryn expressed a high level of discomfort with such situations but focused mainly on frustration with her own inability to understand. Pursuing understanding was something we expected of all discussion participants, yet our own desire for understanding sometimes caused us to take over the very roles we wanted students to adopt.

Sharing the responsibility for questioning and clarifying with students was a critical part of the process of facilitating discussion. Motivating students to pursue understanding of each other's ideas was important, as was leading discussion in ways that would help shift the role of questioner from teacher to students. Kathryn and I made liberal use of strategies suggested in the literature for engaging students in questioning and clarifying. We sometimes struggled, however, with sharing responsibility for pursuing understanding with our students, especially when we were the ones who did not understand. Engaging students in questioning and clarifying involved a shift in roles for us as well as for students, since we had traditionally been the ones to engage in these activities. Having discussed our efforts to engage students in sharing and listening and in questioning and clarifying, I now turn to the third feature of the process of facilitating discussion: agreeing and disagreeing.

Engaging Students in Agreeing and Disagreeing

Engaging students in agreeing and disagreeing with each other's ideas was an important component of Kathryn and my efforts to facilitate discussion. As with sharing and listening, and questioning and clarifying, students' engagement in agreeing and disagreeing was partially dependent on developing ideas prior to discussion and motivating a need for discussion. In addition, Kathryn and I attempted to stimulate debate by (a) positioning students' ideas in opposition to, or agreement with, one another, (b) inserting a problem or question, and (c) pushing position taking.

Positioning Students' Ideas

This strategy for engaging students in agreeing and disagreeing with each other is similar to creating participant frameworks (Forman, et al., 1998; discussed in the "Using Discourse-Enhancing Strategies subsection of Chapter 2). We often simply created lists of students' answers or ideas on the overhead projector and then asked, "Are there any of these answers you want to agree or disagree with?" We generally did not attach names to ideas because we wanted to focus students' attention on assessing ideas, not people; however, students might have been more apt to address their comments directly to each other, rather than looking at the teacher, if we had included names next to ideas.

Positioning students' ideas was a strategy we used throughout discussions. After initial lists of ideas were commented upon, students' reasons for agreeing or disagreeing with particular solutions became ideas to be positioned with respect to one another, through questions such as, "Curtis says the second and third equations both represent the same relationship because . . ., Mia says they don't because . . . . Would anyone like to comment on these ideas?" Responses were then positioned with respect to Curtis's and Mia's ideas. We found that keeping a written record of ideas was useful both because students could more easily keep track of the ideas that arose and because they could more easily refer back to other students' comments.

Positioning students' ideas in these ways was helpful both for focusing students' attention on particular mathematical concepts and relationships and for encouraging students to analyze and evaluate each other's ideas. This strategy did not encourage students to speak directly to one another, as we would have liked them to, but instead may have led students to speak to us, since we were the ones asking for comments.

Another concern for us was whether positioning students' ideas discouraged students from listening to each other. We did not want to encourage the following pattern:

> A student speaks to the teacher
> A student speaks to the teacher
> A student speaks to the teacher
> The teacher presents students' comments to the class
> A student speaks to the teacher
> A student speaks to the teacher
> .
> .
> .

Our efforts to minimize unintended consequences such as these included (a) using wait time after students spoke, (b) asking students to speak up and address their comments to the class, (c) using reference objects that could be positioned around the room (solutions written on butcher paper, for example), and (d) motivating listening (previously discussed in this chapter).

Analyzing discussions after they occurred, Kathryn and I found that we sometimes restated and positioned students' ideas because the class as a whole was not engaged. In other words, when students seemed unattentive, we used the power of our voices to get their attention. In these instances, although we may have moved the discussion forward, we may also have communicated to students that listening to each other was not necessary. Reflecting on discussions was valuable in part because it helped us to identify instances, such as these, when our reactions to student behavior did not support our goals for discussion. Overall, positioning students' ideas seemed to be a useful method for encouraging agreeing and disagreeing. We recognized, however, that this strategy must be used carefully, or students may learn not to listen to one another.

Inserting a Problem or Question

Data analysis suggested that inserting a problem or question, as a method for encouraging disagreement, was something Kathryn and I engaged in throughout the semester. However, it did not become a strategy we consciously applied until one day at the end of September, when I found myself in need of a way of pushing students past merely sharing their ways of thinking. The following excerpt is from a discussion-focused meeting with Janet that occurred after school on that day.

> Task: Students are given a set of 64 linear and nonlinear equations. Students are asked to think about what the graph of each equation would look like and then put equations that they expect to have similar graphs into groups. Next students graph the equations and reanalyze their groupings.
>
> Note: The discussion excerpted below occurred after students had put their equations into groups (i.e. categories) but before they had begun graphing. Kathryn began the Period 3 discussion by asking students to describe one of their categories and a few of the equations that would belong in it.
>
> Rebecca: So today Kathryn started having a discussion with Period 3 about their categories. . . . And the students were sharing and there was some [off task] talk going on . . .
>
> Kathryn: It wasn't something that was capturing their attention. . . . One group didn't really care to hear how somebody else classified [their equations] . . .
>
> Rebecca: And I was sitting in the back thinking, "I've got to lead discussion next period. What can I do?" . . . I was trying to think of something that they could disagree about. So I thought of giving them an equation that had a lot of different [operations in it]. And . . . I thought about asking them, "What category do you think $y = \frac{3(x)^2 - 9}{3}$ belongs in and why?" But then that didn't seem to be "disagreeable" enough. . . . So instead I asked, "What is the most important characteristic of this equation in terms of what the graph will look like?"
>
> .
> .
> .
>
> Note: My choice of $y = \frac{3(x)^2 - 9}{3}$ was somewhat arbitrary. However, my goal was to combine into one equation several of the operations and symbols (exponents, parentheses, multiplication, division, subtraction) that the students would have seen previously. The use of three in the denominator was deliberate; I wanted to

see whether any students would realize the equation could be reduced (no one mentioned it).

Kathryn: And then she put these up. $y = \frac{3(x)^2}{3}$, $y = \frac{3(x) - 9}{3}$, $y = 3(x)^2 - 9$ and said, "What do you think about the graphs of these? Which do you think will look most like $\frac{3(x)^2 - 9}{3}$?" And Adam said he thought it was [$y = 3(x)^2 - 9$] because, "all you're doing [differently in the original equation] is dividing by three. So they'd be parallel." And then Pat said he, "thought it would be [$y = \frac{3(x)^2}{3}$] . . . because at higher numbers [the nine wouldn't matter]."

Rebecca: Like if it was a hundred, the [minus] nine wouldn't matter.

Kathryn: And then Brittany said, "Well, what if it's small numbers?" It was like calculus.

Rebecca: It seemed to be so much better, at least today, to give them something a little bit new and have them use their previous work to look at something new, . . . and give them something to disagree about.

Kathryn: It was something we talked about before, there has to be something for them to disagree about, some conflict.

Rebecca: But I think before [today] the conflict was built into the activities. The mystery graphs, the conflict was built in before we talked. But in this case there wasn't a conflict in the activity, but we could still find a way to insert some conflict to start the discussion. . . . (9/29'/00 joint reflecting meeting with Janet)

In this instance, inserting a new equation for consideration seemed to stimulate students in a way that sharing their work had not. At the end of Period 3, after watching Kathryn struggle, I came to the front of the class and presented the new equation. The new equation seemed to engage those students as well. One drawback of this strategy, as used above, is that students did not have the opportunity to think about the question, "What is the most important characteristic of this equation in terms of what the graph will look like?" prior to discussion. Some students may need more time to reflect before they are willing to share their thoughts. This issue could be addressed by putting forth the new problem or question and then having students think and talk in small groups before beginning the whole-class discussion.

Deciding on a new problem or question to insert into the discussion is something that teachers will likely find they need to do on the spur of the moment, as I did in this particular situation. Kathryn and I also found it helpful to develop problems or questions ahead of time. One benefit of collaborating as we did was that we had opportunities to watch each other lead discussions. The observer was able to pick up useful ideas about what to do, and consider how to make changes to improve future discussions. Reflecting on discussions helped us examine the various strategies we had tried more critically and consider how best to utilize them in the future.

Pushing Position Taking

Kathryn and I knew that students' previous experiences in mathematics classrooms would likely not have included arguing for their opinions in an effort to achieve consensus. Therefore, part of our effort to facilitate discussion involved pushing students to take positions. Several aspects of facilitating discussion that I have already discussed helped encourage students to do this. These aspects include structuring tasks in ways that lead to position taking and debate (see Appendix B, Mystery Graphs activity for an example of this), encouraging talk and bringing students' ideas to the fore, and positioning students' ideas in opposition to one another. In addition, we stated explicitly that achieving consensus was our goal, we frequently asked students to either agree or disagree with other students' comments, and we tallied the number of "votes" for particular positions.

Pushing students to the edge of their comfort zone with respect to position taking and debate was part of negotiating classroom norms and communicating our expectations. Of course, students have expectations of their own and ways of communicating them to the teacher when they believe the teacher's pushing has gone far enough. In the following excerpt, Kathryn described the results of a recent attempt to push students to take a position.

> Kathryn: In the fourth period class I asked a question and they just kind of shut down. I was just trying to drive a point home . . . but they were just sitting there like, "We are not answering this question." And I even had one girl say, "You make us raise our hands every five seconds," because I'm always saying, "Who agrees with this and who disagrees?" (9/15/00, joint reflecting meeting with Janet)

Reflecting on this experience, Kathryn indicated that she felt that students "shut down" because she tried to engage them in debating something that they felt had already been settled. In this instance, it might have been more productive to end discussion and consider what kind of task or question could be used to push further students' thinking. Pushing position taking was something we tried to do prior to, as well as during, discussion. Reflecting at the end of the semester, we both felt that incorporating position taking into the structure of tasks motivated agreeing and disagreeing better than waiting and asking for it only during discussions.

Students' work on tasks often led to disparate outcomes and understandings and we used whole-class discussion to engage students in comparing and contrasting their ways of thinking. In order to encourage students to agree and disagree with each other's ideas we asked them to look at task solutions generated by individuals or small groups and choose ones to defend or reject. Responses became ideas to be evaluated by the class as well. When lists of students' answers or ideas did not stimulate debate, we sometimes inserted a new problem or question, or we attempted to push position taking in some way. We found that we needed to be somewhat careful, however, to avoid inadvertently causing students to direct responses to us, rather than the class, and to avoid pushing students so hard that they disengaged from discussion.

Summary

Features of the process of facilitating discussion I have discussed include (a) engaging students with tasks, (b) engaging students in sharing and listening, (c) engaging students in questioning and clarifying, and (d) engaging students in agreeing and disagreeing. Engaging students with tasks in ways that supported learning through discussion involved careful planning and consideration of how best to utilize discussion further students' mathematical thinking. We were also concerned with helping students develop ideas and opinions prior to discussion, motivating in students a need for discussion, and using differences in students' understandings as a resource for discussion.

Our efforts to engaging students in sharing and listening were multifaceted. We restructured the physical setting, shared our expectations with students, and endeavored to make sharing and listening integral parts of the learning process. We sought methods for encouraging more students to talk and, at the same time, found ways to incorporate quieter students' ideas into discussion without requiring that they speak. We analyzed our own patterns of interaction with students to better understand how we might be supporting, or hindering, sharing and listening. We were particularly concerned with motivating listening, and an important part of the process of facilitating discussion involved developing and testing methods for addressing this issue.

Engaging students in questioning and clarifying involved motivating students to pursue understanding of each other's ideas, and leading discussion in ways that would help shift the role of questioner from us to students. We found it particularly difficult to push students to question each other when we, ourselves, did not understand a student's comment. Analyzing discussion transcripts, and reviewing the literature for useful strategies, helped us become more cognizant of our own behaviors and more aware of alternative ways of interacting with students.

We frequently encouraged agreeing and disagreeing by generating lists of ideas and asking students to choose items to defend or reject. Responses could then become objects for reflection as well. We also inserted new problems or questions as a way to stimulate debate. Our experiences suggest that incorporating a need for consensus into the tasks themselves may be a better way to push position taking than asking students repeatedly if they agree or disagree.

My findings suggest that using strategies commonly indicated in the literature for improving discussion, such as using wait time or asking high-level questions, is only part of the process of facilitating discussion. Facilitating discussion also involves activities such as restructuring space, developing ideas, and motivating listening, questioning, and position taking. The process of facilitating discussion was a complex and difficult one for Kathryn and me. I turn now to the identification and characterization of our struggles, and activities that supported us, as we attempted to use whole-class discussions as a tool for learning mathematics.

Struggles and Supports Related to Facilitating Discussion

In the previous section of this chapter, I described key activities, or features, of the process of facilitating whole-class discussion in mathematics classrooms. In this section, I describe three areas of struggle for us that spanned part or all of the process of facilitating discussion: managing internal dialogue, managing time and curricular constraints, and managing students' expectations. In addition, I discuss two activities that supported our efforts to facilitate discussion: collaborating during planning, implementation, and reflection, and observing one's own students engaging in discussion.

Struggles

Ideally, Kathryn and my efforts to facilitate whole-class discussion would have occurred within an inquiry-based mathematics program that was well-articulated across grade-levels and adopted by all mathematics teachers in the district. Norms and expectations appropriate for the inquiry mathematics classroom would have been developed in previous years, and school wide and state level assessments would match inquiry-based instructional practices. Of course, this was not the case. Most mathematics teachers at Kathryn's school taught using traditional methods, the textbook did not support an inquiry approach, and the departmental semester exam was based on the textbook's content and approach and was beyond our control to change (see Appendix A for sample questions from this exam). Although these constraints hindered somewhat our efforts to facilitate discussion, we recognize that most teachers will likely face similar constraints.

Our struggles reflect both external constraints and our own uncertainties and shortcomings. For example, prior to the school year, we did not think about how best to incorporate alternative forms of assessment into classroom activities. Looking back, we realized that we were still tied to individual, written quizzes and tests as a major determiner of students' grades. This may be the very reason why we did not initially spend time thinking about assessment. We also wrestled with an external constraint related to assessment, the departmental semester exam.

Another area of tension and uncertainty for us involved our desire for common understandings among students. We were comfortable with divergence in students' investigations, solution methods, and ways of thinking, but we were uncomfortable with the notion that some students had developed what we viewed as important understandings and other students had not. This issue was important to us, because we wanted to give all students the same quiz or test, and because we felt there were certain things that students had to be able to do before leaving algebra, such as writing an equation for a line given two points.

To a certain extent, the struggles discussed in the following subsections are idiosyncratic to our situation and to us. They are the issues that arose repeatedly throughout the semester, but each one is of a very different sort from the other two. Managing internal dialogue was a problem only during discussion and was largely the result of the newness of the process to us. Managing time and curricular constraints, and students' expectations, were concerns during both planning and implementation. These struggles were the result of both external factors and inadequate planning. Although other teachers and researchers may uncover different struggles in the process of facilitating discussion, this discussion may be useful to shed some light on interplay among external constraints, teacher beliefs and practices, and the process of facilitating discussion.

Managing Internal Dialogue

> Kathryn: I'm up there I have this constant running commentary going on in my head and I'm thinking how do I respond to that, what do I say, do I respond or do I not respond, do I summarize what this student has said or do I just let it lie . . . It's like this constant thing and a student says something I don't understand, and well what do I do now, what do I do with an incorrect answer, what do I do with a correct answer, you know, it's just constantly going on and it feels, unnatural.. . . I'm constantly questioning myself. (8/29/00 joint reflecting time with Janet)

Facilitating discussion caused Kathryn and me to rethink our ways of interacting with students, ways that had become almost automatic, and we both felt a high level of internal dialogue whenever we led discussions. This self-consciousness about our own actions was due, in large part, to the newness of the process and to the fact that we observed each other. The multiple, sometimes conflicting, aspects of discussion we were engaged in managing added to our internal dialogues, as Kathryn expressed in the following excerpt.

> Kathryn: I just feel like there are so many things to concentrate on and I'm spending a lot of time just concentrating on what I'm going to say next or trying to concentrate on what they're saying and also on trying to make sure I'm calling on a variety of kids or drawing out the kids who are not talking. [It] just feels like too much. (11/10/00 joint reflecting time with Janet)

Managing discussion required both being a part of the discussion, in the sense of listening carefully to each speaker and participating in the development of ideas, and acting on a meta-level, in response to the substance and flow of the discussion as a whole. The metaphor of an orchestra conductor may be appropriate in that conductors must attend to both the parts of individual musicians and the ebb and flow of the music. Conductors must listen and respond at both levels, just as teachers must listen and respond to individuals as well as to the group.

Chazan and Ball (1999) describe three aspects of discussion that teachers must manage: mathematical value, direction and momentum, and social and emotional tone. They characterize teacher actions as “the product of subtle improvisation in response to the dynamics and substance of student discussion” (p. 7). In addition to being attuned to multiple aspects of discussion, and being responsive and improvisational, Kathryn and I wanted to share with students responsibility for the content and process of discussion. Frequently, our internal dialogue during discussion centered on how to shift responsibility for questioning and pursuing understanding onto the shoulders of students. In the following excerpt, I reflect with Kathryn on this issue, using an example from the transcript of a discussion we had been examining.

> Rebecca: It’s a quandary because we want students to pursue each other’s ideas, but they aren’t necessarily used to doing that or motivated to do that, so [then] students don’t take up each other’s ideas and then we do it and then we become the person who does it. [We become] the person who’s responsible for making sure that ideas get understood that then short circuits [the process]. But there’s a good reason for it, because the other kids [don’t do it]. . . . [For example] when I had my seven second pause and I finally said how come, there I am, even if it’s just two words . . . it’s me deciding that yet again I’m going to become the person who asks for more information. (11/7/00 joint planning/reflecting time)

We did try to overcome this quandary by using questions such as “Does anyone have a question for ______ ?” or “Can anyone add to what _______ said?” We also tried wait time, as I alluded to in the excerpt, because we wanted to encourage students to take responsibility for asking questions without a direct prompt from us. Imagine a ball bouncing from speaker to speaker with the students’ attention moving with the ball. We wanted the ball to bounce around the room, but when we spoke, even if it was only to ask students to respond to each other, the ball bounced back to us.

Even when students spoke directly to each other, we still had much to think about during discussion. Although the level of internal dialogue did not seem to diminish over the course of the semester, it gradually became more natural. At the end of the semester, we agreed that a high level of internal dialogue was a necessary, if difficult to manage, part of the process of facilitating discussion.

Managing Time and Curricular Constraints

> Kathryn: I am always conscious of the clock and the time that I have and trying to balance things and fit everything in . . . [and] knowing that there's this mass of things that has to be accomplished, how can you do all of that? (8/29/00 joint reflecting time with Janet)

Kathryn was responsible for teaching the material in the first six chapters of the Heath Algebra 1 textbook (Larson, et al., 1998) during our semester together, and the Algebra 1 teachers gave a common exam, developed in a previous year, at the end of the semester (see Appendix A for sample exam questions). The exam and the textbook emphasized students' ability to memorize and apply specific algorithms to decontextualized, symbol manipulation type, problems. The textbook did contain application problems, but these were not viewed as part of the "core curriculum," and were not well represented on the semester exam. Balancing teaching through posing non-routine tasks and facilitating discourse with the substance of the textbook and the semester exam was very difficult, but we recognized that many teachers feel such constraints, to a greater or lesser extent.

> Kathryn: I think time is a real issue for people and I think the way we restructured [the semester's topics] can . . . save some time.
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> Rebecca: Yeah, . . . looking at the textbook and the curriculum and what we were responsible for, and looking at the big ideas rather than looking at it section by

> section, and then looking for activities that would have a lot of different [concepts] in them. (11/28/00, joint planning/reflecting time)

Restructuring the content of the course was an important strategy for managing time constraints; however, we still found it challenging to meet the requirements of the textbook and semester exam. With respect to facilitating whole-class discussion, we found ourselves struggling to decide "is this something that I really want to open up to a discussion" or "do I just want to make the point and move on?" (Kathryn, 8/29/00). Making decisions about the mathematical value of discussion at a particular point in time is always necessary; however, our decisions were often influenced, in part, by our concerns about time and the semester exam.

We were attempting to blend student-centered, inquiry teaching methods with specific, predetermined learning outcomes; however, inquiry methods did not lead all students to the same understandings.

> Kathryn: With the grades coming up here, I'm feeling like I can't fail anyone . . . because there are some loose ends here. . . . [T]here were a couple quizzes in the middle that were mixed results. Some of the kids didn't do well on either one of them. But on both of them I guess I was still just feeling like I was giving the quiz to get information on where they were, but sort of knowing already that there were kids that weren't where I wanted them [to be]. . . . I feel like some kids are getting it but there are still some that are out on the fringes and I know that they're there and what have I done really to try to get them off of the fringes? . . . [Also] when you're in this mode of trying not to tell, but then you give a traditional test or quiz, then you're telling. You're telling them whether it's right or wrong. And I guess I'm wrestling with meshing the traditional pencil and paper tests and quizzes with what we're trying to do.
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> Kathryn: I wish we had more time. I wish I didn't feel the pressure of this next nine weeks to get things done. (10/17/00 joint reflecting with Janet)

Kathryn's words reflect several issues related to time and curricular constraints that we struggled with throughout the semester. First, the "loose ends" to which Kathryn referred were the mathematical concepts or understandings that had, as yet, not been developed by all students. Finding ways of tying up these loose ends within our time frame and at the same time adhering to our inquiry-based approach to teaching was challenging. Second, although our interactions and observations provided us with evidence of student knowledge, we had not collected the type of individual, in-class, written assessments expected by our students. We, ourselves, felt a certain need for traditional assessments both as evidence of students' ability to solve problems without assistance and as preparation for the semester exam, yet we felt uneasy because some students were "still on the fringes" and in need of more tasks and more time. Third, we wanted to shift onto students some responsibility for determining the correctness of their solution methods. During classroom activities, we would avoid evaluating students' ideas, instead asking questions or otherwise directing students' efforts in ways that would help them uncover inconsistencies in their thinking. When we used quizzes and tests, at least in their traditional configurations, as assessments, we took back the responsibility for evaluating correctness.

The issue of how to assess the learning that occurs during whole-class discussions, both for making instructional decisions and for assigning grades, is a difficult one. Although our attachment to written assessments was strong, by the end of the semester, we had begun to wonder whether we ought to be trying to use them to measure growth in understanding.

> Kathryn: When we were having the discussions and giving the kids the projects, I think there were so many good ideas that came out. I remember there were days after class when we were just, 'wow, did you hear what so and so said' or 'this is what this kid thought about this problem' and, but that's not easily-
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> Rebecca: Quantified.

> Kathryn: And then you know I have to give them a grade, so I have to have something to base a grade on, so I have to give them some kind of a written assessment and feeling like [the written assessment] doesn't completely match what we were thinking about or doing in the classroom and how do you effectively measure everything that was going on, or what gains they made in their thinking as you were having a whole class discussion or as they were working on a project? It feels like something that's just like out of your grasp. (12/15/00 interview of Kathryn)

We needed to assign grades, but our way of teaching did not fit well with the accepted forms of evaluation, namely quizzes and tests. Still, how does one assess the development of understanding *as it occurs*? Measuring the having of ideas seemed to be beyond our grasp.

Our teaching methods had provided us with opportunities to learn much about students' understandings and ways of thinking. We realized, without a quiz, that some students had not yet developed understanding of certain concepts. Other students had developed understandings, and these students were ready to move on. We wanted somehow to bring everyone to the same level, establishing some knowledge that everyone who was intellectually engaged during class and doing their homework would possess. What were our options? We could establish knowledge of procedures for finding solutions by telling them to those students who had not developed solution methods of their own. We could individualize instruction and pose different tasks to different students. The second option seemed more consistent with our beliefs about teaching and learning; however, we felt pressured by time and curricular constraints to move on to other topics before the semester ended.

What we tried to do was be more directive, without reducing the problem to the memorization of a procedure.

> Kathryn: I was having them do this matching activity and we're supposed to give nine-week tests and I didn't really want to give a summary exam, . . . so we had

> kind of talked about, well, we'll just give them something similar to this matching activity, so I wanted to make sure they really had some understanding of this because I'm going to QUIZ them on this.
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> Kathryn: [So] at first I felt like I was doing a lot of telling . . . but I'm not sure it was so much telling, it was a lot of repeating though. I was still taking their ideas about, "well if there's a plus two, then I know it has to cross at two on the y axis". But then I would kind of take that and really emphasize it. And the same thing with some of their [other] ideas. So I really felt like I was repeating a lot. That I was really pushing . . . (10/17/00 joint reflecting with Janet)

This "pushing" Kathryn described is reminiscent of the "cued elicitation" (Edwards & Mercer, 1987) pattern described in "The Teaching Process" section of Chapter 1. Kathryn used her tone of voice to emphasize ideas she wanted students to pick up on, thereby relieving students of the obligation to decide which ideas had merit. Although she did not "tell" in the strictest sense, she did provide students with clues about which ideas were the ones they needed to learn, such as "if there's a plus two, then I know it has to cross at two."

Pushing students in this way, in order to prepare them for a quiz or test, was not satisfactory to either Kathryn or me. Unfortunately, we were not always successful in using whole-class discussion to bring students' to consensus and develop classroom-level understandings. Issues raised during discussion were sometimes left unresolved because we didn't want to tell students who was right and who was wrong. Achieving consensus would have been useful not only because of time and curricular constraints, but also because the path to consensus could involve deeper examinations of various ways of thinking. We also did not make as much use as we might have of alternate forms of assessment, such as classroom observations, which could have alleviated somewhat the need to "emphasize" students' ideas during discussion in preparation for assessment.

Our goal was to teach algebra in ways that would encourage understanding of concepts and relationships among concepts. We believed, and continue to believe, that providing students with procedures for getting answers short circuits this process. Yet, at the end of the semester, we still struggled with how to teach without telling, given time and curricular constraints.

> Kathryn: The telling part of it is still a real struggle. I think if you are giving them a problem, dot patterns, or some other problem that you really want them to explore, if you end up doing the telling you've kind of missed the point of the whole thing, if you're trying to figure out what are the kids thinking and what are their approaches then you don't want to tell them what your approach is or what your ideas are about it, but there are certain things we have to teach that given time limits I'm not really sure how you teach it without just telling them. I don't think there's enough time for Jenny to figure out everything on her own in the space of a semester. I don't even know about some of the other, really bright kids. (12/15/00 interview of Kathryn)

Managing time and curricular constraints was an integral part of the process of facilitating discussion. We felt strongly the tensions inherit in teaching through inquiry and reaching specific, predetermined, common learning goals during whole-class discussions. We sometimes found ourselves "pushing" or "emphasizing" in an effort to help students reach these goals within our time constraints. We also felt pressure to curtail what might have been valuable discussions due to time pressures. Encouraging students to achieve consensus and using alternative forms of assessment may be helpful for managing these constraints.

Managing Students' Expectations

> Kathryn: The point I was trying to make [today in class was] I think the thinking that you're doing here is probably going to be more helpful to you than just drilling you on skills. If you can think through a problem on your own and know how to attack it and how to test whether your answer is right then you're going to be much better off. And I really do believe that, but it does still concern me that they think "she's not getting me ready for ISTEP". . . . (9/15/00 joint reflecting with Janet)

Note: The “ISTEP” to which Kathryn refers is the Indiana State Grade 10 Graduation Qualifying Examination for English/Language Arts and Mathematics.

Students’ expectations about what should happen in Algebra class were influenced by their previous experiences with school mathematics, their knowledge of what friends in other Algebra classes were doing, and their desire to be prepared for both the semester exam and the graduation qualifying exam. When our teaching methods and goals for learning did not meet students’ expectations, they began to exert pressure on us to modify them. This pressure took various forms including not engaging with tasks, or asking “Why are we doing this?” or “When are we going to do algebra in here?” or “Is this on the ISTEP test?”

Problems with students’ expectations did not arise immediately. After the first few weeks of school, Kathryn and I were pleased, for the most part, with students’ engagement with the tasks we had chosen and their willingness to share their ideas with each other. We began the school year with an activity (see Appendix B, Typical/Ideal Mathematics Classroom Activity) designed to bring forth students’ perceptions about mathematics classrooms and to help us explain our vision of the ideal classroom. Instead of beginning the year with a review of some of the previous year’s material, as is often done in mathematics classes, we began with new and, we hoped, exciting explorations (see Appendix B, Developing Graph Sense activities). By the middle of September, however, we began to hear rumblings of discontent, mostly from students who were struggling with the current tasks (see Appendix B, Writing Equations activities).

Kathryn: I’m concerned because at least for Curtis and there are a few others like Jenny, Amy, Melissa in the other period, I’m feeling like they haven’t gotten a whole lot this week and they haven’t had anything really that they could feel like I’m getting this, I’m successful, so I think that’s probably where some of the frustration is coming out, so you do start to think how have I really helped

> those students, if they've gone the whole week and really haven't had a success?
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> Kathryn: But then on the flip side I feel like because we haven't done any traditional teaching we've pretty much established the expectation that, you know, every day is pretty much like this, we're going to expect you to share your thinking and we want to hear different ideas. And we purposely started with those because we felt like it was going to be essential to establishing that climate to have tasks that would lend themselves to discussion and many different answers and so forth. So I don't know. I'm not sure how I feel about it. On the one hand, I think it's been good but on the other hand, especially this past week, I think I've started to lose a few. (9/15/00 joint reflecting with Janet)

When students struggled with a task, they frequently looked to us for more direction. We, not wanting to do the thinking for students, would ask questions or raise issues. Of course, the students often wanted more direction, and previous experience suggested to them that we should provide it. We did not, and frustration mounted. This is not to suggest that most students were unwilling to try; rather the time and sustained effort required to work through some of the tasks we chose was great and many students were not accustomed to putting forth such effort on a single problem.

Another point of concern for some students was that we were not following the textbook, as some of their friends were in other classes. Students who raised this concern were not necessarily the students who struggled with some of the tasks we posed. We began to suspect that some of the high-achieving students believed that the book problems were more difficult and therefore wanted to work out of the book. We also wondered whether their ability to find solutions to tasks without being given a procedure led them to believe that the tasks were easy.

Unwilling as we were to tell students how to solve problems or move through the textbook section by section, we felt that students' concerns were reasonable, and wanted to find ways to address them.

> Kathryn: I thought I might go back through some of the old ISTEP tests, and I know there are questions on there that are writing equations and writing formulas, . . . so maybe what I need to do is find some questions that are algebra questions from the open response part and just show them what some of them look like. (9/15/00 joint reflecting with Janet)

In addition to explicitly linking students' current work with the semester and graduation qualifying exams, we looked for ways to incorporate textbook problems into classroom activities and homework. We hoped students would begin to see that they were, in fact, learning the content in the textbook and much more.

The most difficult student expectations to manage were those of struggling students who, in their view, had been successful in previous years when teachers had given them step-by-step procedures for finding solutions that they could memorize. Concern for these students led us to sometimes be more directive than we would have liked. At the same time, we tried to provide additional supports for these students, such as after school help sessions. These sessions provided students and us with more time to probe their thinking and ask questions or raise issues that could help them think more deeply about the task at hand.

Managing students' expectations was not a struggle for us only with respect to facilitating discussion; it related to the whole of teaching algebra. Yet, it was a discussion-related issue for us because if student frustration rose too high, they would not engage in discussion. Also, as I described in Chapter 4, the development of ideas and interest prior to discussion was critical to the quality of discussion. Therefore, managing students' expectations was necessary, if discussion was to occur.

Reflecting in December on the semester's work, I asked Kathryn if anything had surprised her about the students during the semester. She said:

> I don't know if this goes in the surprise category, but realizing that you can't ignore their pre-conceived notions of what a math classroom should look like. To change things drastically, for them, is really difficult. . . . Even though we knew they were doing algebra, their perception was that they weren't doing algebra. And I don't know exactly how to address that. But I think that [if] some of them think they learn best by a different method, then somehow that needs to be honored or addressed. (12/15/00 interview of Kathryn)

In the end, Kathryn and I realized that managing students' expectations involved much more than clearly communicating to them our expectations. It involved finding ways to be true to our beliefs about teaching and learning while at the same time respecting their experiences and their beliefs about what they needed to be successful.

Summary

Kathryn and I experienced three major areas of struggle related to facilitating discussion: (a) managing internal dialogue, (b) managing time and curricular constraints, and (c) managing students' expectations. Reflecting at the end of the semester, Kathryn and I recognized ways that we might have been able to better organize activities to better manage time, but finding ways to help students develop proficiency with specific procedures would still have been difficult to incorporate with the rest of our activities. Had we done a better job of incorporating alternative assessment, we might have struggled less with both curricular constraints and managing students' expectations. However, as long as students' previous experiences and the experiences of their friends in other classes, and the semester exams and state assessments, deviate widely from what we attempt to do, struggles such as those we experienced are likely to occur.

Having described the struggles Kathryn and I experienced I turn now to those activities that seemed to help us become better at facilitating discussion, or at least better able to meet the challenges of teaching in this way.

Supports

Our efforts to facilitate discussion were supported in two ways. First, we shared ideas, and planned and reflected together. Second, we took turns leading discussion and learned from observing each other. As sharing ideas and learning from each other during implementation are tied together, I discuss them concurrently in the following paragraphs, rather than separating them into subsections.

Kathryn and I worked together to plan, implement, and reflect on instruction. Working together in this way allowed us each to have ownership over and responsibility for facilitating whole-class discussions, rather than one of us performing and the other one evaluating.

> Kathryn: And what I was most worried about before we started . . . was having the strength to do it. I think it does take some strength to have someone watching you all the time. So that was something I was concerned about going into it. But as we've talked about before, I liked . . . that you were teaching some lessons and I was teaching some lessons and that we were planning pretty much everything together and so it was OUR idea that we were testing out and not MY idea. Not like, I'd do a lesson and then in retrospect, "well, Kathryn that was a bad idea or a bad way to do it." You know? Like waiting for things to go wrong, kind of. So I didn't have that sense that it was "I'm watching you." (12/15/00, interview of Kathryn)

Kathryn and my efforts to facilitate whole-class discussion were complex and the demands on us were great (see the first section of this chapter for descriptions of features of the process of facilitating discussion, and the second section of this chapter for descriptions of struggles we experienced). Having someone with whom to share ideas and concerns was valuable, both practically and emotionally. Although co-teaching is not possible for many teachers, some may find like-minded colleagues in their departments with whom to share ideas and reflect; other teachers, such as Kathryn, may find that they need to look outside their schools.

> Kathryn: [During an inservice session yesterday, someone] brought up the issue of students not understanding when they're done [writing an equation given two points] . . . that the x and y stay x and y in the final equation. That you're not putting the specific values in. And someone else said "well, I just kind of avoid that whole problem, I never let them put a point in for x and y to solve for m and b, I use the point-slope form instead so they never have to do that." And I was kind of sitting back listening to that and I just thought "but isn't that something you really want them to understand? That putting a point into that equation makes that equation true and how can you just take that away [from them]?" . . . sometimes I feel like I'm on another planet from [the people in my department]. (10/17/00, joint reflecting time with Janet)

Kathryn's beliefs about teaching and learning often caused her to feel isolated from her mathematics department colleagues. Our collaboration provided professional and emotional support that she could not have found within her department. Throughout the semester, Kathryn repeatedly commented on the importance of having "someone to talk to" and "share ideas with."

In addition to sharing responsibility for a range of teaching activities, Kathryn and I took turns leading whole-class discussions. As we watched each other, we thought about what the other person was doing and what we ourselves might do.

> Rebecca: When I watch you I'm thinking about what you're going and then I'm trying to think about what . . . I want to do because I see what happens when you did something. [For example] asking about agreeing or disagreeing and would someone like to respond to that? So I make a little note of a phrase or something that I want to remember to try to use. And at the same time I'm trying to start to think about what are the things that I do that I need to think about, what things I want to change or add . . .so all those things are running through my mind. (8/22/00, joint planning/reflecting time)

Being a discussion leader myself, I was concerned not only with what Kathryn was doing, but also with what I might "change or add," often in the next class period.

> Kathryn: (to Rebecca) I was telling [Janet], third period I had a discussion that went nowhere and [then in fourth period] you had two beautiful discussions. . .
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> Rebecca: (to Janet and Kathryn) Before period three we talked about what we thought we should try to do with the discussion and . . . [then] I went and sat down and you did it. And then I watched you and I thought "Ok, that's not working at all. Now what can I do [differently]?" (9/29/00, joint reflecting time with Janet)

Again, many teachers will not be able to teach together the way Kathryn and I did. If they teach the same subject at the same school, however, they may be able to occasionally observe each other leading discussion, and use the insights they gain to inform their own practices.

Asked what had been particularly helpful to her, and what might support other teachers' attempting to engage in the process of facilitating discussion, Kathryn commented:

> Having someone to talk with, and to model, and being able to sit in the back of the classroom and watch. And not just being able to sit in the back of any classroom but being able to sit in the back of your classroom with your own students and watch. Because I've been up in front of them and I know the feel of that perspective, and then to be able to sit in the back and watch how things might be different with the same group of students, I think was a really valuable experience. And being able to talk to someone about it. (12/15/00, interview of Kathryn)

The opportunity to observe one's own students engaged in discussion was a valuable experience for both Kathryn and me. We each noticed the affects of the other's ways of interacting with students; for example, how Kathryn encouraged reluctant students to share their ideas or how I inserted a new idea or problem to stimulate discussion.

Reflecting on discussions after they occurred provided us with yet another perspective on our work. We found that written transcripts were particularly useful because we could look for patterns in the talk. For example, how many different students' talked, or what was the nature of our questions or comments and to what extent did they stimulate discussion? We found that using frameworks for analyzing the quality of discussions (e.g., Mendez, 1998) helped us to think about the extent to which we had been effective in engaging students in sharing their reasoning and building upon and analyzing each other's ideas. After analyzing transcripts, we sometimes watched videotape of the discussion; videotape was useful for gaining insight into the tone of the discussion and the degree of engagement of the class as a whole.

Each facet of our work supported our efforts to facilitate whole-class discussion. Planning and teaching together provided us with a sense of shared ownership and responsibility. Leading discussions and watching each other lead discussions gave us a chance to see discussion from the front and the back of the classroom. Kathryn felt, and I agreed, that watching one's own students engage in discussion with someone else leading was valuable. Because we were both discussion leaders, watching each other was both reflective and anticipatory; we were often able to make changes in the next period based on observations from the previous one.

Clearly, it is not possible for many teachers to develop a collaboration of the type Kathryn and I shared. It is unlikely that Kathryn would have found someone of like mind in her own school with whom she could collaborate; even if team-teaching or release time had been possible. Teachers might develop, however, smaller scale collaborations with colleagues that utilize some of the aspects described here. For example, teachers might observe each other leading discussions during planning time and perhaps collaborate on the development and implementation of a unit, including analysis of whole-class discussions using audio or videotape.

Summary

I have described three areas of struggle for Kathryn and me: managing internal dialogue, managing time and curricular constraints, and managing students' expectations. Each of these areas was problematic throughout the semester, albeit to differing extents. By the end of the semester, we had become more accustomed to high levels of internal dialogue during discussions, although we still felt the strain of juggling a range of considerations while trying to remember to make use of the various strategies we had learned. Student expectations, which had not been a large problem during the first month, became increasingly problematic for us as the semester went by. We also felt time and curricular constraints more acutely during the second half of the semester, although we spent considerable time attending to them throughout.

The activities that supported our efforts were relatively simple and straightforward. We shared responsibility for planning and implementation of classroom tasks and observed each leading discussion. We also used written transcripts and videotape to help us understand the patterns of teacher-student and student-student interaction occurring during discussions. Having a partner with whom to work, and observing one's own students engaging in discussion were particularly helpful.

Conclusion

Analysis of Kathryn and my experiences facilitating discussion suggested four major features of the process: (a) engaging students with tasks, (b) engaging students in sharing and listening, (c) engaging students in questioning and clarifying, and (d) engaging students in agreeing and disagreeing. I have subdivided each of these features into several aspects and discussed in the preceding chapter a range of discussion-related issues such as physical structure of the classroom, the development of ideas and opinions, and the use of various discourse-enhancing strategies to encourage sharing, questioning, and debate. In addition, I have identified three areas of particular struggle for Kathryn and me (a) managing internal dialogue, (b) managing time and curricular constraints, and (c) managing students' expectations. Sharing ideas, planning, implementing, and reflecting on instruction, and observing each other leading discussion supported our efforts.

Chapter 5: Conclusions and Recommendations

In this study, I examined the process of facilitating discussion as experienced by two teachers, Kathryn Thomas and me, in order to answer the following questions:

- What are the features of the process of facilitating whole-class discussions in secondary mathematics classrooms?
- What do teachers struggle with as they attempt to facilitate discussion?
- What supports teachers in their efforts to facilitate discussion?

In the previous chapter, I identified features of the process of facilitating discussion including developing ideas prior to discussion, bringing forth students' ideas during discussion, motivating listening, encouraging questioning, and pushing position taking. I also discussed struggles Kathryn and I experienced, such as managing external constraints and student expectations, as well as activities that supported us, such as observing each other engaging students in discussion. In this chapter, I relate these findings back to Simon's conceptualization of the teaching process (Simon, 1995, 1997; discussed in Chapter 1), and I discuss how my findings connect to the literature on classroom discourse (discussed in Chapter 2). I describe how this study has affected my view of classroom discourse and suggest areas for further study.

The Process of Facilitating Discussion and the Mathematics Teaching Cycle

Simon's Mathematics Teaching Cycle (Figure 1, Chapter 1), based on the emergent perspective described earlier, provided me with a way of thinking about the process of teaching mathematics that included planning and teacher knowledge development, as well as interaction with students. Teacher reflective activities, such as our discussion-focused meetings, while not explicitly addressed in the cycle, can be thought of as feeding into teacher's knowledge and thus into planning and future interactions with students (Figure 3).

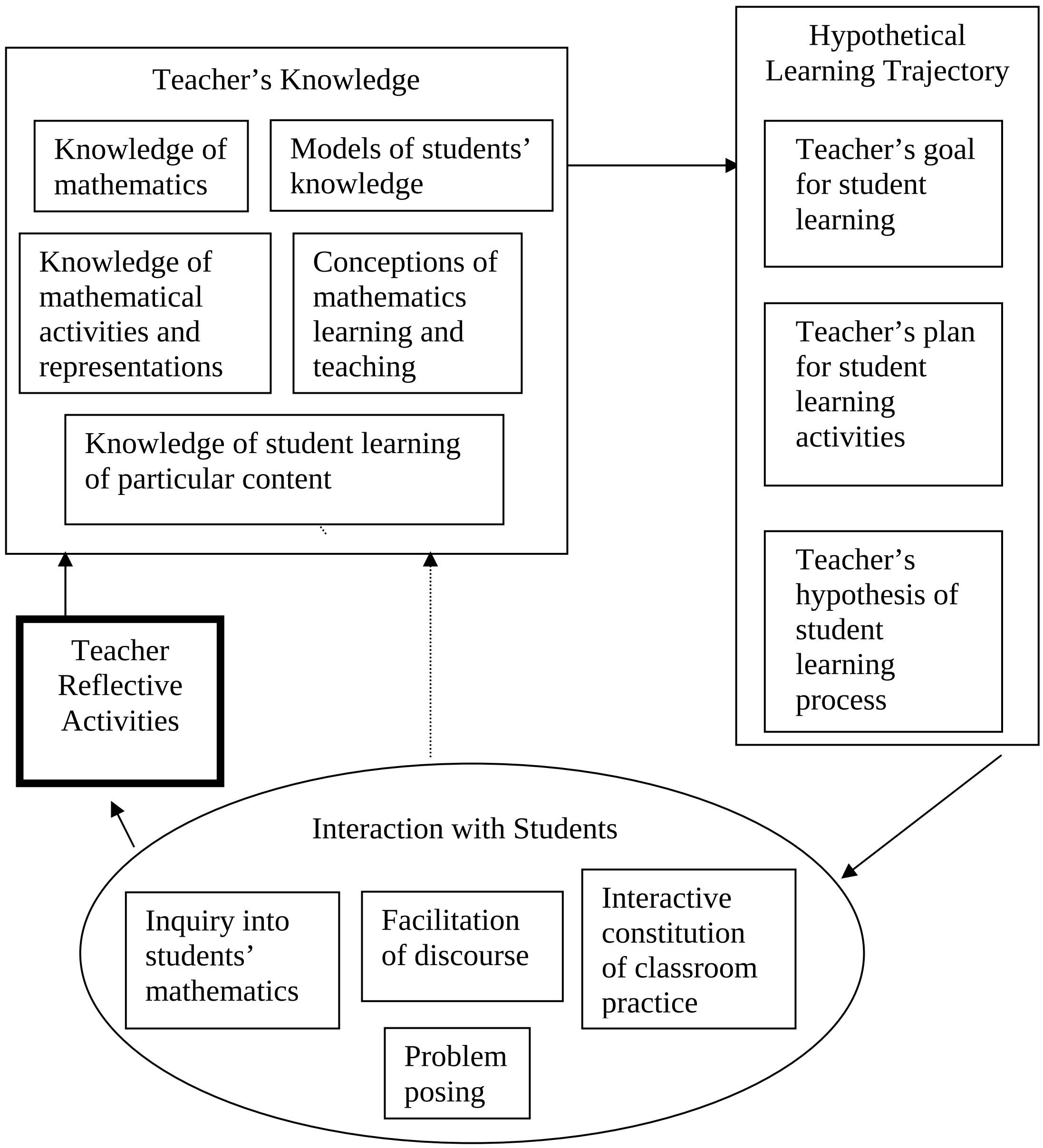

Figure 3. The Mathematics Teaching Cycle with Teacher Reflective Activities (Simon, 1995, 1997)

Traditionally, facilitating discussion has been synonymous with "leading" discussion and has referred to what a teacher did or might do *during* a given discussion. I have conceptualized the process of facilitating discussion more broadly to include teacher activities that relate directly to discussion but occur before or after it. Therefore, I examined data not only from discussions themselves, but also from planning and reflective activities, in order to identify a wider range of discussion-related issues imbedded in various parts of the teaching process. In the following paragraphs, I use the language and structure of the Mathematics Teaching Cycle to relate aspects of facilitating discussion to the larger process of teaching mathematics.

Deciding how best to use whole-class discussions to further students' mathematical development and how to engage students in tasks in ways that motivate a need for discussion are two aspects of the "engaging students with tasks" feature of the process of facilitating discussion. Consideration of these aspects occurs as teachers develop hypothetical learning trajectories for their students. Hypothesizing about the learning process may also involve thinking about differences in students' levels of understanding and how to use these variations as resources upon which to draw during discussion. Attention to students' engagement with tasks also occurs during the "interaction with students" and "reflective activities" phases of the teaching cycle, which feed into teacher knowledge, and lead to the refinement or alteration of hypothetical learning trajectories.

Aspects of the other three features of the process of facilitating discussion, engaging student in sharing and listening, questioning and clarifying, and agreeing and disagreeing, can also be found in the development of hypothetical learning trajectories and teacher reflection areas within the Mathematics Teaching Cycle. When teachers reflect on and hypothesize about the learning process, they may consider how to gather up and re-present students' ideas to the class, what questions or issues to raise during discussion, and what strategies to use to encourage talk (e.g., using wait time, redirecting students' questions back to the class, or positioning ideas). In each case, teachers hypothesize about how students will learn through discussion and how they can best lead discussion to support that learning.

Making decisions about the physical positions of people and objects in the classroom can also be thought of as part of developing hypothetical learning trajectories, since these positions will likely influence how students interact and what they learn. Teachers may alter the arrangement of desks in the classroom, their own positions in the classroom during discussions, and the placement of reference objects, such as diagrams or three-dimensional objects (e.g., bottles). For example, Kathryn and I conjectured that the placement of reference objects could be a useful tool for shifting attention away from the teacher and helping students focus on each other's words. Our interactions with students supported this conjecture and, in subsequent learning trajectories, we incorporated object placement into our plans for classroom activities.

In addition to features of the process of facilitating discussion, I identified in the previous chapter three struggles associated with the process: managing internal dialogue, managing time and curricular constraints, and managing student expectations. The development of hypothetical learning trajectories is strongly influenced by the second and third of these issues. With respect to managing time and curricular constraints, teachers are frequently expected to devise hypothetical learning trajectories that move students from varying starting points to common endpoints in relatively short amounts of time. Interactions with students and reflective activities cause teachers to re-evaluate these trajectories, but revised trajectories must fit into ever narrower time frames. As this cycle continues, teachers may find it increasingly difficult to respond to time and curricular constraints and at the same time help students develop understandings beyond the memorization of procedures.

When teachers hypothesize about the learning process, they imagine students engaged in various activities, such as solving problems, listening, making suggestions, and debating ideas. Students, however, have their own expectations about the content and process of learning mathematics. Through their interactions with students, teachers come to know the extent to which students' expectations differ from their own. As teachers' knowledge of students' expectations grows, hypothetical learning trajectories can be revised to incorporate or address these expectations.

The development of hypothetical learning trajectories can be improved through activities that support the process of facilitating discussion (e.g. collaborating on instruction or observing other teachers leading discussions). Through observations and reflections, teachers can delve more deeply into the process and content of discussions and use their understandings to improve their hypothetical learning trajectories. In addition, teachers' hypotheses about the learning process should contain references to their own behaviors, since these behaviors influence student learning. With respect to whole-class discussion, teachers may identify patterns in their own speech, such as repeating students' comments, that may inhibit discussion. Teachers can also identify or learn ways of interacting with students that may support discussion. When teachers consider the impact of their own actions on discussion, they increase the likelihood that learning through discussion will occur as they hypothesize.

Although whole-class discussions occur during the “interactions with students” phase of the Mathematics Teaching Cycle, consideration of their placement and focus occurs during the development of hypothetical learning trajectories, as does consideration of teachers’ and students’ roles during discussion and the physical positions of people and objects. Management of time and curricular constraints, and students’ expectations, must be considered as well. After teachers have put their hypotheses to the test and engaged students in discussion, reflective activities, such as those Kathryn and I engaged in, help teachers make sense of the discussion process and decide how to alter their hypothetical learning trajectories.

Situating the process of facilitating discussion within a framework for thinking about the larger process of teaching seems useful to me for several reasons. First, we can think of the process of facilitating discussion not as separate from other teacher activities, but rather as occurring within and through the teaching process. Second, we can fit examinations of other aspects of teaching within the cycle and explore relationships and connections more easily. Finally, as suggested by Simon (1997), we can think of the teaching cycle not only as an encompassing framework, but also as a tool for refining a single aspect of teaching, a single aspect of the process of facilitating discussion. Having situated the process of facilitating discussion within the larger process of teaching mathematics, I now consider the relationships among my findings and the literature on classroom discourse.

The Process of Facilitating Discussion and the Literature on Classroom Discourse

Kathryn and my efforts to facilitate whole-class discussion were informed by the literature on classroom discourse. We familiarized ourselves with traditional patterns of classroom discourse and strategies for altering patterns of discourse (discussed in Chapter 2), and we used frameworks and concepts suggested in the literature for judging the quality of discussions to help us reflect and improve. In the following paragraphs, I describe how particular aspects of the process of facilitating discussion fit within and inform the literature on classroom discourse.

I have suggested that making decisions about when and how to incorporate whole-class discussion into classroom activities are important aspects of the process of facilitating discussion. Teachers' beliefs about the nature of classroom power and authority relationships, the nature of mathematical activity, and the nature of the learning process influence their decisions, as do external concerns such as time and curricular constraints and students' expectations. Teachers may agree that discussion can be a valuable tool for learning mathematics, but vary widely in terms of when and how they make use of discussion in the classroom. The interplay among teachers' beliefs, external concerns, and the incorporation of discussion into classroom activities is not well understood and may be a valuable focus for future research.

When teachers encourage students to develop ideas and opinions, and when they engage students in ways that motivate discussion, students may come to view discussion as an integral part of the learning process and a legitimate mathematical activity. The importance of motivating a need for discussion, as part of the process of facilitating discussion, cannot be overstated, as motivation influences the extent to which students share, listen, question, and debate ideas. When discussion is well-motivated, students engage not only because the teacher requires it, but also because they see discussion as beneficial to their own learning (see Chapter 2, "the ritualization of discourse," for a description of what can happen when this is not the case). Part of Kathryn and my difficulty with motivating discussion likely occurred because of a mismatch between instruction and assessment. Kathryn and I engaged in informal assessment as students worked in small groups and during whole-class discussions, and this type of assessment was particularly useful as we planned for instruction. The formal assessments we used, however, and upon which Kathryn partially based students' grades, sometimes did not capture or reflect the everyday classroom activities in which students engaged – such as small group work and whole-class discussion. This was particularly true of the departmental semester exam, which emphasized decontextualized symbol manipulation, rather than sense making. If we had been better able to align instruction and assessment, some aspects of the process of facilitating discussion may have become less problematic (e.g., managing students' expectations, motivating listening, and managing time and curricular constraints).

I have suggested that facilitating discussion involves engaging students in sharing and listening, questioning and responding, and agreeing and disagreeing and that motivating a need for these activities is crucial. In addition, teachers can utilize various strategies for altering patterns of discourse (discussed in Chapter 2) including sharing authority over the content and process of learning, developing norms and expectations for discussion, and using discourse-enhancing strategies, such as establishing participant frameworks or providing think time. Kathryn and I focused mainly on establishing norms and expectations for discussion and using discourse-enhancing strategies. In addition, even though we did not explicitly focus on sharing authority with students, we frequently allowed discussion to move in directions suggested by them.

Recent research suggests that teachers interested in using discussion as a tool for learning mathematics will need to set up appropriate norms and expectations for discussion in their classrooms. Suggested strategies for establishing norms and expectations include (a) requiring that everyone participate (this need not mean that everyone talks), (b) pointing out when students refer to each other's ideas, (c) holding students accountable for learning from discussion, and (d) reflecting with students about their behaviors both before and after discussions (these suggestions are drawn from the work of Manouchehri & Enderson, 1999, and Sherin et al., 2000). Kathryn and my efforts to establish norms and expectations began with an opening day activity designed to bring forth students' visions of mathematics classrooms and classroom activities, and communicate our vision for the classroom to students. We also demonstrated to students that if we repeated their words, they would not need to listen to each other, and asked them to help us remember not to repeat. Subsequently, our main strategies for establishing norms and expectations were to make whole-class discussion a regular and necessary part of the learning process, and to use discourse-enhancing strategies, such as wait time, to encourage talk. In retrospect, Kathryn and I realize that, although we attempted to set up appropriate norms, we probably should have spent more time explicitly sharing and reviewing our expectations with students. The fact that students' expectations became problematic as the semester progressed suggests that norms and expectations had not yet been completely negotiated.

Kathryn and I made extensive use of discourse-enhancing strategies such as using wait time, redirecting students' questions back to the class, and presenting and positioning students' ideas. These strategies were particularly useful for encouraging students to question, and agree or disagree, with each other. They were less useful for encouraging sharing, and generally not useful for encouraging listening. Our experiences suggest that when students have developed ideas and opinions, and have some desire to engage in discussion, discourse-enhancing strategies can be useful for encouraging students to respond to and debate with each other. On the other hand, if students' have not developed strong opinions or do not feel that a need for discussion exists, discourse-enhancing strategies are not sufficient for encouraging discussion. Further research is needed to confirm these conjectures and to better understand when various strategies for encouraging discussion are most useful.

Mendez (1998) developed six dimensions along which one can judge the quality of a particular whole-class discussion (Figure 2, p.37). Kathryn and I used these dimensions, particularly the levels of justification and building, as tools for reflection. During discussion-focused meetings, we frequently looked through the transcripts of particular discussions for instances in which students explained, disputed, or built upon each other's ideas. We then discussed what might have caused students to engage (or not engage) in these behaviors and how best to encourage students to use each other's words as "thinking devices" (Wertsch & Toma, 1995). Reflective activities such as this were a regular part of the teaching process as well as an important part of our efforts to facilitate discussion. More research is needed, however, to determine the relationships between various types of reflective activities and teachers' ability to facilitate discussion.

In summary, our efforts to facilitate discussion were influenced by descriptions in the literature of purposes and patterns of classroom discourse, and strategies for altering and judging the quality of discussion. Analysis of our experience with facilitating discussion suggests that altering one's own behavior during discussion and utilizing discourse-enhancing strategies are important, but not sufficient, for engaging students. Teachers must also engage students with tasks in ways that allow opinions to develop and motivate a need for sharing and debate. Researchers suggest that a high level of effort may be needed to establish appropriate norms and expectations for discussion; our experience supports this position. Further research is needed that examines how the process of facilitating discussion is influenced by teachers' beliefs, how teacher reflection influences efforts to facilitate discussion, and how and when various discourse-enhancing strategies may best be used.

Reflections on the Process of Facilitating Discussion in Mathematics Classrooms

As an insider in this study, I have had the opportunity to plan, lead, and reflect on whole-class discussion, as well as analyze and report on our experiences. My goals were both to describe the process of facilitating discussion and associated struggles and supports, and to grow as a mathematics teacher and mathematics teacher educator. It was my hope that I would become better at facilitating discussion and at helping others to facilitate discussion. In the following pages, I reflect on my work and these goals.

I was surprised at what a complicated and difficult process it is to use whole-class discussion as a tool for learning mathematics. Each part of the process presented particular challenges. Deciding when and how to incorporate discussions required thinking both about students' engagement with tasks and when talking as a class about our work would be most beneficial. Once we had imagined the sequence of events, we considered how, with respect to the particular task and goals for discussion, to best bring forth students ideas. Would we depend on students to share? Would we collect ideas and present them to the class? Would students present their ideas more formally? We also had to imagine how discussion might proceed, that is, what ideas might come up and what we might say to encourage students to question and build upon each other's ideas. We considered whether students would see a need for discussion or care about how other students were thinking. Should we incorporate some accountability measure, we wondered, or would exerting that type of pressure inhibit rather than support discussion? The issues I have mentioned are just a sample of those we thought about as we planned for instruction.

Kathryn and I knew that we needed to be both planful and responsive. We needed to think about which mathematical ideas might arise and what mathematically valuable turns the discussion could take, but we also needed to allow students' ideas and interests to influence the content and process of discussion. At the same time, the pressure of time and curricular constraints increased as the weeks went by. Sharing authority with students for the content and process of learning was important to us, but the content students' chose to focus on did not necessarily match the content Kathryn was assigned to teach.

Sharing authority for the process of learning was difficult as well. Some of the students wanted a much more traditional learning environment than either of us was comfortable with. I believe strongly in inquiry-based instruction, but students believe strongly, too. Teachers must respect, and respond to, students' expectations and, at the same time, engage students in activities that are likely different from their previous experiences. This requires great finesse since pushing students too far can cause them to disengage with tasks and resist our attempts to engage them in discussion. Perhaps had we been more successful in establishing norms and expectations at the beginning of the school year, we would not have had this problem.

Understanding and managing the ebb and flow of classroom activities and interactions involves attentiveness to many variables and the ability to make appropriate decisions moment by moment. Responding to and incorporating students' ideas and expectations is a critical part of the process, as is attention to the mathematics under investigation. Some researchers have begun to describe the complexities of this process (e.g., Chazan & Ball, 1999), but it remains an important area for further study.

Keeping in mind and deciding when and how to use various strategies for enhancing classroom discourse was also challenging. Our habits of teaching are often just as deeply ingrained as students' habits of learning. When we attempt to alter our habits, we are likely to be especially conscious of them and struggle with heightened levels of internal dialogue. Our careful, thoughtful efforts to plan and set up discussion contrasted with our experiences as we led discussion. As I described in the previous chapter, a multitude of thoughts raced around in our heads, and we found it difficult to make quick decisions about what to say and do.

Facilitating discussion was not easy for us. This is not to suggest that the process was not worthwhile. Engaging students in discussion helped us learn about students' ways of thinking and, although it was not the focus of my research, seemed to help students develop deeper and more meaningful understandings of mathematics. Over the course of the semester, we developed a sense of the process of facilitating discussion in all its complexity and many ideas about what to do and not do. Were we to collaborate again, we would spend more time explicitly working with students to establish norms and expectations for discussion, we would push students harder to take positions and achieve consensus, and we would frequently engage students in writing about discussion or, in some other manner, hold them accountable for learning from discussion. In terms of learning about students' ways of thinking, we found that their thinking varied greatly. Students sometimes had only partial or very limited understandings of the most basic concepts. This troubled us greatly, but also helped us to make better decisions about instruction. Traditional methods of teaching seem to mask these issues; many teachers may not realize how much their students do not understand.

We engaged students in discussion because we believed that they would benefit from sharing, listening to, evaluating, and building upon each other's ideas. We were surprised by how much we benefited from discussion, in terms of understanding students' ways of thinking. Our knowledge of students' mathematics increased markedly through our efforts to facilitate discussion. Teacher's knowledge of students' mathematics was not the focus of this study; however the ways in which teachers learn about and use the information they gain through whole-class or small-group discussion would seem a profitable area for further research.

In addition to developing knowledge about the process of facilitating discussion and better understanding my own strengths and weaknesses as a discussion facilitator, I hoped this research would push my thinking about how to assist other mathematics teachers in using whole-class discussions as a tool for learning mathematics. Of course, it is my hope that the descriptions of the process of facilitating discussion I have created will be of some help. Kathryn indicated how useful it was to her to sit in the back of the room and watch her own students engage in discussion as I facilitated. This type of activity may be useful as a professional development tool; however, partnerships of this type can be intensive and time consuming. More research is needed to understand the extent to which Kathryn and my experience will translate to other settings.

I cannot state strongly enough the benefit to me both as a mathematics teacher educator and as a mathematics education researcher of going back into the high school classroom and teaching mathematics. Although researchers gain and lose something no matter how they position themselves (see Chapter 3 for a discussion of how I managed my position), I felt strongly the benefits of the insider position. It seemed especially helpful to me, as a researcher of the process of using a particular type of activity to learn mathematics, to combine engaging in the activity myself with observing someone else doing it. I was not merely told what the problems and struggles were, I experienced them for myself. Another way of approaching this study that could have been similarly beneficial would have been for each of us to teach one class period and observe the other. Romagnano (1994) conducted a study of this type and was able to juxtapose his experiences with those of his teacher/collaborator. I could not engage in that type of juxtaposition because Kathryn and I worked together to teach both class periods. However, we did frequently watch each other lead the same discussion with different groups of students. For example, Kathryn would lead a discussion in third period and I would lead a discussion on the same topic in the following period.

In the same way it was useful to me to engage in teaching, it was useful, I believe, to Kathryn to think about the methodical examination of practice. Our discussion-focused meetings, in particular, provided us with opportunities to consider both the details of specific discussions and the scope and substance of the process as a whole.

Seeking not only to change the nature of events in the classroom, but also to understand them, can be an empowering experience for both teachers and researchers.

To conclude, let me revisit the goals and outcomes of this research. Having struggled myself to facilitate whole-class discussions, and having witnessed the struggles of others, I was interested in examining the process more closely. I chose to focus on the experiences of two teachers and looked across multiple data sources for themes or patterns in the teaching process related to facilitating discussion. What developed from this were the features, struggles, and supports, of the process of facilitating discussion that I discussed in Chapter 4. I hope that the results presented here will be useful to researchers investigating teaching, and teachers attempting to engage students in whole-class discussions. For me, this research represents a first attempt to make sense of the complexity of teacher actions related to facilitating discussion. I plan to follow up this work with an analysis of the interplay between specific tasks and ensuing whole-class discussions. Ultimately, I hope that work such as this may help bridge the gap between the vision of classroom discourse set forth by NCTM and others, and the struggles of teachers working to bring this vision to life in their classrooms.

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Appendices

Appendix A. Selected Problems from the Algebra 1 Semester 1 Exam

The problems below were selected as representative of all the problems on the exam. There were 56 problems on the exam and students were given 90 minutes in which to complete it.

Evaluate the expression $(4v + 3w)^2$ for v = -1 and w = 5.

A. 361 B. 121 C. –11 D. 241

Write an expression to represent the following: “ –4 times the sum of g and h.”

A. (-4)g + h B. –4(g + h) C. –4g + h D. –4(gh)

Simplify: 6x – 1 + 3x + 5.

A. 3x – 6 B. 3x + 4 C. 9x – 6 D. 9x + 4

Solve: 6x + 5 = x + 2.

A. $\frac{11}{6}$ B. $-\frac{3}{-5}$ C. $\frac{3}{5}$ D. $-\frac{5}{3}$

Find the slope of the line passing through the points A(-1, -5) and B(4, 8).

A. $\frac{13}{5}$ B. 1 C. $-\frac{1}{13}$ D. $\frac{5}{13}$

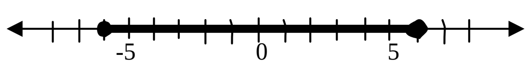
Solve: $|x + 6| = 2$.

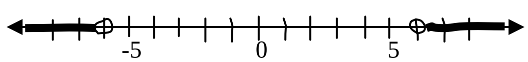
A. {8, 4} B. {-4, -8} C. {4} D. {-8}

Write an equation, in slope-intercept form, of the line that passes through the point (-2, 1) with slope –4.

A. y = -4x – 7 B. y = 4x + 1 C. y = 4x – 7 D. –4x + 1

Graph: $|x + 1| \geq 5$

A.


B.


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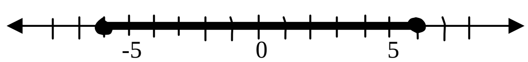

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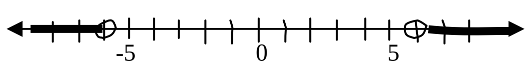


A rental car agency charges $13 per day plus 8 cents per mile to rent a certain car. Another agency charges $17 per day plus 6 cents per mile to rent the same car. How many miles will have to be driven for the cost of a car from the first agency to equal the cost of a car from the second agency?

Make a table and graph y = 3x – 2.

Appendix B. Task Descriptions

Typical/Ideal Mathematics Classroom Activity

Students are given several cartoons depicting various types of working environments and classrooms such as:

A classroom with students sitting in rows and the teacher pointing and the chalkboard
A doctor consulting with a patient
A group of people holding a discussion around a conference table
A laboratory with some people working together and individually on a variety of tasks

Students write up and share responses to the following questions:

1. Which cartoon shows a typical math classroom? Explain you choice.
   Describe the roles of the teacher and the students in the cartoon you selected.
2. Which cartoon best depicts your vision of the "ideal" math classroom? Explain your choice.
   Describe the roles of the teacher and students in the cartoon you selected.*

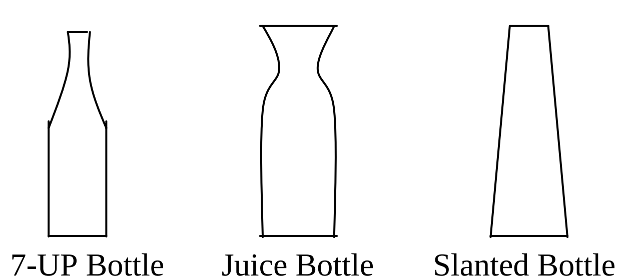
Adapted from Fleener, M. J., Dupree, D. N., & Craven, L. D. (1997). Exploring and changing visions of mathematics teaching and learning: What do students think? *Mathematics Teaching in the Middle School, 3*(1), 40-43.

<u>Developing Graph Sense</u>

Bottle Filling Experiments

Activity 1

Three bottles are presented to the class, two cylindrical (except for the top) and one that slants inwards. We ask the class is asked to think of mathematical questions that they might ask about the bottles. Students are also asked about what similarities and differences they can notice in the shapes of the bottles. After students share their ideas, we tell them that they are going to investigate the relationship between the number of scoops of water in the bottle and the height of the water.



Next, we lead students in setting up the graph for the experiment. We graph the number of scoops of the horizontal axis and the height of the water (in cm) on the vertical axis. We measure the height of the tallest bottle and estimate the volume of the largest bottle to determine appropriate scales for each axis. We also encourage students to label the axes and later we label each of the three graphs and also title the graph.

We ask groups of four students to come to the front of the room to fill each bottle. One student pours the water, one student measures the height of the water, one student plots points on the graph (on the overhead projector), and one student records the information on a chart on the chalkboard. Students at their seats have charts and graph paper on which they record the data. Students use three different color pencils so that we can put all three graphs on the same set of axes and easily compare them.

Using a bowl full of water (colored with food coloring), a half-cup measuring scoop, a funnel, and a ruler, one student adds water to the bottle one scoop at a time while another student measures the height. Before each scoop is added, we ask students to predict what the change in the height will be based on the graph, chart, and bottle shape. After the last scoop is added, we discuss whether or not to connect the dots and how to connect them (smooth curve or straight segments between the points).

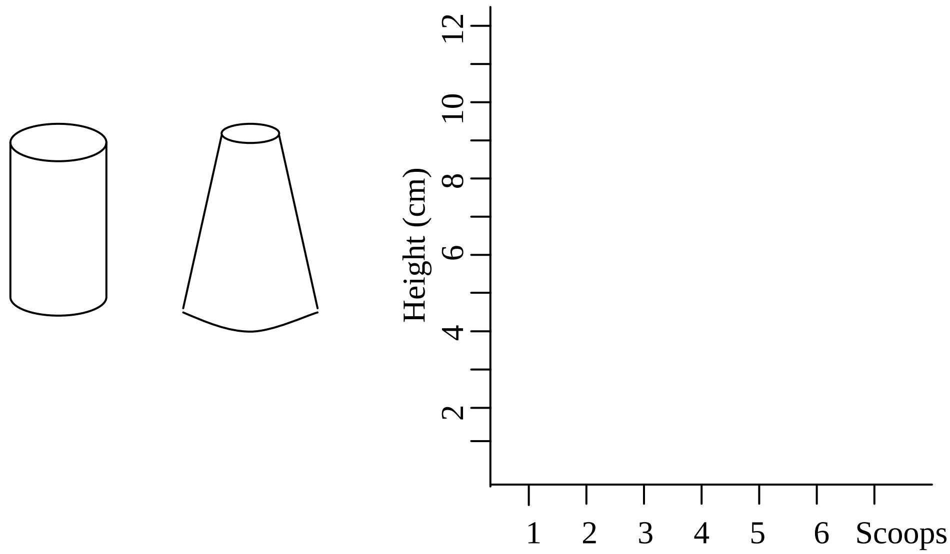
After graphing the first bottle, students are asked to think about what the graph of the second cylindrical bottle will look like (both shape and location with respect to the first bottle's graph). Similarly, after graphing the second bottle, students predict the shape and location of the graph of the third bottle. Finally, similarities and differences among the shapes of the three graphs and bottles can be discussed.

Activity 2

Task: Graph the data for each bottle. Examine the graphs and the bottles. Decide which bottle is bottle A and which is bottle B. Explain your reasoning.

Note: Students first matched on their own, then talked in groups, then wrote their explanations, and now are going to discuss the problem as a whole class)

Bottle A		Bottle B	
Scoops	cm	Scoops	cm
1	2.5	1	1.5
2	4.5	2	2
3	6.5	3	3
4	8.5	4	4.5
5	10.5	5	6.5
6	12.5	6	9

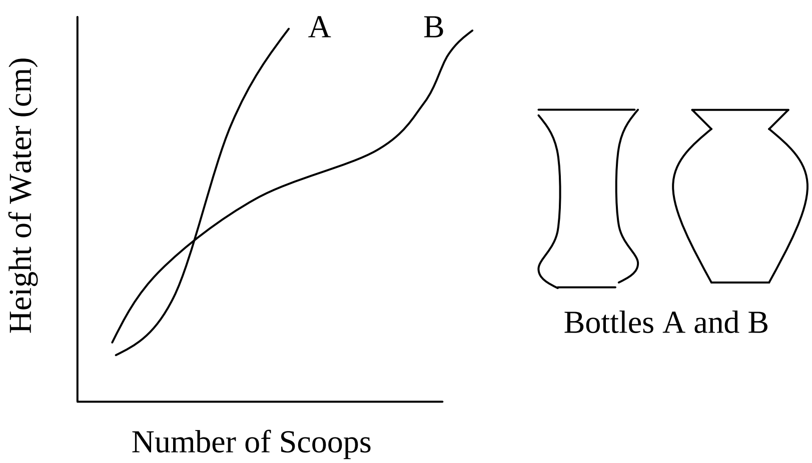


Activity 3

Small groups of students are given a bottle and asked to predict and sketch the graph (scoops of water versus height of water) in as much detail as possible. Students then fill the bottles with water and sketch the actual graph on the same paper as their predictions. Finally, students compare the two graphs and describe how they differ and why.

Activity 4

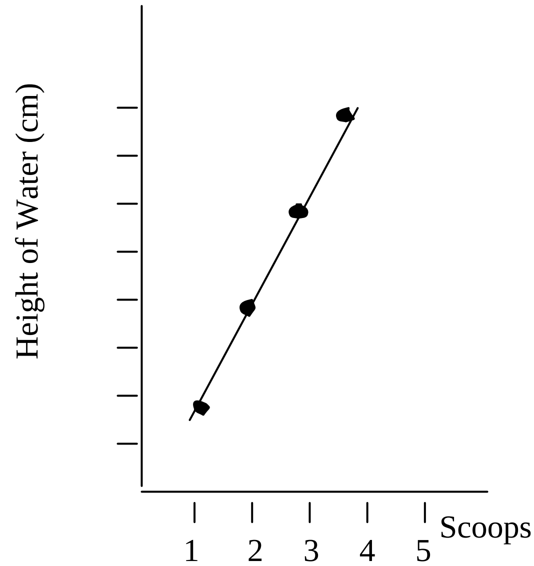
Students engage in a think-pair-share activity to decide which bottle is A and which bottle is B and to identify various characteristics of the graphs that can be used to help make this decision.



Bottles A and B

Activity 5

Students are given the graph shown at right depicting the number of scoops of water versus height of the water in a certain bottle. Students are asked to describe the rate of change of the water in the bottle as scoops are added. The teacher lists students' descriptions and the class discusses different methods for representing rate of change.



CBL Motion Detector Experiments

A. For each of the situations below, students sketch a graph and discuss in small groups what a walker must do to obtain the graph. Students then take turns walking in front of the motion detector. Students write descriptions of what the walker had to do to obtain the graph.

2 Line with an upward slope
3 Line with a steeper upward slope
4 Line with a downward slope
5 Line with a less steep downward slope
6 A horizontal line
7 A U-shaped curve opening up
8 A U-shaped curve opening down

B. Students are asked to think about the graphs they have made and answer the following questions.

0 What does a graph look like when a person is not moving?
1 How can you tell from a graph whether a person is walking or running?
2 How can you tell which direction a person is moving from the graph?

How can you tell where a person started from by looking at the graph?

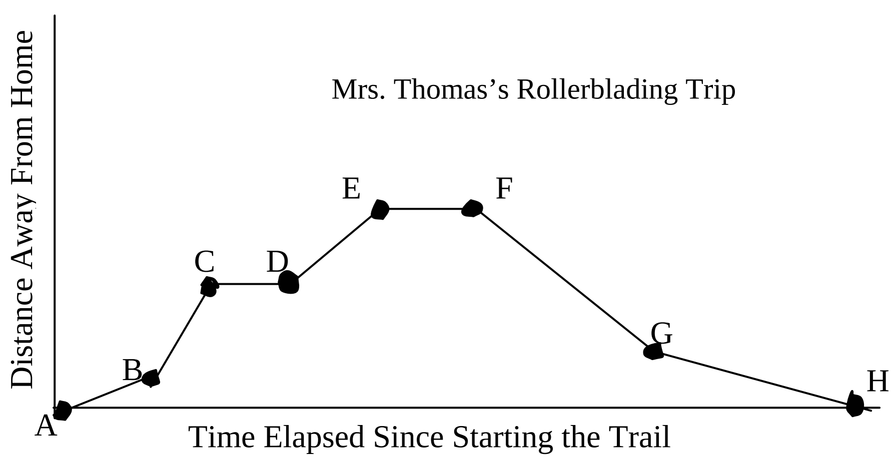
Mystery Graphs Activity

Small groups of students are each given a different real-world situation, such as “The number of in (fill in the name of a nearby city) watching television.” or “The number of people at the McDonald’s drive through window.” Each group of students is to create a graph that represents the given situation. Students are told that the graph must show a 24-hour period (midnight to midnight) on the x-axis and number of people on the y-axis (so that the graph shows number of people as a function of time). Students draw their untitled “mystery” graphs on butcher paper and post them around the room. Next, students are given a list of all the given situations and attempt, in their groups, to match the situations and graphs. Discussion about which situation matches which graph and why concludes the activity.

- The number of people in the South Jackson High School hallways on a school day.
- The number of people at South Jackson High School on a school day.
- The number of people at movie theaters in the U.S. on a weekday.
- The number of people on school buses in the U.S. on a school day.
- The number of people asleep in the U.S.
- The number of people at McDonaldj’s restaurants in the U.S.
- The number of people watching TV in the U.S.
- The number of people in the South Jackson High School cafeteria on a school day.

Analyzing Graphs

Activity 1



1. As I skated from Point A to point E, between which two points was I skating the fastest?

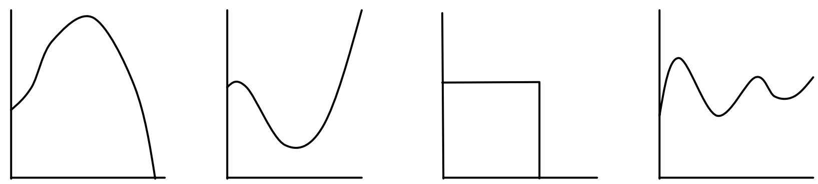
2. What was I doing between points C and D?
3. Why does the graph start to come back down again at point F?
4. Was I skating faster between points F and G or between points G and H?
5. Write a story about Mrs. Thomas’s Rollerblading Trip. Be creative. Be sure to write about each segment of the graph.

Activity 2

Adapted from Van Dyke, F. (1998). *A visual approach to algebra*. White Plains, NY: Dale Seymour.

Students select among several graphs the one that best matches a given situation and then discuss their answers in small groups and as a class.

Example: An animal population first increased then decreased until the animals became extinct



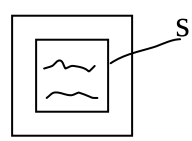
a b c d

[time is on the x-axis and population in on the y-axis]

Writing Equations

Pool Tiling Problem

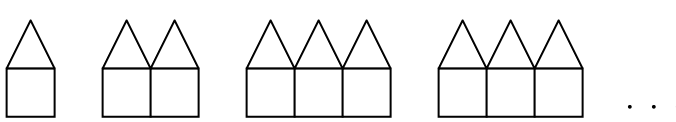
A square pool has sides of length s feet. There is a border of tiles (1 square foot each) around the outside of the pool. Write an equation that will allow you to find the total number of tiles needed for any size pool.



Shape and Dot Patterns

Students are given patterns of shapes such as those shown below and are asked to write equation that could be used to determine some specified characteristic of any shape in the sequence.

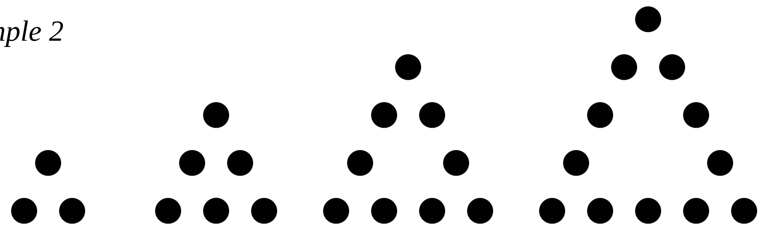
Example 1



Shape Number = 1 2 3 4 . . .

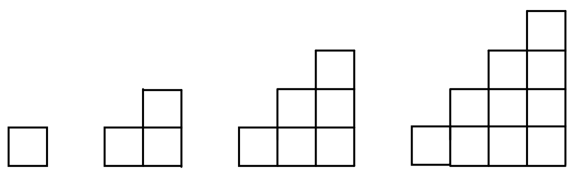
Write an equation that allows you to find the perimeter (p) of the figure for any given shape number (n).

Example 2



Write an equation that describes the relationship between the number of dots (n) on one side of a dot triangle and the total number of dots (t) in the triangle.

Example 3 – Staircase Problem

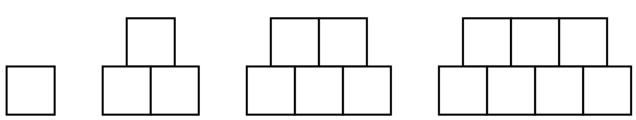


The drawing shows staircases 1, 2, 3, and 4 steps high. How many blocks would be needed for a staircase 100 steps high? For n steps high? Write an equation that you could use to find the number of block for a staircase with n steps.

Mystery Equations Activity

Small groups of students develop their own "growing" shape patterns and then write equations to represent the relationship between two quantities that vary in their patterns (see examples below). Students are asked to use x and y as their variables.

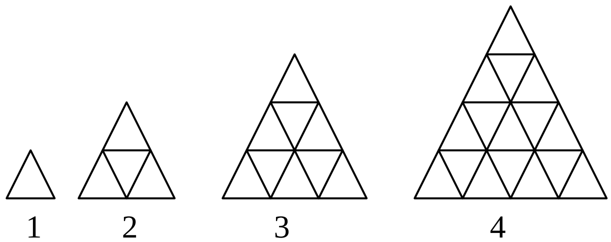
Example 1



x = side length (across the bottom)
y = total number of squares

$y = x + (x - 1)$

Example 2



x = shape number
y = total number of lines

y = ((n(n+1)/2) times 3

Students put their patterns and variable definitions on butcher paper and post around the room. The teacher creates lists of students' equations and the students try to match each equation with its "mystery pattern." A whole-class discussion of which pattern matches which equation and why concludes the activity.

Equation and Graph Relationships

Classifying Equations Project

Students are given a set of 64 linear and nonlinear equations. Students, working in pairs, are asked to think about what the graph of each equation would look like and then put equations that they expect to have similar graphs into groups. Next students graph the equations and reanalyze their groupings.

Accuracy in graphing was important if students were to analyze characteristics of the graphs such as slopes and intercepts. At the same time, we did not want students to spend a large amount of time creating their graphs. Our goal was to engage students in thinking about a range of graph characteristics and developing connections among operations and numbers in equations and the shapes and locations of graphs, not to engage them in practicing graphing. Therefore, we had students use graphing calculators to create their graphs and asked them to copy their graphs onto small grids we gave to them (with appropriate axes and scales provided). Also, students worked in pairs and were thus able to share the creation of graphs with a partner.

Although Kathryn and I were pleased with this project in general, we did feel that it took too long for students to begin reanalyzing their groups after graphing. Reflecting on the project, we agreed that some alterations would need to be made were we to use this project again in the future.

Writing Equations for Lines Given Data Points

Students are given a few collinear data points in a simplified context and are asked to find an equation that describes the relationship among the quantities.

Example 1

> Right now I am 2 miles from home. In 1 hour, I will be 5 miles from home. In 2 hours I will be 8 miles from home. Graph these points (with time from now on the horizontal axis and distance from home on the vertical axis). If the trend continues, plot a few more points that would be on this line. Write the equation of this line, define your variables.

Example 2

> Two days ago the temperature was -6 degrees, yesterday it was -5 degrees, and today it is -4 degrees. Assume the trend continues and find an equation that describes the relationship between the day and temperature.

Matching Activity (Graphs, Slopes, Intercepts, and Equations)

Each students is given 3 x 5 card with one piece of information on it (either a point, a graph of a linear equation, a slope, or a y-intercept). Students must find the three other people whose information fits with their own (i.e. the point and y-intercept are on the graph and the slope and graph match). Each set of students then works together to write the equation that goes with their information.

Solving Equations in One Variable (1- and 2-Step)

Analyzing Bones

This activity was adapted from Lappan, G., Fey, J. T., Fitzgerald, W. M., Friel, S. N., & Phillips, E. D. (1998) *Moving straight ahead: Linear relationships*. .Menlo Park, CA: Dale Seymour.

Students are given a set of equations (used by forensic scientists to estimate heights) that show the relationships among the lengths of various bones and the height of males and females. Students are asked to use the equations to find heights given bone lengths and bone lengths given heights. Below is one example.

Males: $H = 27.5 + 2.24\,F$ $\qquad$ H = height in inches
F = femur length in inches

If a man is 65 inches tall, about how long is his femur?

It is important to note that this problem was not given to students after they had learned a procedure for working backwards to solve for F. Rather, students had to figure out a method for finding F given H. Some students had learned a procedure in pre-algebra, but many had not or did not remember it.

Appendix C. Whole-Class Discussion Transcripts

<u>8/17/00 Period 3</u>

Discussion of ideal math classroom led by Kathryn

(Kathryn asks students to raise hands if they picked #1, #2, etc.)

Kathryn: So, number three, we have a couple of people that picked number three. Would you like to share with us why you picked number three?

Kathryn: Uh, Tonya?

Tonya: No

Kathryn: No. Any thoughts about why you picked that one, uh, Joey?

Joey: I picked it because, uh, I don't know, it was my choice.

Kathryn: Why is it your choice?

Joey: I don't know.

Kathryn: What did you write on your paper?

Joey: It's just, it's better than a normal classroom (inaudible)

Kathryn: Ok. Tonya, do you want to add anything to that?

April: I'm April.

Kathryn: April. I'm sorry.

April: No, not really. I just think it would be a lot more funnier than just sitting here and like taking notes off an overhead or something.

Kathryn: Alright. Thank you. I think that was it for number three, right? How about number four? Someone who selected number four as the ideal classroom would you like to share with us why you picked that one? (Tonya raises hand). And you're Tonya.

Tonya: Uh, I picked it because it seems like everyone's on task, doing there own thing, working at their own pace, so the teacher is basically helping the other student on the same problem or question. Everyone's working hard. Stuff like that.

Kathryn: Alright. Someone else like to add to what Tonya said? (Laura raises hand)

Laura: Uh, I picked number four because (inaudible)

Kathryn: Ok. Alright. Anyone else like to add to that? Rob.

Tom: It's more of a hands-on activities, kind of, you know, they're not just sitting there like looking and answering a problem, just on paper and things, they get to actually do the problem with the actual things instead of, and the teacher instead of just standing in front talking to everyone is helping them and doing the problem with them.

Kathryn: Anyone else? Who picked number four, anyone else want to add anything that was a different thought they'd like to contribute? (2 sec wait time) Alright. Number five I know was a popular one, so. Jackie.

Jackie: Uh, I chose that one because I think it would be easier to learn 'cause the teacher is basically showing step-by-step how to do whatever they're learning. And I think that it's easier and the kids look more attentive in the class.

Kathryn: Anyone like to add to that? Other reasons why you picked number five? (3 sec wait time) Lots of you picked this one. (6 sec wait time). You all basically agree with what Jackie said? Did you say something different? (2 sec). Number six, there were a few people who selected number six also. Would someone be willing to share why you picked number six? (Brad raises hand) Brad.

Brad: Because clowns are funny. Because they're doing their work and they're learning but they're still having fun while they're doing it.

Several students overlap:
They look like they're-
They're yelling at each other-
Yeah

Brad: How are they yelling at each other, man?

Several students overlap:
Look at him pointing his finger-
And the guys like slamming his hand down-

Brad: Well that's what you-

Kathryn: Ok, Jackie. Let's listen to Jackie. What were you saying?

Jackie: Oh, that they're pointing at each other right? Telling each other what to do basically and arguing and you're not going to get any work done by doing that. That's my personal opinion.

Kathryn: I thought someone else was disagree with what Brad said. Was there someone else who was responding to what Brad said? (1 sec wait time). Alright, uh, Sarah?

Sarah: I think it would be an ideal classroom because, uh, it seems like there's no teacher and if there was, uh, the teacher's like interacting with the students, uh, and the students are like all clowns so it will be fun (laughter) And it looks like they're learning and discussing problems, that looks fun.

Tonya: Well like, uh, like any discussion people are going to be disagreeing, so that's what they're doing probably, having a discussion, they're disagreeing and trying to make their point.

Kathryn: Uh, Ben, you look like you're dying to say something.

Ben: I said because I like clowning around.

Kathryn: That's why you picked that one? (Ben nods). Anyone else? (five sec wait time). When we started this activity yesterday I shared with you that one of the things that I hoped would come out of it and I think I have, I've learned quite a bit about you, I think as, uh-

Student: We don't know nothing about you

Kathryn: You don’t know anything about me?

Student: Except for you’re the teacher and we’re not supposed to make you mad.

Kathryn: Oh, (laughs) you think that’s what the rule is? You’re not supposed to make the teacher mad?

Student: You’ve got those green sheet things to send us to the office.

Kathryn: So, I’m the boss, huh? Uh, and I did tell you that the other thing that I wanted to get out of this is I also wanted to share with you what some of our expectations may be of the class. So, uh, I guess it’s interesting to me to see the choices that you selected for the ideal classroom and I’m not surprised that there was a variety of answers. And if you look at number 5, many of you picked that as the typical math classroom, you also picked it as the ideal math classroom. Four and six, as I listened to the reasons why you picked it, I heard some of the same reasons, that you saw the students working together and talking and working in groups, and you saw that as the ideal situation, that you got to kind of explore, I think I heard someone say hands-on, uh activities. It used to, when I was in school, even when I started teaching, I started teaching twenty years ago, that pretty much every classroom looked like number five. The teacher stood in the front, the students were in rows, and they worked and listened.

Student: Yeah, but twenty years ago they would beat you with a ruler.

Kathryn: They would beat you with a ruler?

(laughter)

Kathryn: Let’s see, twenty years ago when I started teaching you couldn’t do that-

Student: They still do, at my cousin’s-

Kathryn: Where do they?

Student: At my cousin’s school they hit them with a ruler.

Kathryn: Do they? Yeah, there may still be some schools, most schools you can’t. But about ten years ago, about ten years ago the National Council of Teachers of Mathematics, which is the professional organization for math teachers, came out with a set of standards. They wanted to look at what was going on in math classrooms and look to see if what we were doing was effective or not. And at that point they did start encouraging teachers to do more hands-on type activities with their students. But, as you can imagine, it’s hard to change. Teachers who had been teaching for a long time, it’s hard to all of a sudden do something different in your classroom. So change has been taking place slowly. So there still are a lot of classrooms that look like number five, but there are also more classrooms that are starting to look like four and six, so as I said, it doesn’t surprise me that maybe in your experiences when you picked the typical math classroom that you picked some that looked like four and six or you picked one that looked like five. And then as far as your ideal math classroom, everyone had different ways that they feel that they learn best. Some of you feel like you learn best in a classroom situation that looks like five, well organized, everybody’s listening, some of you feel that you can learn better in the other situation. And what our goal is, I think, is to, uh, recognize the fact that you have different ways that you feel like you learn best and recognize the fact that you know some of you said that you learn better like number five, one better with four or six, that we’re going to try to address those. And so, uh, we are going to be trying to do activities where we get you actively involved. Ms. McGraw and I feel like if you’re actively involved that you ARE going to learn better and so we want to try to create situations where we get you actively involved, but then there are also going to be times when we have to talk about what we’ve done so there’ll be times when the classroom looks like number five also. So it’s not going to be one of all and none of the other. So that's kind of I guess like I said what we wanted to communicate to you, or what I wanted to communicate to you about the classroom and that this is a learning process for me too. I'm going to be doing some things that I haven't done before. Ms. McGraw is going to be helping me with that. So we're all going to kind of be learning at the same time and hopefully helping each other through it. Do you want to add anything?

Rebecca: I don't think so. Sounds good to me.

End of transcript

8/17/00 Period 4

Discussion of ideal math classroom led by Rebecca

Rebecca: Let's just get down a few of your ideas for what you put down for the ideal classroom. And I know different people picked different cartoons for their ideal classroom, but, but, that's ok. Let's see if we can get some ideas about what you'd actually LIKE to have in a math classroom. Who'll start us off? I'm just going to do a list. (2 sec wait time). Who's brave enough to go first?

Student 1M: I will-

Rebecca: And say something about what they wrote. Yeah?

Student 1M: Alright. Hands-on activities.

Rebecca: Ok. What cartoon number did you pick?

Student 1M: Four.

Rebecca: Number four? Which one was that?

Student 1M: The dogs.

Rebecca: The dogs? Ok. And you put down hands-on is what you would like. Somebody else? Who will? Yeah?

Student 2F: I said number four as well because uh I feel that, I liked it because all the students were working together.

Rebecca: Ok. Working Together (RM says it as she writes it on the overhead). Yeah?

Student 3F: I picked number four also. The teacher is working on (inaudible)

Rebecca: So one-on-one help, that would describe it?

(I imagine that Rebecca did something to nominate 4M- gesture or nod)

Student 4M: I put number four also and I said students all work at their own pace.

(Rebecca records students' response on the overhead)

Rebecca: Over here? Did somebody have their hand up? Not necessarily for number four. Mm hm?

Student 5M: I picked number five because (inaudible)

Rebecca: Which one was number five?

(Several students say ducks and chickens)

Rebecca: Ok, did anyone else pick number five? Yeah?

Student 6M: I chose it because the teacher is going step-by-step and

Rebecca: And that would be something that might be ideal is getting that kind of explanation where you know exactly what to do?

Student 6M: (inaudible)

Rebecca: So the teacher's in front? (RM writes this down) Anything else for number four or number five and then we'll move on to other ones? (1 sec wait time). Whose got something on their paper that would be an addition to either of those lists? (6 sec wait time) Or who can think of something to add.

Student 7F: Well four number four I put that the classroom looks like, well anyway, there's a window and right outside (inaudible)

Rebecca: Homey?

Student 7F: Yeah.

Rebecca: Not like a, you know, concrete square? With actual windows where you'd actually see some light. Anything else for number four or five that we could add to our picture of the classroom? (2 sec wait time). Who picked something else besides four or five? (10 sec wait time)

Student 8F: Uh, I don't know because I thought it was working at their own pace (pointing out something I wrote on the overhead that didn't actually represent what a previous student had said). Right here, right? Is that what you're saying? You're right. I was thinking it right but I wrote in wrong, I wrote it the exact opposite of, not everyone working at the same pace but each person. Thanks.

Student 8M: I picked number three because it was kind of an original teaching style, the game show.

Rebecca: Oh, ok, it was the game show one. (Rights students ideas on the overhead.) So it might be more interesting because it wouldn't be what you had every day in every class. Anybody else pick number three or one of the other- yeah?

Student 9F: I picked number three because it was like (inaudible)

Rebecca: Because? It was fun. Anybody else pick three or two or one. Was there a six? (20 second+ wait time). These look very different to me, what do you think, do you think you can have both? Yeah?

Student 10F: Uh, I like number four because it seems like you don't have to work on doorknobs all the time, it's not like, you don't just have to work on what you're supposed to be doing, in math sometimes it gets really frustrating, it seems like you're allowed to do something else (inaudible), it seems like one of those classrooms where you're not tied down and you don't have to sit there and write.

Student 11F: I think about number three, like I think there is some students work better when it is something they like to do instead of just (inaudible)

Rebecca: (writes on overhead) How's that? Does that capture it? (pause) Which classroom would you rather be in? I mean some of you picked four and some of you picked five so I think we've kind of established that some of you, but uh, what is it about five, let's ask this about each one, 'cause I think this is important, what is it about five that isn't good? I mean we put up about five what is good. Yeah?

Student 12F: (inaudible – something to the effect that some students might not ask questions)

Rebecca silently nominates students, several in a row, they aren't audible.

Rebecca: What about, yeah?

Student 13F: The teacher is up there explaining what's going on but maybe some of the kids just don't get it but you can't just (inaudible)

Student 14F: (Inaudible – some students might not ask questions)

Rebecca: How about number four, the dogs, is there anything you can think about the dog scenario that might not be so good, you know, for you if you were thinking about you in math class?

Student 15M: The teacher might not have as much control.

Rebecca: Chaos might break out.

Student 16M: Yeah, uh, if the problem (inaudible)

Student 17M: Uh, it seems like they're all just making the doorknob, that's all they're getting done, they aren't doing anything else, (inaudible)

Rebecca: That's ok, we can come back. Angel did you have something?

Angel: Well I was going to say they're all like on different paces like if they wanted to move on to another level, like the one who is still looking at the board, I mean he's not going to know what to do because he's still looking at the little graph thing that the teacher drew, and then the other, like this guy, they're all on different levels.

Student 18F: Sometimes (inaudible)

Rebecca: That is a big, big, challenge for teaching, if you try to have kids, uh, doing different things, if you try to have people working on different things so that they can be challenged but then you want to bring everyone back together. It is a challenge. Other things about number four that might be problems?

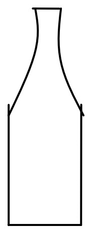
Student 19F: (inaudible)

Rebecca: Yeah, I think something that somebody back here said about, uh, sometimes it looked like the doorknob problem might be easy. Sometimes something else that's hard to do, in a classroom from a teacher's perspective, when a teacher is in front of the classroom doing that it can be hard to try to do a problem where everybody is for their level and everyone can understand and get it and you don't have people sitting their in silence not understanding, and that happens all the time, I know. And on the other hand, when you have everybody working on an activity more, you know, hands-on, and doing that at their own pace, then sometimes it's hard to make it challenging for everybody and not have it be, you know. . . . Ok. Does anybody want to add anything else to this? (2 sec wait time). Basically I think Mrs. Grant and I hope that in this class we'll get this going on (circle the number four list) and try to deal with some of the problems that can happen with this, with you all doing activities and being involved and not just sitting here all facing forward. And there will be some of this too (circle the number five list), I mean of course there is, I mean right now, right? I'm up here talking. So there will be some of this and there will be some step-by-step explanations, and, but, but I think that you'll see more of this one (number four) in this classroom than sometimes you see in math, but you'll also see some of that (number five). So I hope that everybody can be comfortable with the activities that we do in here and everybody can feel free to ask questions and talk and you don't have to feel like you just have to sit here in silence. Do you want to add anything?

End of transcript.

8/18/00 Period 4

Discussion of where second point on graph will go



Kathryn: Alright. Before we have Pete put in the next scoop, those of you who are sitting at your desks, what I'd like for you to do is just take your pencil, you're going to use it as a pointer, so don't write anything with it, and I want you to point to the spot on your paper where you think the next dot is going to go, just point, and I'm going to come around and see where you're pointing and then we're going to talk about some of the things I see as I walk around. (Kathryn walks around room)

Kathryn. Alright. These are some of the answers I saw as I walked around. (Kathryn puts row of dots on the graph). Would anyone like to argue for or against any one of those? Brittany.

Brittany: We've already done the first scoop.

Kathryn: We've already done the first scoop? Alright, so what is, so which point are we arguing for or against here? (2 sec wait time) Which point should we take away?

Brittany: The one above the one we did.

Kathryn: The one above the one we just did? We've already done one scoop? alright? So we'll take that one away. Adrianna, no wrong, Beth.

Beth: Take away the one on the zero.

Kathryn: On the zero? And why do you want me to take that one away?

Beth: We started on the one scoop

Kathryn: Alright, we already started on the one scoop, this one would stand for zero scoops. Uh, Angel?

Angel: (inaudible)

Kathryn: Take the three one away? Alright. Uh, I found it interesting that everyone who had the point that was at two scoops put it at eight. Somebody want to explain to me why you did that, why you picked eight? Amy.

Amy: (inaudible)

Kathryn: Ok. Alright. Let's give it a try. The nice thing about this one is we can actually do the experiment and see. Alright, so let's go ahead and put that next scoop in.

End of discussion.

Kathryn: Before we put the next scoop in how about if we just kind of throw out some numbers for this next one. Nine? Nine point five? Anybody else? Eight point five? Anybody else? Any other guesses. Would anybody like to argue against or defend any one of those numbers? We've got 8.5, 9, and 9.5.

Student: Eight

Kathryn: Eight you think? We've got another guess. Anyone like to defend or argue against any of those numbers?

Student: 8.5.

Kathryn: Eight point five? Why 8.5 you think?

Student: (inaudible)

Kathryn: Eight? Alright, why eight?

Student F: It just seemed like a good number.

Kathryn: It just seemed like a good number? Alright. How about the nine? Who said nine? Mindy?

Mindy: Uh, I didn't say that but I'm saying that because it looks like it's about to be constant so it should come up less.

Kathryn: It should come up what?

Mindy: Incrementally.

Kathryn: Incrementally?

Mindy: Yeah. The bottom was sort of like something that you wouldn't know because there could be different things at the bottom, but it looks pretty even, you know, until once you get at the top.

Kathryn: Ok, so incrementally, we just said just a minute ago that this went up two point five-

Mindy: So I'm thinking another 2.5.

Kathryn: Another 2.5? So if we went another 2.5 that would put us at nine? Right? How many of you agree with what Mindy just said? How many of you disagree with what she said? And how many of you are just not sure? (lots of hands go up) Alright, let's go ahead and put the next scoop in and see what happens.

End of discussion

Kathryn: Another question we might ask is now what are we going to do with this? Somebody asked earlier about, you know, are we going to make a bar graph, what were we going to do with this? And one question that comes up if you don't make a bar graph, because you have bar graphs and you have?

Students: Line graphs

Kathryn: Line graph? So the question might be should we turn this into a line graph, should we connect these points or should we not connect these points?

Students: Softly say connect and not.

Kathryn: Let's here some arguments one way or the other, either to connect or not to connect. Crystal? You said not connect? Is that what you said?

Crystal: Yeah. I don't think so because it goes straight up first and then all of a sudden it starts curving so I don't, I can't see it being connected.

Kathryn: Crystal, tell me a little bit more about why since, what I think I heard you saying was that these were kind of going along in a pattern and all of a sudden these are not in the pattern?

Crystal: Yeah.

Kathryn: So that's why you think not? Tell me a little bit more. (six sec wait time). And Mindy said well if just make sure you kind of go in order, dot to dot. So I guess that might be, what if I decided to connect and I thought about not connecting right in order dot to dot, what if I connected uh say these two, what if I hadn't looked, what if I hadn't collected all of this data in here? What if I had just put in one scoop and then quickly put in all the way up to eight scoops and this information?

Jessica: Uh, it would be inaccurate. Your saying that two scoops would be about seven but it should be six and a half. And at three scoops it didn't, hit twelve, and it would be inaccurate all the way up.

Kathryn: Ok. So, so it doesn't match the data we collected if we do that? But what if we followed Mindy's suggestion and went with connecting them in order and went right along, what if I connected say these two?

Student F: Connecting those?

Kathryn: Do you think we are inaccurate there? What Jessica was pointing out was when we did this she kind of went in the middle and she looked at well if we did that four scoops would say it was ten but we know that that's not right, we know that four scoops was twelve. What do you think about it here? What if we did the same thing that Jessica did, kind of pick a point – go ahead you wanted to say something

Jessica: I was going to say it's only doing two of the examples we had so it's still inaccurate. It's like starting in the middle and not having a beginning or an end.

Kathryn: Alright. Anybody have any thoughts about this one? If I can connect two that are right next to each other, the fourth scoop and the fifth scoop? (five sec wait time) What do you think about that? (seven second wait time) Is this OK? Is this not ok?

Student M: I think that if you did that that just represents a certain distance, if it's only certain points, because if you were to have any other ones with that distance you could make that line. So you could like label different things with different lines because of the distances between how many scoops (inaudible)

Kathryn: So, ok, I'm not sure I'm entirely clear. Are you thinking that that's ok to go ahead and connect or not ok to connect those two that are right next to each other?

Student M (same one): Well just to connect those, not ok, if you were to connect other ones that were the same distance apart

Student F: Dot to dot.

Kathryn: The same distance apart?

Student M (same one): Uh, with a, the amount of water, how much water was

Kathryn: So here you mean how much water was going up?

Student M (same one): Yeah

Kathryn: Same amount going in each time. So here you mean how much water was going up, the same amount going in each time, so here we were saying 2.5 and then we got 2.5 again, here we were getting still 2.5, that, am I understanding correctly on that?

Student M (same one): Yeah.

Kathryn: So you think that it is or is not ok to connect them?

Student M (same one): not ok.

Kathryn: Alright. And why again?

Student M (same one): because if you just connect that one it's inaccurate like Jessica says, if you're just connecting those you don't know.

End of transcript

8/21/00 Period 3

Kathryn: Remember we talked about the fact that we want to try to get some conversations going in class. So there are a couple things I'm going to ask as a favor of you, to help us getting conversation going, one is if you would raise your hand before you respond. And the reason for that is if everyone kind of shouts out answers, one thing, it's hard for me to hear, if I have answers coming from all different directions, and the other thing is if one person always shouts out the answer, then everybody else gets to kind of sit back and do nothing. So I want to try to avoid that. And I want to make sure that I have a chance to call on a variety of people through the class too. So when I ask a question, for the most part, and I may and if I am at a point where I want anybody to answer I'll give you a signal, I'll say "ok, just shout answers" or "anybody?" or something like that. But for the most part, when we're having a conversation, if you could raise your hand, I would appreciate that. The second thing is, Ms. McGraw and I talked about this the other day, I have a tendency, and this is a habit that is going to be hard for me break because I've been teaching for so long, I have a tendency to repeat what you said, and if I want you to talk to each other, if I repeat everything you say, then you don't need to listen to each other do you? So, for instance, suppose these two girls are having a conversation, say something-

Student (to student beside her): How are you doing?

Kathryn: How are you doing? Now say something (directed at second student)

Student (to first student): Good.

Kathryn: Good. Now do they have to listen to each other?

Students: No

Kathryn: No. And who are they going to tend to listen to?

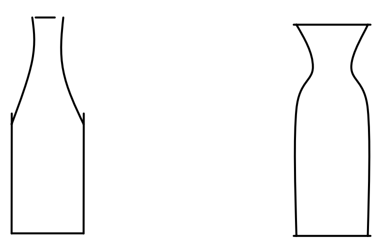
Students: You.

Kathryn: Me. Because I'm probably the loudest and I'm dominating, right? So if I want you to to talk to each other, then I want to try not to get into the middle of your conversation. There may be times that I'll want to make sure that everybody's heard something or I'll want to emphasize a point but I'm going to try really hard, and I'm going to ask you to help me too, I'm going to try really hard to not repeat everything you say. So what that means is you are going to have to talk loudly enough so that everybody in the class can hear. Because one of the reasons that I tend to repeat is students tend to talk a lot more softly than teachers do and if you say something very quietly I'm afraid that somebody else isn't going to hear it so I repeat it to make it louder. So if you make it loud enough to start with then I won't have to repeat. So this is something I'm trying to work for me, but what I need for you to do to help me out is to make sure you're talking so that other people can hear you. And I'll try to remember, I'll remind you, and I hope I don't drive you crazy reminding you to speak up, maybe if I just give you a little signal, you know like a thumbs up or something like that, alright, that'll mean speak up so that everybody can hear you.

(returns to conversation about connectedness and physically putting half scoops in the bottle and would it make sense)

(NOTE: Kathryn tells students to connect in straight segments, which is not correct? for curved graphs – slant bottle, where you would need to look at the overall shape of the graph and make a smooth curve)

(NOTE: in proceeding discussion the graph of the 7-up bottle that the class filled and graphed the previous day is shown on the overhead projector. Also the 7-up bottle and juice bottle are on the front table where students can see them)



7-UP Bottle Juice Bottle

Kathryn: I didn't get to this point with the boys, but I did with the girls, so let me limit this question to the boys for right now, if I asked you to make a prediction about the graph, any prediction that you might come to mind about the graph of this one (the juice bottle) is there anything that you might say or predict-

(Brad starts to speak)

Kathryn: Raise you hand. Dion?

Dion: It'll take more scoops.

Kathryn: Alright. It's going to take more scoops. (a student says you repeated). I did. I just repeated (laughs). Brad?

Brad: Uh, I don't know.

Kathryn: How about the shape of the graph? This question could be for boys or girls. I'm not sure that I asked that question. What prediction might you make about the shape? (3 sec wait time) Tom?

Tom: (inaudible)

Rebecca: Tom, I can't quite here you back here.

Tom: Oh, ok. Uh, at the top of it it's going to hold a little, or it's going to start filling up quicker because it's narrow.

Kathryn: Sounds great Tom, thank you.

Tom: Ok, cool

Kathryn: Sarah, you had your hand up. (Sarah indicates she doesn't want to contribute right now) Brad?

Brad: Well maybe the line will be a little bit straighter because the bottle's a little bit more even until you get to the top, then it'll probably go, the line should go a little bit straighter because the amount it goes up, it should go about the same, it should be better than that one (the 7-up bottle)

Kathryn: Alright. And what do you expect to happen at the top?

Brad: It'll go closer and closer.

Kathryn: Can everybody in the back here him when he's talking? (some students indicate no and Kathryn motions for Brad to talk up)

Brad: Talk up?

Kathryn: Andrea?

Andrea: Uh, you can't really fill it all the way up to the rim because there's a spout and it'll pour out.

Kathryn: Alright. What would you, how would you describe the bottom part of this graph (the 7-up bottle) as far as the shape, if we just look at this part? (5 sec wait time) Cassie?

Cassie: It'll get a gradual increase because the bottom part of the bottle (7-up) is the same size until the top where it got narrow.

Kathryn: And then what happened to the graph when the bottle (7-up) got narrower?

Cassie: It changed drastically, the line.

Kathryn: Ok, alright. Tom?

Tom: On this one when we plotted it, when we were here yester-, er, last Friday, we were using the ruler to put in on the line to get the curve or whatever, and because, uh, it wasn't always real accurate because of the shape of the bottle so for this one, for like quite a while we'll be able to just put the ruler at an angle (on the graph) and the dots should be adequate-

Tom: Alright. I think what I hear you saying is that for this part (lays her ruler on the graph of the 7-up bottle) you're saying that these were, we were using our ruler and these were lining up

Tom: Yeah

Tom: And you're saying that you think we're going to be able to do the same thing-

Tom: For like most of the time it will work until it gets to that, where the shape is changing

Kathryn: And then something will change?

Tom: Yeah.

Kathryn: Alright. What I'd like for you to do next is again use your pencil as a pointer and I'd like you to point on your graph, don't actually put a dot down yet because we'll actually do the experiment and test, point, on your graph, to where you think the first point on the graph of juice bottle is going to end up. Just use your pencil. The point where you think the first dot is going to go. (Kathryn circulates to look at what students are pointing at)

Kathryn: As I walked around these are some of the responses that I saw (draws the points on the overhead graph). I saw some that were down in this area, they may have been at four (cm), they may have been at 3 and 1/2 (cm) but basically they were down here below where this point was for the last bottle. I also saw some responses that were up here, maybe here, here, something like that and I also saw some responses that were something like this (out on scoop 2, 3, etc.). So what I'm going to ask about this, is, is there anyone who would like to argue either for or against any of the points that you see? (3 sec wait time) Nicole?

Nicole: It can't be the one on the 3 or the 4 because we're not on that scoop.

Kathryn: Alright. We're on scoop one so we'll have to take those out. Daria?

Daria: It can't be up there at the 6 (cm) or the four (cm) because the bottom of that container (the juice bottle) is bigger than the 7-up therefore it'll take more scoops to fill up the bottom than the 7-up did.

Kathryn: Could everyone hear her when she was talking?

(some yeses)

Kathryn. Alright. Would anyone like to add anything to that? Yeah?

Deanna: I was going to say that the juice bottle is wider at the bottom than the 7-up bottle is so it'll be less than what the 7-up bottle is.

Kathryn: How many agree with what Deanna just said?

(Several hands go up, someone says "what did she say?", others laugh)

Kathryn: Remember, you have to listen, because I may ask you these questions every once in a while. How many disagree with what Deanna said? (few or no hands go up) How many-

Tom: I have something to add.

Kathryn: Alright. How many aren't sure? (few or no hands go up) Alright. Tom, you wanted to add something.

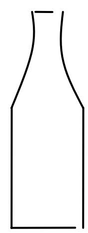
Tom: Another thing is that the indentation in the bottom of the 7-up bottle, so it like took away quite a bit, but since the bottom of this (the juice bottle) is flat, it'll be a lot lower.

Kathryn: Alright. Let's go ahead and try this experiment and see what happens. Where's a group where no one has been up to the front yet? Alright. Let's take this group back here.

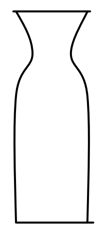
End of transcript

<u>8/21/00 Period 4</u>

(NOTE: in proceeding discussion the graph of the 7-up bottle that the class filled and graphed the previous day is shown on the overhead projector. Also the 7-up bottle and juice bottle are on the front table where students can see them)



7-UP Bottle



Juice Bottle

Kathryn: On Friday we were working on the 7-up bottle, we just about finished it, but there are a few little details that I want to make sure that we get cleared up before we move onto the next bottle. Remember that we were filling this bottle one scoop at a time with a half cup scoop and then we were measuring the height and then graphing the results and we had information on a table and we also put information on the graph over here. And I believe we ended the period with a little bit of a discussion about what we should do at this point now that we have these dots. Should we connect the dots or should we not connect the dots? (Slight pause). Alright. Looks like somebody has an opinion. Were you thinking about it? Uh, Anna.

Anna: I think we should connect the dots.

Kathryn: Why?

Anna: Because I like the dots connected. (laughter)

Kathryn: Jessica?

Jessica: I think we should connect the dots so that, uh, when we do the other, when we fill up the other containers we can tell the difference between them, not like looking at the dots and we can figure out just by looking at the line.

Kathryn: OK. Uh, Amy?

Amy: I think we should connect the dots because like Jessica said when we did it it will be less confusing than having dots all over the place.

Kathryn: Chrissy?

Chrissy: I think we should connect the dots because shows a trend over time, it shows it going up.

(Kathryn motions to Chrissy to speak up)

Chrissy: Well they need to be connected to show a trend over time.

Kathryn: Alright. All of those are reasons why you might want to connect the dots. You might want to kind of make sure you are kind of showing the trend. Now over time, do we, do we really have time involved?

Chrissy: Well like over scoops.

Kathryn: Over scoops? Alright. So we might want to show the trend, we might want to make sure that we don't get the points on this graph confused with the points that we put for the other one. So those are all certainly things that you might think about and take into consideration, but another thing that is, and we could spend a lot of time talking about this but I am choosing not to so we'll just kind of talk about this briefly. Another thing though that you want to take into consideration when you're trying to decide whether to connect dots on a graph or not is if I connect those then what I'm really saying is that all of these points that are on this segment also make sense for our experiment. So for instance if I pick, if I pick a value (draws segment and picks point on overhead graph), if I pick a point somewhere in the middle right here, can someone tell me, for this particular point, how many scoops have we put in the bottle and how high did the water go? (5 sec wait time) Everyone see that? (short pause) Raise your hand when you think you know an answer. How many scoops of water have we put in the bottle at this point and how high is the water? Raise your hand when you think you know an answer. I have two, only two? Three? Four? (5 sec wait time) Five? Crystal, what do you think?

Crystal: Uh, three and a half scoops and (inaudible)

Kathryn: I'm sorry we're making a lot of noise with pencils over here. Three and one half scoops?

Crystal: Yeah and ten point five centimeters.

Kathryn: Ten point five centimeters? So the question that we would want to consider would be would that information make sense? For instance, three and a half scoops. Could we physically put three and a half scoops into this bottle?

(some students say yes)

Kathryn: Anybody?

(more yeses)

Kathryn: Yes. Alright? That would make sense. Could the height of the water after we do that be ten and a half? Would that sort of an answer make sense, that fraction, ten and a half?

(some students say yes)

Kathryn: Yes. Alright? So that number ,those numbers would make sense. Sometimes you might have graphs and an answer of three and one half might not make sense. If you're talking about people, alright? You want to talk about whole number of people, it may not make sense for your experiment to talk about a half of a person, but a half of a scoop we can talk about. Another question that might be involved with something like this is now or after you've decided that "yeah it's ok to connect those, those points that are in between make sense" is what should the shape of this connection be? Should I connect it with a straight line? Or should I be connecting it with a curve somehow? Should I curve it this way? Should I curve it this way? Should it kind of wiggle as it goes up? Or should it just be a straight segment? So that's another thing you might take into consideration. Anna?

Anna: Uh, I think it should be a straight line because if it was all curvy then the line would look kind of nasty and it's kind of hard to like follow it up.

Kathryn: Alright. If it's all curvy, she doesn't like the way that it looks if it curves.

Jessica: I think it should be straight because if you make it all curvy there might hit some points that don't make sense.

Kathryn: Joe?

Joe: Well if the line is curvy, just like she said, it'll say that, if you make the line curvy it'll be like two scoops it went up to six but then it came back down and then, I don't know, it's just moving around. It'll misrepresent the actual numbers.

Kathryn: Oh you mean, alright. I think what you're saying, you're saying like if I don't connect them in order, if I went from two to six and then-

Joe: Not even that just the scoops

(a couple other students say "scoops"

Joe: Dipping back and forth it wouldn't follow the path-

Kathryn: Dipping back and forth here we might get some points that weren't accurate? Yeah. And we could get into a big discussion about if I connect this with a straight segment will all of these points be accurate? If I connect these with a straight segment, will they all be accurate? But I think what we'll do is we'll just agree for right now we're going to connect these with segments and then move on to the next bottle. Uh, so go ahead and if you haven't done this already, you want to connect them, and in order, I think that discussion came up the other day, scoop one to scoop two to scoop three and so forth. (pause) And then we didn't quite finish this up. (goes into discussion of adding a horizontal line to the end)

Kathryn: The next bottle we are going to do is the juice bottle. And just a reminder of some of the things we talked about the other day when we were comparing the things that were the same and the things that were different with these bottles, we noticed that the 7-up bottle is a little bit taller. Both of them start out at the bottom shaped like cylinders, I believe that term came up. They both narrow. And then as far as the circumference, alright, we noticed that the circumference of the 7-up bottle is a little bit smaller than the circumference of the juice bottle. With all that information in mind, could you make a prediction, any prediction, about what you think the graph of this juice bottle might look like? (13 sec wait time). Joe?

Joe: Uh, first can you bring it over here so I can see if the bottle has a dome?

(Kathryn takes bottle to Joe)

Joe: I think it'll be like, go straight up to a certain point, like the angle of the line won't change, until it gets to the nose and then it'll change.

Kathryn: OK. How many of you agree with what Joe just said? (several hands go up) How many of you disagree with what Joe just set (no or few hands) How many of you aren't sure? (Some hands go up).

Jessica: I have a question.

Kathryn: Yeah, Jessica?

Jessica: You mean the line will just go straight up like this or the line will go straight this way?

Kathryn: I don't know. What did you mean Joe?

Joe: It'll stay constant but it'll be at an angle.

Kathryn: Anna?

Anna: I think that the, uh, angle of like the line is going to be like, it's not gonna go straight up as fast, it's gonna be like basically like straight over because it's not gonna, the measurement of the height or whatever, the water, it's not gonna be as much.

Kathryn: So are you saying, you're saying, I think what I hear you saying is it's going to be something like this (puts ruler on graph to show lower slope).

Anna: Yeah. It's going to be more like angled straight towards you.

Kathryn: Alright. How many agree with that? (pause) How many disagree with that? (pause) How many aren't sure? I don't't think everybody raised their hand there. Alright. Let's give it a try.

End of transcript

8/22/00 Period 4

(NOTE: Students have been working on worksheet, first problem, matching two bottles with two graphs and writing explanations. Students first matched on their own, then talked in groups, then wrote their explanations, and now are going to share in whole group)

Kathryn: These don't look exactly like ones on your paper (drawings of two bottles on the overhead). Could I have everyone turn their attention up here to the overhead? But similar ideas. That we have one that is shaped like a cylinder, that word came up in some of our discussions, this one, if we imagine it going on, kind of like a cone. Alright, so this bottle, how many of you labeled this bottle B? (no hands) Nobody. How many labeled it A? (many hands up) Everybody. So by process of elimination the other one is B. So it looks like everyone is in agreement as to which one is which. Now is there someone who would be willing to share what you wrote on your paper or what your group discussed? Maybe someone that I heard from yet today. Brittany?

Brittany: Uh, I labeled the first one bottle A because the bottle goes straight up and so does the line on the graph. (laughs) Basic.

Kathryn: Alright. Would anyone like to add to that? Rachel.

Rachel: Ok. I said the same thing she did but , I can't remember your name-

Rebecca: Ms. McGraw

Rachel: Ms. McGraw says that bottle B's slant goes straight also but if you look at it from a different angle besides that one angle it wouldn't be straight.

Kathryn: If you look at bottle B at a different angle?

Rachel: Yeah. Because we're only seeing it an that one angle.

Kathryn: So what do you think she meant then when she said that B was also straight?

Rachel: She meant looking at it at one angle you would see it straight but if you look at it from different angles it wouldn't be.

Kathryn: What might be a better way then that we could describe those bottles instead of saying that A is straight and B is not straight? Is there a better way we could describe that? Beth?

Beth: Uh, for A on the graph (inaudible)

Kathryn: Ok. That's true. Now you're talking about the graph.

Beth: Yeah

Kathryn: Alright and what I was asking was, what Rachel brought up I think was that we can say, some people might say that the sides of bottle A are straight and they might say that the sides of bottle B are not straight but the point that I think was being made was that when you look at this, just look at that (slanted) side, it's a straight line. So when we say that this is straight and this is not, maybe that's not the best way to describe that. So is there a better way that we can describe the sides of those bottles so there isn't going to any confusion? Selma?

Selma: (inaudible)

Kathryn: Ok, alright.

Student: What'd she say?

Kathryn: Oh yeah, speak up.

Selma: The first one has a consistent graph like on the graph it goes up in a straight line.

Kathryn: Alright. Now again we're back talking to the graph again. Eric?

Eric: Ok. On A it's a cylinder so when you pour the water in it's going to go up as a constant. On B, it's not a cylinder, the shape changes so the amount and the heighth of the water it's not going to be the same every time, so all you have to do is look at the graph and see which one is constant.

Kathryn: Now again, Eric talked about the graph in relation to the bottle. Let's concentrate for just a second still on the bottle. He said that bottle a is a cylinder and bottle B is not a cylinder, alright? I can't remember, did you give it a name?

Eric: Uh no.

Kathryn: Ok. It's not a cylinder? So what is it that makes bottle A a cylinder and bottle B not a cylinder? Chrissy?

Chrissy: Well when you look at it as a two-dimensional figure, the lines are parallel to one another.

Kathryn: On which bottle?

Chrissy: On A.

Kathryn: And on B?

Chrissy: They're not. They're going to intersect eventually.

Kathryn: Ok. Alright. Uh, Anna?

Anna: I think that bottle B looks like a cone but cut in half. So.

Kathryn: A piece of the cone missing. Alright. So she's given a name to that thing that's not a cylinder. Amy?

Amy: It looks like an upside down-

Kathryn: I'm sorry. Speak up.

Amy: B looks like a cup.

Kathryn: An upside down cup?

Amy: Yeah.

Kathryn: A doesn't look like an upside down cup?

Amy: No I mean, never mind.

Kathryn: I know what you're saying, I'm just teasing. Joe?

Joe: (changes his mind about having something to say)

Kathryn: Jessica?

Jessica: The difference between A and B, I mean like one's a cylinder and the reason you know it's a cylinder is the lines are always going straight up and B they're, basically like she said, they're not parallel and they're going to intersect.

Kathryn: Alright. So she's bringing out the same idea that Chrissy had about the sides, if you look at it as a two dimensional figure and these being parallel and these not parallel. Now you said these are going straight up, you said something else too.

Jessica: Yeah.

Kathryn: Alright. So these are going straight up but these are not straight up?

Jessica: Right.

Kathryn: Anybody expound on that? (2 sec wait time) What does straight up mean? Robert?

Robert: I was about to say that uh, something that had to do with that maybe on bottle B the circumference of the bottle is different than A so that might affect which way the lines are going.

Robert: Sure, uh huh. Ok. I think where I wanted to go with this discussion about the bottles themselves was just being precise about you language, too. And like I said words like straight maybe are not very precise and maybe you can think about what are better ways you can explain that so that there is no confusion. Alright. Now I heard as we were discussing that, I heard some things where you were talking about the graph and I kind of put you off a little bit because I wanted to hold up on that discussion about the actual graph and Robert just said that he feels like the circumference at the bottom may have something to do with what the graph looks like and Selma, I'm going to come back you again and would you share once again what you said about the graph because I put you off and kind of put you on hold. (pause) I think you were explaining why bottle A was the one that it is?

Selma: The bottle, A, has a consistent graph.

Kathryn: What is it that's consistent about the graph?

Selma: It goes straight.

Kathryn: And the other one's not straight?

Selma: (agrees)

Kathryn: Robert?

Robert: On A, the water's rising at a consistent number. But on B it doesn't. Like on A the line is straight but on B the line is curved.

Kathryn: Now you said A it rises at a consistent number, what is that?

Robert: Uh, if it goes from 2.5 to 4.5 then the difference between those two numbers.

Kathryn: And what is the difference between those?

Robert: Two.

Kathryn: Two. Alright. So you're saying that each time it goes up by that same amount.

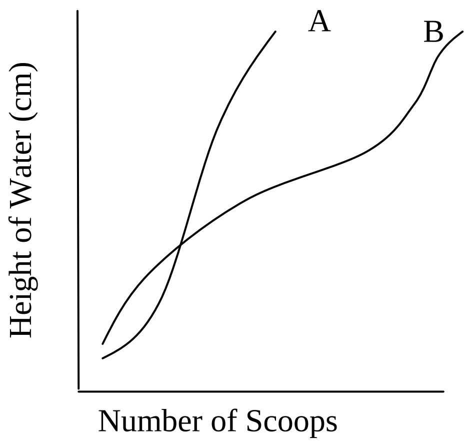
Robert: Yeah.

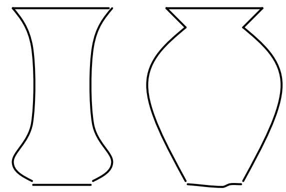
Kathryn: Anyone else? Any other thoughts or ideas to add? Alright.

End of transcript

8/24/00 Period 3

(discussion is share part of think-pair-share which Kathryn explained to students at beginning, discussion relates to how to match bottle shapes and graph shapes – see below)





Bottles A and B

Kathryn: Let me just start out by taking a poll. How many of you thought that this one was A? Raise your hand if your group thought this one was A (many hands go up). Alright. Was there still some disagreement in this group over here? We're not sure? Over there too? Alright. Now, what I'd like to talk about are what are some of the clues that you gathered from this graph that helped you make your decision. Katie, you want to start with your group?

Katie: Yeah. Uh, the one beside, the right side, that one (2 above), A has, goes like straighter and the other one is B, kind of like the line moves like that (motions) and the line moves like that too.

Kathryn: Alright. So you're saying that this one (2 above) is straighter in here, is that what you mean?

Katie: Yeah, it like stays, yeah.

Kathryn: Alright. And so what do you expect the graph to look like then for that straight part?

Katie: To go up straight.

Kathryn: Alright. And then over here on this one (1 above)?

Katie: To curve

Kathryn: Alright. Now if I, if I take my ruler and, so we're looking kind of the middle of the graph, if I take my ruler and line that up on graph A, alright?, we can kind of see that in the middle section of this graph that those points seem to be lining up in a straight line. What about the middle of this graph, graph B?

Student: I got a question.

Kathryn: Alright. Gosh, it looks to like middle part of that is straight line also. So I guess what my question is is how do you know that this one (2 above) goes to A? Because if these are both straight lines. Tina?

Kathryn: Ok. Hold on. Let's everybody listen. I know there are lots of people that want to share but let's listen first and then you can have a chance.

Tina: The top of curve B, it's like, it jumps from 15 and a half up to 21 and so it's like on B, that one (1 above) it gets real skinny so it's gonna go up faster than A does.

Kathryn: How many agree with what Tina says? (many hands go up) Alright. Anyone like to add anything else? How about back to this question about this is a straight line and so is this (middles of both graph lines) so how do you that this one (graph line A) goes with this bottle (2 above)? Jeff?

Jeff: Because A is steeper and that means that the bottle is more narrow.

Kathryn: How many agree with what Jeff says? Tonya, you're not sure.

Tonya: I don't understand what he's talkin' about. But I was gonna say-

Kathryn: Alright, go ahead.

Tonya: Ok. It's real simple. Everything, this is a real simple project. It's like the circumference. The smaller the circumference, the more centimeters it's going to be. So if it's this small (makes a small circle with her fingers), 1 scoop is going to fill up real quick so it's going to be like this much (shows height with thumb and forefinger). About, hmm, 5 centimeters. (lots of laughter). If it's real wide, this big (shows big circle with fingers) and you put 1 scoop in, it's gonna be real small, like 1 cm because the water has to stretch out. (some laughs here) So B is the first glass, the one that's real clear (1 above).

Kathryn: This one is B?

Tonya: Yeah, that one is B because you see how the little dots get real small, like right beside each other, real small, that's because the circumference in the middle of the bottle is real big so the water has to stretch out making it smaller, so it gets real small. And then on the other glass (more laughter here), the circumference is real small so it has to take big steps. See what I mean? (question is to all students who seemed to have been listening attentively?)

(spontaneous applause from the class for Tonya's explanations)

Kathryn: I take it that you're all in agreement. And guess what Tonya, you said the same thing Jeff did.

(class laughs)

Tonya: I had to expand it.

Kathryn: You did, you did. Alright. Uh, go ahead, Brad, you wanted to add something.

Brad: Well, my group had the glass with the clear glass (1 above) so and the graph looked like B so we can deduct that B is the clear one so there is only one more left so A is the other one.

Kathryn: You know what they're saying? This is the bottle that they filled yesterday so they knew that one because they filled it yesterday. Alright. Oh and you did, you did this one?

(side discussion of who had what bottle yesterday)

Kathryn: Anyone else? Where they any other clues that you used? (Brad speaks out and Kathryn reminds him to raise his hand) Could you have used the height of these bottles as a clue?

Students: (several say no, they are about the same height)

Kathryn: They are about the same height? And to be honest, I fudged a little bit.

Student: (jokingly) You lied.

Kathryn: I fudged just slightly. Could you have used the number of scoops that go into those as a clue?

(lots of yes's)

Kathryn: Which bottle took more scoops to fill? Ben?

Ben: The fatter one (1 above) The clear one because it was wider, it had a lot of circumference.

Kathryn: Ok. Is it, is, do you think it's obvious to everyone just by looking at these bottles, do you think you would have been able to guess which one was going to take more scoops than the other?

(lot's of yes's)

Kathryn: Do you? You might have had a pretty good hunch. I'm not absolutely positive when I first looked at 'em I thought this one's kind of going out, this one's going in, but this one's narrower on the bottom than this one is. I wasn't really sure. So, uh, you may or may not have been able to use that. But if you look at the graphs, alright, can you tell from the graphs which one took more scoops to fill.

(a couple yes's)

Kathryn: Yeah. Which one took more?

Student: B

Kathryn: B. Alright. Anyone like to add anything else to that? (4 sec wait time) Now, notice that on this one, I still gave you some clues down here with numbers. Did you really use those numbers when you were making your decision?

(a couple of no's)

Kathryn: Did any group use those numbers at all?

(a couple of no's)

Kathryn: No? So if I took the numbers away it wouldn't both you?

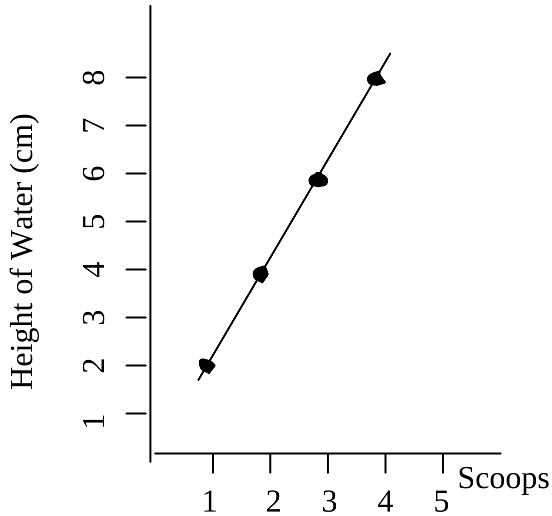
(a couple or no's and right's)

(moves on to matching graphs without numbers)

End of transcript

8/25/00 Period 3

(Kathryn puts sketch below up on the overhead)



Kathryn: Right now, I just want you to look at problem number one. This one, once again, represents a bottle graph. And the first question I would ask is is that bottle filling at a constant rate?

(lots of yes's)

Kathryn: What rate would we say that's filling? (10 sec wait time). Tonya?

Tonya: 2 centimeters per scoop

Kathryn: (as she writes) 2 centimeters per scoop. Would anyone say that a different way? Tom?

Tom: Two to one.

Kathryn: (as she writes) Two to one. Two colon one? Mia?

Mia: Two over one.

Kathryn: (as she writes) Two over one. Katie?

Katie: Could you say two out of one?

Kathryn: Two out of one?

Katie: Yeah.

Kathryn: (writes it down).

Alysha: Two percent.

Kathryn: Two percent? (writes it down) Any other ways that we could say that?

Student near Alysha: How did you say two percent?

Kathryn: Can you answer that? Why would you say two percent Alysha?

Alysha: I don't know.

Kathryn: Jeff?

Jeff: It'd be 200 percent.

Kathryn: It would be what?

Jeff: It'd be 200 percent.

Kathryn: Two hundred percent? (as she writes it down)

Sarah: Two out of one, two percent, and two hundred percent don't make any sense.

Kathryn: Say that again Sarah.

Sarah: And two over one.

(students quietly begin to talk at once about their opinions)

Kathryn: Go ahead Cassie.

Sarah: Oh yeah, two per one, ok, I get it.

Kathryn: Ok, hold on. One at a time. Cassie, go ahead.

Cassie: Because if you're comparing something, two percent doesn't make any sense because you don't know what the other thing is and two percent makes you think two over a hundred, and that's not what the ratio is. And then two hundred percent, I don't think works either because you have to do it, if you want to add 'em, the ratio would have to be one hundred over two hundred, or two hundred over one hundred.

Kathryn: So two hundred percent means two hundred out of one hundred?

Cassie: Yeah. Neither of those is the ratio that we have. The ratio that we have is two over one. I know (to student near her), if you would reduce it-

Brad: You already have it reduced

Cassie: But, right-

Kathryn: If you reduce this one, this one would be two to one (the 200 over 100)? If you reduce this one (the 2 over 100)?

Cassie: It would be one to fifty.

Kathryn: One to fifty (as she writes it).

Cassie: That's not the ratio that we have.

Kathryn: That's not the ratio-

Cassie: And then two out of one, we're talking about centimeters per scoop and-

Sarah: Two centimeters out of one scoop-

Cassie: Yeah, it doesn't make any sense-

Sarah: Yeah, it does. Two centimeters out of one scoop.

(another student starts to speak)

Kathryn: Speak nice and loud so everyone can hear you.

(NOTE: Cassie, Sarah, Alysha, and Brad all sit together on one side in the front near the overhead and Kathryn)

(Sarah and Cassie exchange a couple more remarks that aren't audible)

Sarah: Two scoops out of one centi- Two centimeters out of one scoop.

Kathryn: Alright. Alright. One thing that I want to zero in on is Sarah was looking at this two out of one. Alright? And then it sounded like maybe to clarify it to yourself you said "two out of one, two centimeters out of one scoop".

Sarah: Right.

Kathryn: When she was clarifying it for herself, she put these words in here, the centimeters and the scoop, do you think those words are important?

(several yeah's)

Kathryn: Yeah. Because if we talk about two out of one, is it clear what we are talking about?

(one or two no's)

Kathryn: No. Or if we say just two to one? And there are three different ways, these are all different ways to write ratios. But when you write a ratio and you don't have any units in here, what that means is the units are the same. Alright? If you are comparing points to points or if you want to compare distances, you know, if you want to say one person traveled 10 miles and another person traveled 5 miles, and we could compare those distances by saying the ratio is 10 miles to 5 miles. If those units are the same, then we can actually take them back out again. Alright? And we can say the ratios ten to five or we could simplify it to two to one. But when we have these two different units we need those units in there to clarify which is which. So I think it's important that we have those in there. Somebody said down here that two hundred percent will simplify to two to one. That's the same ratio that we had up here, but again do we have the units included? Are we clear about what the two and the one stand for? And then the one to fifty, what about that one? Does anybody think that that one should stay in the list?

(several no's)

Kathryn: Yeah, Sarah? Yeah, Tom?

Tom: On the two out of one, wouldn't, you could show, it's not really right, I mean not really wrong, if you used just different numbers like different, instead of scoops like say sixty miles out of one hour, that's pretty much the same thing, it's talking about the distance or a height and then like an hour scoop equals, er, sixty miles out of one hour doesn't make sense.

Kathryn: The wording you think doesn't make sense?

Tom: Yeah. ‘Cause you can’t go sixty miles of an hour.

Kathryn: Out of one hour?

Tom: You can go sixty miles AN hour, but you can’t, and then, so, you can gain two centimeters per scoop but you can’t gain two centimeters out of one scoop.

Kathryn: Ok, so you just think the wording is kind of funny on that one.

Tom: Yeah.

Kathryn: Same numbers and maybe getting at the same idea but just kind of funny wording.

Sarah: Like, ok, two centimeters out of one scoop and then, for the next one, would you put like four centimeters out of two scoops, or would it be the same?-

Kathryn: Well let’s talk about that. Good question. Alright. So your looking at perhaps the next, the next one and your saying we got four centimeters out of two scoops?

Sarah: Yeah. Would it be that or would it be two centimeters out of one scoop? Because that’s the rate.

Kathryn: Ok, actually, this is kind of a special case. Alright? And if these numbers were slightly different then it might not be working out exactly the way that you’re thinking right now. But if you’re talking about four centimeters for two scoops (writes down four cm/two scoops-

Sarah: Oh, you just, you just, like-

Kathryn: Well, ok. For this particular graph, alright, it’s just kind of a coincidence that for two scoops it went up a total of four centimeters, uh, and I could change this graph and still have it going up two centimeters for each scoop but not quite have that same information, but let me clarify, when we’re talking about the change, the rate that this is changing at, if we’re talking about between here and here (over 1 up two on the graph), alright? We can say it did go up two centimeters for this one scoop, didn’t it? Alright? But suppose I went up two scoops, alright? Or I put in two scoops of water, then I’m looking at going from this point all the way up to this point (outlines over two up four on the graph), aren’t I?

Sarah: Mm, hm.

Kathryn: So if I put in two scoops of water, then how much did the height go up?

Sarah: Four.

Kathryn: Four. So you’re right in that, that for putting in two scoops of water it did go up four. Now we said that the rate was two centimeters for one scoop but now it looks like we have information that it’s four centimeters for two scoops.

Sarah: So you just simplify it?

Kathryn: Alright, if we simplify? Yeah. If you write a rate as a ratio, and remember I mentioned before, and hoping that this is something that you remember from previous class, that when you have a ratio, you can treat it like a fraction and that you can simplify it. So I can simplify this four centimeters to two scoops to the two centimeters per one. Alright? How do you go about simplifying a fraction? Why can I say that, that this would simplify to this (4cm/2scoops to 2cm/1 scoop)

(about 1/2 the class appear to be listening to Kathryn)

Sarah: Because two divided four and two divided by two equals two centimeters-

Kathryn: Two divided by four?

Sarah: Equals two centimeters.

Kathryn: Two divided by four equals two?

Sarah: Yeah.

Kathryn: How many agree? Two divided four equals two?

(Two or three students from around the room say "four divided by two equals two")

Sarah: Ok, yeah.

Kathryn: I know that's what you meant. Alright? So if I take the four and divide it by two and then divide this by two, I can simplify that to two to one. Alright. And it would be the same thing. Uh, one other kind of definition that I can throw in here is we said that this is a rate (four cm to 2 scoops) because we are comparing two quantities with different units. This is also a rate (two cm to 1 scoop) but this is what's called a unit rate. A unit rate. Why do think it's called a unit rate? (8 sec wait time) Two people? Anybody else? Danaand Tonya I need to make sure I have your attention back there, sit up please.

(laughter)

Student: Tonya's still asleep.

Kathryn: Nicole?

Nicole: Because, uh, it's just how much it's going by one thing, by like one how high it goes.

Kathryn: Alright. Because it's comparing the height to one scoop. Alright? What does the word unit, what does the word unit mean? (2 sec wait time) Sarah?

Sarah: (two words inaudible) Like centimeter is a unit, scoops is a unit

Kathryn: Ok, yeah, you're right. It could be a unit, yeah. When we think of the word unit, this kind of prefix, uni-, means one. Alright? So that's why unit rate means exactly as Nicole said, for one scoop. So this is a unit rate because the denominator is one.

End of transcript.

<u>8/29/00 Period 7 Janet</u>

Discussion of T-Shirt and Cokes Problem

Janet: Now, I would like to ask you guys some questions because I was able to walk around and I could only here some things going on, and what I noticed and what I'm curious about is what is the very first thing your group did when you got together and read the problem? You guys want to go first and just kind of tell me what you guys did, the first thing?

Student: Ok, I think we first looked at the one with the forty-four and we like, because we really didn't understand the question, and we divided it in half and we divided the other things too.

Janet: Good. And what was the first thing you guys did when you got together as a group, and I'm kind of being, I know you're with the guys and you're with girls, so what was the first thing this group did, Aleta?

Aleta: We decided on a problem And then we (inaudible)

Janet: Ok. Now you say you decided on the problem. Can you be more specific?

Aleta: Well there's three of 'em.

Janet: Oh, I thought I told you just to do the T-shirt problem

Aleta: Oh, you did? Oh, well (laughs)

Janet: Ok, that's alright. You were verifying what I was asking within your group first and that's good. That's good. Ok. Guys, yes?

Student: We just looked at 'em and tried to figure what prices (inaudible – the student was speaking very softly, I wonder if other students could hear?)

Janet: Now, how'd I do this?

Student: Us four.

Janet: Oh that's right, you four. What was the first thing you guys did?

Student: We kind of, uh, we thought like what would be a reasonable price for the things, and then we kind of, we just kept guessing different numbers.

Janet: But I like your idea, you thought of something reasonable. For example, if you buy fruit-of-the-loom T-shirts, how much do they cost?

Students: Five bucks, a dollar

Janet: Five bucks each, a dollar. Alright. Then if you go into Gap, or Abercrombie & Fitch, or American Eagle, how much do those T-shirts cost?

Students: Thirty, twenty five, forty

Janet: Thirty, forty, fifty, yeah, somewhere in there, twenty. So you're saying you started with a realistic idea first. Thank you. What did you guys do first?

Student: We guessed and checked?

Janet: Right at the beginning?

Student: Yeah

Student: No.

Student: We actually guessed numbers for it all.

Janet: Right at the beginning you just jumped right in and said I think it's this.

Student: No, we all talked about it first.

Janet: Ok. What did you talk about?

Student: What we were guessing were, they thought, they thought that coke or whatever wouldn't five dollars and we talked about it

Janet: Good. Ok. Did you want to add something?

Student: We, uh,

Janet: You don't have to if you don't want to, it's ok. It's ok. Ricky do you want to help her out?

Student: He wasn't in our group.

Student: He was in our group.

Janet: Oh, he was in your group. No wonder you [Ricky] don't want to help her out. Can you help her out Melissa or Aleta? What's she trying to share with us?

Student: We, uh, we listened to each other's ideas and then like we (?) the original amount and we figured it out, we tried to work those out.

Janet: Ok, so the first thing you did, let me summarize, you talked about the problem, you thought ok, how do we get started, and do you remember the very first thing you started on besides talking about the problem? Was it to see what was reasonable? Is that what you were going to say Melissa?

Melissa: Yeah, and how to do the problem. Like, how to guess and check.

Janet: And how to guess and check. That's powerful. Ok. What did you guys do first?

Student: We kind of did the same thing Susan's group did. We were thinking well how much would the T-shirt cost and then we started the whole problem like, well, if it costs this much then, you know, you have to multiply that by two because there are two T-shirts in the first part and then we guessed how much we thought the coke would be and see if it added up to the forty four dollars.

Janet : Ok. So, this is the second time I've heard this reference to two. So we had two t-shirts plus two cokes equals forty four dollars

(Janet writes on the overhead projector 2 t-shirts + 2 cokes = $44)

Janet: Now you are the second person to mention something about two. What are you saying? What are you trying to tell me?

(inaudible but I believe a student says 2 t-shirts and 2 cokes)

Janet: Right. There are two t-shirts and two cokes, but I keep, I hear, I don't want to put words in your mouth, but there have been two statements made that intrigue me. Makes me want you to tell me more. What did you do with the two?

Student: I think, well everybody thought it like, that, uh, like, ok, like on the top row and the bottom row, the cokes and the t-shirts didn't have to be the same-

Janet: Oh, ok.

Student: And then you came around and told us that, so we found out it did, but that's I think.

Janet: Ok. So, I see. I remember that. Some of you thought that these cokes were different prices and I said no, this coke would be the same as this coke cost, this t-shirt cost would be the same as this t-shirt cost, so what you were really asking me is do the variables represent the same thing.

Student: Well what we did, 'cause we just picked something for each, like just any dollar amount for the uh, two shirts together, like if they were 15 dollars, that would make 30, which would make the cokes like, I think that's 14 dollars, so that would have to make the cokes like 7 dollars each, but then we had to go dow-, we referred to the bottom and just tried to see if it worked on the bottom, but it didn't, so we had to go back to the top and try it again.

Janet: I like your explanation of your process. That was very good. Alright, let's go on to the next question. This is something I'm curious about also. Let's get back to what we kind of talked about up here. What mathematical ideas did you use, or find, in this problem? What mathematical ideas or, did you use or find? Go ahead, Martin.

Martin: For one, we tried to start at different points in the problem. Instead of starting at the beginning, we started in the middle and did the second half first, to see if it would match up to the first.

Janet: So you started here?

(Janet write 1 t-shirt + 3 cokes = $30)

Janet: What made you do that?

Martin: Well, I couldn't figure out the top part of the problem and so I just skipped on down to the lower half and tried to figure that out, so, and I figured if I could get the bottom half the I could try to work on the top half afterwards.

Janet: I like that. I like your, the way you presented that process of thinking. Thank you. Go ahead, Susan.

Susan: Uh, like we found, uh, numbers that weren't exactly equal to 44 or 30 but were around there and we liked worked with that and like subtracted a dollar and added a dollar and subtracted fifty cents-

Janet: Ok. So you did spend some time with the fifty cents. Some of you were concerned with the cost being the same. Martin, you said that you didn't start with the first one, you actually started with the second one and that was because you didn't have success with the first one. Did it look easier, or?

Martin: Yeah. I think it did 'cause there was only one t-shirt there and the cokes were smaller amount of money, so, I just tried working with that.

(Janet writes Cokes cost < T-Shirts cost)

Janet: So I'm hearing that we had an assumption that the cost of the cokes were less than the t-shirts and that was a very important assumption and again we were basing that on reality. This one we had lots of questions about and that was a fair thing to say and you were also testing these possibilities to see what would work, what wouldn't work. Now, I think I've heard how you've found what was reasonable. I'm gonna go back though, what other math ideas, what other thought processes did your group through that has not already been mentioned? Susan?

Susan: Well we kind of had to think about like what kind of coke it was. Like if it was a big coke or a can of coke.

Janet: So this has to do basically with how much. So size came up, so that's another mathematical idea. Ok, uh, reasonableness, estimation, that's really what I'm hearing about whether the t-shirts would have been 15 dollars or 800 dollars. Uh, what other math ideas? By the way, this discussion is the best I've had so far with these two problems, you guys are doing great. Sharing. I love this. Fantastic. Yes?

Student: Uh, well, the shirts say Olympics on it. But when you go to an event they're usually kind of expensive. So they're not going to be like a five dollar shirt.

Janet: Ok. So you were also questioning did it have a logo?

Student: Yes.

Janet: Ok. Again you were trying to find a beginning place. I'm curious. What was your beginning guess? I think I heard some groups say they tried 15. Ok, that was our beginning guess for the t-shirt. And your beginning guess for the cokes?

Student: Like seven dollars. 'Cause that was the only thing that really fit with 15 dollars.

Janet: Ok. What others? Yes? Melissa.

Melissa: Well we thought the cokes were like a dollar at first because we didn't know how expensive they were going to be and so, they were pretty expensive when we found out the answer.

Student: For the top one, by myself, uh.

Janet: That's alright. We've all been there. We know. And I admire your courage.

Student: I thought that the shirts were 20 dollars apiece. And the cokes were three dollars.

Janet: That is good, it is good. Then what happened?

Student Well, when we had figured out, ok, when you split it in half the cokes were 11 dollars.

Janet: Split in half?

Student: The 44 dollars.

Janet: You split this in half?

Student: And then half again. So everything would be equal then, the shirts and the cokes-

Janet: Ah. So you checked out whether the t-shirts, can I just call them T's?

(Janet replaces the words she's written with Ts and Cs)

Student: Yeah.

Janet: Whether the T's and the C's were the same?

Student: Yeah.

Janet: So you thought of dividing by two?

Student: Yeah

Janet: What'd that tell you?

Student: That we were wrong.

(laughs)

Janet: But look at your idea. If you had two here, this was divisible by two, this was divisible by two. That was divisible by two. So you divided each part of this expression, this equation by two. So you might have ended up with one t-shirt plus one coke equals 22 dollars.

(Janet writes 1T + 1C = 22 under the other equations)

Janet: How would that have helped you here? If you knew that one t-shirt plus one coke equals 22 dollars. Jenny?

Jenny: (inaudible)

Janet: Ok, keep talking.

Jenny: (inaudible – but just two or three words)

Janet: Ok. We know we have one t-shirt and three cokes is 30 dollars and from what Tammy shared we know that one t-shirt plus one coke is equal to 22 dollars. We know that we have one t-shirt plus three cokes is equal to 30 dollars. What can we do with this? (5 sec wait time) What do you think two cokes would cost? (3 sec wait time) Dana?

Dana: Don't look at me, I don't know.

Janet: You have the light bulb look on your face. You went "oh my gosh, you almost saw it, didn't you?"

Dana: Almost, yeah, sure.

Janet: That's what it looked like. Ok, guys, we know one t-shirt and one coke is 22 dollars. One t-shirt plus three cokes is 30 dollars. Aleta?

Aleta: You can add another t-shirt and another coke to the top and see if it works, if it makes 44.

Janet: Ok. What were you going to say?

Student: I was going say that one t-shirt and, uh, two cokes were 32.

Janet: Tell me where, say that, you're going to say, 1 t-shirt plus two cokes (writes it on the board) is equal to? That was a question in your mind? Tell me what you were thinking again.

Student: I thought you asked that question.

Janet: Ohhh. Ok. Well that's a good one to think about. Melissa I saw your hand up. What were you going to say?

Melissa: Well I was going to say that you know that's wrong because if one t-shirt and one coke is twenty two dollars and then you add another two cokes, then it would be over 30.

Janet: Think so? Ok. Let me do something for a minute. Let's call this [1 T plus] 1 coke plus two cokes is equal to 30. And we know what this piece is (points to 1T + 1C), don't we?

(Janet writes 1T + 1C + 2C = 30)

(Some yeses)

Janet: Aleta?

Aleta: Yeah, it's 22, or-

Janet: Yeah, it's 22. So if I put in 22 dollars in place of the one t-shirt and the one coke that Tammy started us thinking about plus two more cokes is equal to 30 dollars. Now what? Susan?

(Janet writes 22 + 2C = 30)

Susan: Now you just subtract 22 from 30.

Janet: Wow. Ok. (Janet subtracts 22 from each side of the equation on the board) Twenty-two minus twenty two is?

(students say zero)

Janet: Zero. And zero plus two cokes is still two cokes. 30 minus 22?

(students say eight)

(Janet writes 2C = 8)

Janet: Eight. What do I do now?

Student: Divide

Janet: Divide it by two. (does it on the board) And what do cokes equal?

(students say four)

(Janet writes C = 4)

Janet: Four dollars. If a coke is four dollars, what's your t-shirt?

(students say 18)

Janet: Guys, do you know what you just did?

(many students say no, and laugh)

Janet: You guys this is really awesome. Do you know that you solved a system of equations by substitution, by solving equations, by coming up with a solution, going back and find the other solution, you guys we are not going to do this until close to December or January, but you just did it.

(some "wows" and one "cool")

Janet : This is called simultaneous or a system of equations. You'll do more with these types of equations at the high school. Does that make you feel pretty neat? It makes me feel pretty neat. And you know what? No other class of mine, including the honors class, did that. Your thinking was magnificent. And Tammy and Heather started us off with thinking oh, the two. Now most of the others of you spent good time thinking, I saw a lot of you write, at least earlier today, about guess and check, well here's another way you guys. Now, oh, there is one other question I'm curious about. What do you think of this process? First of all working in groups. Second of all writing about it, talking about mathematics, solving, problem solving, what's your opinion?

Student: Uh, I think the groups is a good because some other person can have a different idea and that you wouldn't have thought of.

Janet: In fact our principal has a saying about teams, do you know what it is?

(some seem to know)

Janet: Together, everyone achieves more. Look at what we just did as a class. What were you going to say Dana?

Dana: I thought that it's a good idea to have groups because one person has an answer then you challenge them to compromise.

Janet: Yes, that's a good word, compromise. Aleta, what were you going to say?

Aleta: Uh, I think it's like a good idea with working in groups, 'cause, like, as Tammy said that the other people in your groups come up with different ideas that you wouldn't think about and if you all have like a different idea that you can tell each other and that you all work as a team.

Student: I like it too, because if I'm having trouble then someone else can explain it to me if they understand it and there's always somebody else in the group that knows how to do something so we all get to work together.

Janet: Well said, thank you. Martin, what do you want to say?

Martin: Something that's good about working in groups is say everyone in the group has a different answer then you just find something that's common with all the answers and you start from there.

Janet: Excellent. Plus I want to give you credit too, Martin, you are the one who had us focus on the idea of maybe looking at another part of the problem, and that's good, that's why we're all together, that's why we achieved what we did.

End of transcript

9/1/00 Period 3

Discussion of at end of Mystery Graphs activity

Rebecca: Somebody like to add to that?

Brad: It's the only one that goes down during the day and up at night? Or in the morning.

Rebecca: Is that true?

Student: Yeah.

Sarah: Uh

Rebecca: Uh huh? Sarah?

Sarah: Did they mean to start like way down at the bottom during the night?

Student: No (some laughs)

Rebecca: Do you see what Sarah's saying? (Pause) So group what would you want it to be at midnight?

Tonya: I don't know. I'm tired. It is what it is, that's what it is. (some laughs)

Rebecca: It's nobody asleep at midnight on one end and everybody asleep on the other.

Tonya: Yep.

Brad: Well maybe that's just out partying or something. (some laughs)

Rebecca: Ok, ok. Alright, group, uh, Tonya and company, was your graph the asleep in Indiana one?

Tonya: No, ours was the hallway, I'm just playing.

Rebecca: It was asleep in Indiana. Ok. So give yourself credit if you put down three for A.

. . . .

Rebecca: Alright. Where's graph B? B was the back group? What did some people put, yeah, Laura, what did you put for B?

Laura: Inaudible.

Rebecca: So that would be eight?

Laura: Yeah.

Rebecca: What did other people put? Uh huh? Yeah, April?

April: Uh, the mail in the mailboxes.

Rebecca: The mail in the mailboxes? So that would be number two. Yeah?

Student: Inaudible.

Rebecca: What number? Another eight? What else? So I'm hearing some two's and eight's. Yeah, Melissa?

Melissa: We had four.

Rebecca: Shh. I can't-

Melissa: We had four.

Rebecca: You had four?

Melissa: Yeah.

Rebecca: Ok, interesting. Now what we're going to do is figure out, based on what you all say, whether there are good arguments for just one of these or more than one of these. It may be that you can argue credit for different answers on these and it may be that we come to agreement on one. But let's see. Who will start us off? Yeah, Mia, do you want to argue for one of these.

Mia: I think it's, uh, number eight-

Rebecca: Shh. Listen to Mia please.

Mia: Because school starts at 7:30 and people are in the gym in the morning 'cause they have class or whatever then it goes down because people leave after school and then it goes back up because (inaudible)

Rebecca: Yeah?

Jackie: I think it would be-

Rebecca: Listen please.

Jackie: -number four, McDonald's drive thru because in the morning that's when people like eat breakfast and that's the busiest time for at McDonald's and then if you travel on it continues to be busy

Katie: At night?

Jackie: This is Friday right? Ok. And McDonald's doesn't close til 11:30

Student: It doesn't.

Jackie: I work at McDonald's!! And then from 6 o'clock, around 7, that's around the time it usually gets busy, well at my McDonald's that's how our schedule usually goes.

Brad: She can't see over the counter.

Rebecca: Let me see a hand if you'd like to respond to that. Ok, Katie wants to respond to that.

Katie: Ok. Well, uh, I think that yeah you're right that it gets like, when people start coming to breakfast it gets high but then like four, five o'clock, people after school or like after their work, they start to go, you know? Like doesn't McDonald's like, they're always going?

Jackie: No, it get's slow sometimes.

Katie: I mean I just thought between five and six, I mean it would be really busy.

Rebecca: Ok. Let's get a few more ideas out about the McDonald's thing because we seem to have two positions. We have a position of a time that it gets busy that's a little later and a position that it's a little earlier. Are there other things we can tell in this graph? Do you want a, you want to argue for a different one?

Student: I think it's eight because that's the only one that's Friday.

Student: No it's not.

Brad: Well they know if theirs, obviously they know if their's is

Rebecca: No, it's theirs down there, it's not this groups.

Sarah: Yeah but theirs is the only two Friday's there and since she said it's not theirs, it must be theirs.

Rebecca: I think that's a good point Sarah. Listen.

Tom: I was going say that that's like, you can't really, besides the fact that we know it's not McDonald's now because of the Friday thing, but that couldn't be McDonald's just because you don't have from 7 am or whatever, you wouldn't have people getting, all day, getting stuff from McDonald's til like six pm and suddenly just dive down. It'd be up in the morning and then kind of go down for a little bit and go back up at lunch and then it'd go down and go back up for dinner.

Rebecca: So you're arguing something that maybe looks like (I draw two humps like a camel)

Jackie: Well what if people didn't do their graphs right, so that's not fair.

Rebecca: Oh, no, let me interject. That's a very good point because that's why we're having this discussion because it may be that even though there is an answer, I mean B was supposed to be a certain thing? It may be that the way that they drew their graph, different people could get credit for different answers, depending on how well they argue for their answers.

Student: I have a question here.

Rebecca: Who said two, mail in the mailboxes.

Tonya: We did.

Rebecca: You did? April, do you want to say something about that?

April I said that because like the most mail when people go out around like everybody put's their mail in and that's the time people leave so they're not there getting their mail and then when they come home like everybody's starting to get their mail like at around six o'clock because that's when people get home.

Brad: (Starts to speak.)

Rebecca: You've got raise your hand 'cause otherwise the conversation just takes off and I can't hear. Does anybody see a reason why it wouldn't be mailbox?

Daria: Because like my mail, it don't get there til around two or something and then like at night like around 7 or 9 or something, why would it be high?

(some students start to speak)

Daria: Ok, (inaudiblde

Rebecca: Brad.

Brad: Obviously, the graph, the way they're got it set up, the mailman would have to deliver like all that mail like at one time. It'd have to gradually go up 'cause he can't deliver all that mail in like one hour.

Rebecca: So it should gradually go up between what time and what time? Approximately.

Brad: Not that high. Not straight up

Rebecca: You want it to go up like this (I draw lower slope line on the overhead)

Brad: Yeah, like that. It shouldn't go straight up.

Rebecca: What does that mean (students are talking to each other)- Let me ask a question and then I'll get to, I know you two have important points. What does it mean when a graph goes up really fast like that, for our graphs? What does that indicate?

Student: Probably something happens fast.

Rebecca: It happens fast? So down here we have not very many people and here we have a lot of people in a very short time? Or amount of mail? Sarah?

Sarah: Ok, and it couldn't be the mail one because there is gonna be people who have mail in their mailboxes all the time, over vacation and stuff, so it couldn't be like 11: 30 is when no one in Indiana has mail.

Rebecca: 11:30 at night?

Sarah: Yeah. Everybody, or at least one person has mail in there mailboxes.

Rebecca: April, do you want to respond to any of those arguments (nominated)? I know you're not reading a book back there. Thanks. Yeah, back here?

Jackie: I have to argue about that too because how can that be a South Jackson game on a Friday? I've been to a South Jackson game and a game does not start at 7am in the morning and then it goes down

(several students say no, no)

Rebecca: Let her finish.

Jackie: And then the games are over at 10:30 so that couldn't be right, and the games start at around 6 o'clock or 7, so how could it go down and that doesn't make sense.

Rebecca: Yeah, Tonya and then we'll come over here.

Tonya: But like we have school at 7:30 so it would be like dead from midnight until like 7:30 and when people have class it'll be bumpy and then when it gets like 5:30 on Friday it'll get real tall and then at 10 or 10:30 (inaudible)

(some students start arguing about this one, noise escalates)

Rebecca: Ok, ok. Let me take a few more. I'm really glad that you all have different ideas about this and I think that there are different ways- Hold on, just a sec. I think there are different ways to argue these different points, now (pause) Listen to each other. I'm concerned 'cause we're getting the views out but I'm concerned that some people weren't listening when Tonya was making her point and, but now, you know, so let's all listen. You and then you.

Deanna: I was gonna say kind of like it says on a basketball game Friday, it doesn't mean that the basketball game is happening like right now. So it's going to be the gym until six and there's still going to be a couple of people in there because of like practices for stuff and then there's going to more people in around six and it's going to go down around 11:30 because the basketball game will be later.

Rebecca: So you're saying that you like the way that graph looks for the basketball game. Cassie, did you want to add to that? Ok. Katie, did you still have and then we'll listen to Tom.

Katie: I was going to say that, uh, the same point where it doesn't say it has to be at night.

Rebecca: Tom?

Tom: I agree with it for the most part except wouldn't there, there's gym classes every day like right now and in the morning until the end of school, but to make it a little more accurate they probably would have wanted to add like dips, like going up and down, for the period changes, things like that.

(this comment generates some side talk between Tom and couple of students near him)

Rebecca: Ok. Let's listen to Tonya for a moment. Shh.

Tonya: Ok. What's your name right there?

Deanna: Deanna.

Tonya: Deanna? Ok. So you're saying that there's more people in the gymnasium from 7 o'clock to six o'clock and then it's less people in the gymnasium when the games coming.

Deanna: Yeah because the whole school, there's about 700 people in each graduating classes and there's four classes so there's going to be a lot more people here when it's the school, like during the day for school, than there is for the game.

Tonya: I disagree because there's not going to be that many people in the gym during classes than during the game.

Rebecca: So Tonya's arguing I think, tell me if I'm right, Tonya's arguing that the graph would be more accurate if it did something like that (I draw on the board)

(several students say yeah)

Tonya: Yeah, like that. It'd be more accurate like that because including cheerleaders, everybody that comes to the game, basketball players, the other team and people that came for their class and everybody's gonna be (?)

(something generates a big laugh)

Rebecca: Cassie?

Cassie: Because in each gym period there's not going to be 700 people in there because there's only like,(somebody says something about freshman and gym) what?

Rebecca: So it's not people-

Cassie: You're not going to have more people, you couldn't fill up the whole stands with your gym class

Rebecca: Melissa, do you want to add to that? Same thing?

Rebecca: (points to Daria)

Daria: And besides like in my gym period the guys are playing football not basketball so it can't be high like around 9 either.

Rebecca: SO you're arguing at this time of year if you were looking at it there wouldn't even that many people during the gym during gym class.

End of transcript.

9/1/00 Period 4

Discussion at end of Mystery Graphs Activity

(Graphs of eight events are posted up on the walls, students have just tried to match the list of situations with the graphs of the events created by their fellow students)

Kathryn: Alright. Let's start the discussion, let me ask is there one of the, is there a graph up here that just really jumped out at you, it was just kind of like the first one that you knew absolutely which one it was? Pause. Uh, Anna?

Anna: (starts to speak, hard to hear)

Kathryn: Ok. Let me just remind everybody as we're going through this, kind of remember some of the guidelines that we've talked about for discussion as far as making sure that you're listening to what other people are saying, and also that you're talking loudly enough so that everybody in the group can hear. Ok? I know that everybody is going to have lots of great ideas that they want to contribute but remember to raise your hand and be acknowledged just to keep things orderly otherwise I, nobody can hear what anybody's saying. Ok? Alright. Go ahead Anna.

Anna: Ok, I think that-

Kathryn: I'm sorry, we, now I've lost track of which graph we were looking at. Which one did you say?

Anna: That one.

Kathryn: A you thought was obvious right away? Ok.

Anna: Yeah. I knew that one right away because if you look, it goes down during like the middle of it, so like, it goes down and it goes back up like toward the night area, the evening, so asleep in Indiana, number three, is A.

Kathryn: Alright. So you thought that one was three?

Anna: Yeah.

Kathryn: Alright. Did anyone have that A was anything else other than three?

(students say they had three)

Kathryn: So everyone is in agreement that A is the number of people asleep in Indiana? Would- (some students talking) Remember if you have a comment you'd like to make you need to raise your hand please. (things quiet down). Would anyone like to make any other comments about A, in support of it being the number of people asleep in Indiana? (5 sec wait time). If it does indeed stand for the number of people asleep in Indiana, is there anything you would change or do you think the graph is pretty much ok just the way it is? (5 sec wait time). Ok. Group, why don't you tell us, what was yours?

(some in the group says – asleep in Indiana)

Kathryn: It is asleep. Ok. You want to tell us a little bit about why you designed the graph the way you did?

Brittany: (inaudible) We just went with the majority of people. Early, early in the morning a lot of people aren't awake, so, but then there are some that get up early to go to work, then like towards the middle of the day most people are awake, unless they take a nap, or I don't know, they have a job where they have to sleep during the day, but most people are awake and then towards the later part of the night a lot of people would be asleep (inaudible)

Kathryn: Ok. Anyone have any questions for this group? (2 sec wait time) I have one question. Uh, if you take a look, take a look at there midnight, I guess on either end it's kind of the same amount. And then take a look at, yeah you can be our pointer Jon (Jon is pointing at the graph on the wall above his head), take a look at the number of people asleep at noon. Alright? About what fraction of the people seem to be asleep at noon? Pause. As it's compared to the number of people asleep at night. Jessica?

Jessica: Maybe like 1/3. Or a little bit more.

Kathryn: One third?

Jessica: I can't tell which one-

Kathryn: Can you see it? Can you point to which one is noon again, Jon?

(Jon points)

Jessica: Yeah, a little bit more than 1/3 but less than 1/2.

Kathryn: Ok. What do you think about? What do you think about that ratio? (4 sec wait time) DO you think there's about half the number of people asleep at noon as there are asleep at midnight?

Anna: No.

Kathryn: Anna, was that you?

Anna: Yep.

Kathryn: You want to expound on that?

Anna: Well because a lot of people like work during the day and a lot of kids are going to school, you know? The only people who really are sleeping are people who have night jobs or kids that are sick.

Kathryn: So how would you change that then?

Anna: Uh I would say that like 1/3 or not even that.

Kathryn: You might bring that down a little bit?

Anna: Yeah.

Kathryn: How many would agree with that?

(some? Hands go up)

Kathryn: Any other comments about this one? Jessica?

Jessica: I'd probably bring it down a lot more because uh, most people aren't sick and most people, I'm pretty sure on average, people don't have a night job, so I'd bring it down a lot more.

Kathryn: We might not know the exact numbers but it's just kind of the idea to kind of think about that, think about the relationship between the different numbers you have graphed. Yeah, Brittany?

Brittany: We didn't really go by a certain number, we just estimated by, you know, the dot is really high up that means that there's more people sleeping and we weren't like exactly like oh there's ONE THIRD. In the picture we just estimated.

Kathryn: So you weren't thinking about that? You were just kind of thinking about the ups and the downs and the highs and the lows?

Brittany: Yes.

Kathryn: Ok. Alright. Is there, uh, is there another graph, uh, that kind of jumped out at you, that it was one of the first ones that you knew exactly what it had to be. Jessica?

Jessica: C

Kathryn: C? Ok.

Jessica: I thought it was pretty obvious because school starts at 7:30 and the line goes up around seven thirty and everybody's in the, and, well, and . . . uh, and they, but then they stay there for a minute or two and then they drop because everybody's in their classrooms and they go back up when the period is over.

Kathryn: Alright. How many, so you're saying that you think that that one's number one, the number of students in the hallway. Alright. Are there other choices that people made for graph C? Joe, what did your group say?

Joe: Well I think she's wrong, I think D is for people in the hallways.

Kathryn: Ok, so you think D is number one, alright. And so what did you pick for C?

Joe: Uh, C, in the classroom on a school day.

Kathryn: You think that one's five? Is that right? Ok. Why did you think that one was in the classroom on a school day?

Joe: Because, uh, I don't know, that's what other people in my group got. (a few students laugh)

Student: -an arch in the graph on like homeroom and you wouldn't be in the hallways during homeroom.

Kathryn: Did they label that homeroom?

Student: Holly did.

(someone says oh, oops)

Kathryn: Ok. If that hadn't been there do you think it would have been as easy to make that decision.

Brittany: Maybe not as easy, but probably would have-

Kathryn: Probably still would have picked that? Did anyone pick anything other than one or five for C? Alright. Now, what about D? Because I guess I would agree that they look kind of similar, C and D? So we have a vote for number one for C, uh, number of students in the hallways, did anyone choose anything else for D? Uh huh?

Student: Five

Kathryn: Five? Ok.

Student: They look alike.

Kathryn: They look alike so they flipped them around?

Student: Right.

Kathryn: Ok. Would anyone like to argue one way or the other for those? Uh, Joe?

Joe: D I think is people in the hallway because during the lunch period it stays up instead of traveling down . . .

Kathryn: Alright. Pause. What differences are there I guess in those graphs that might help you make a decision? Adam?

Adam: (inaudible)

Student: (inaudible)

Kathryn: Alright. They labeled the periods?

Student: They didn't say . . .

Kathryn: Is that what those numbers are that are up there at the top? Is there anything else that's different? (six second wait time) Brittany?

Brittany: The graph is more up and down . . .

Kathryn: This one's more up and down? In what way is it more up and down?

Brittany: It just looks like it is.

Kathryn: It just looks like it is? Ok. Angel?

Angel: This one, it goes up . . . and that one it like stops in the middle . . . this one it goes up.

Kathryn: Yeah, that's certainly something that's different about them. Let's go ahead and hear from the groups what theirs where supposed to represent. Anna (nominated), what was C?

Anna: Ok. This is the number of people in this classroom during a school day. And it goes up for period 1 and then it stays up there for like a little bit of time and then it comes back down. And these are the passing periods when there's mostly nobody in here because your usually out in the hallway and we didn't know exactly when you had classes or when you don't have classes so we just kind of like did all the periods anyways. So then it just keeps going back up and back down and the top plateau things are supposed to represent the classroom time that everyone's in the classroom and then the bottoms represent the passing periods.

Kathryn: Alright. Is, uh, Jessica?

Jessica: I was just gonna say the reason our group didn't pick that one, or one of the reasons, is because you had said you teach classes 2,3,4, and 5 so that would mean that, and so people wouldn't be in here during those periods.

Kathryn: Ok. Alright. Because you had asked me specifically when I was in the classroom and this group didn't know so they just kind put all the periods on. Yeah, Jon?

Jon: Did anybody talk about how the, there's slants in the, like between the different times or whatever. (student starts to say something and Kathryn starts to say go ahead and Jon continues) Because like up here there's no like, all the people leave, and so it's just a straight line, nobody's in there. And then that's like to represent the time that it takes for the passing period.

Kathryn: Ok, so you're talking about down here in this, actually up here and then down here in this little valley there's kind of a flat piece and in this graph it just comes to a point (draws on the overhead)?

Jon: Uh, no.

Kathryn: No. Ok.

Jon: The way it, uh, theirs just goes straight up, this one slants.

Rebecca: So what does the slant-

Jon: Alright.

Kathryn: Which, you know what, can you, go point to where you're talking about because I'm not sure exactly where you're-

Jon: (walks over to graph) Like right here. So this is later than down here. And over there, there's barely a-

Anna: So why does that one go so far over and ours goes straight up?

Jon: Yeah. This is like the time that, that's how to distinguish between the two. They're in the hallway or in the classroom.

Kathryn: Ok. So you're saying because it, are you saying because it's steeper-

Jon: Yeah.

Kathryn: Or because, actually that one's not as steep as this one, this one goes up a little more steeply? Ok. Alright. And that helped you decide-

Jon: Which was which.

Kathryn: Which was which? So your group thought this one was what? In the hallways?

Jon: Yeah.

Kathryn: Ok. Alright. And why did you think that in the hallways needed to be less steep than this one did?

Jon: Because, uh, it takes seven minutes, don't they have seven minute passing periods? It only takes like ten seconds to get out of here.

(seven seconds pause)

Kathryn: Ok. So this one took less time and that one would represent the passing periods you're saying?

Jon: No. Uh, the people in the classroom. It takes less time to leave the classroom.

Kathryn: Ok. Anyone else like to, like to comment on what Jon's saying? (seven sec wait time) Alright. Group D, can you tell us a little bit about your graph and why you designed it the way that you did? And this one I guess is the number of the students in the hallways.

End of transcript

9/5/00 Period 4

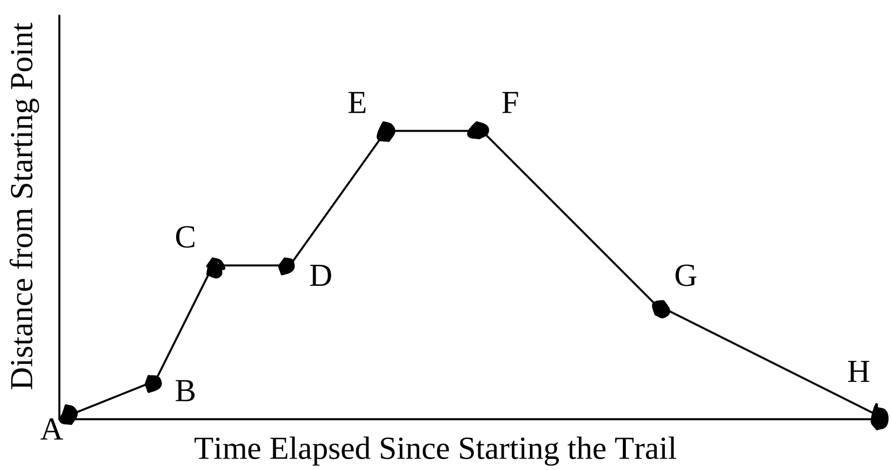
Kathryn: Take about 10 minutes . . . to get together with your group, this is groups time, this is not free time, this is not time to be up across the room, this is time to talk with the other people in your group because you may have missed something that someone else got correct. If you have time, and there are some revisions that you need to make, you could read your story to the group and they could kind of make some suggestions, or help you understand what it is that I'm looking for, alright.

(despite these instructions, some groups are quickly off task)

About 15-20 minutes later the following discussion begins

Kathryn: Were there any issues here that you would like to discuss? (Overhead of Mrs. Thomas's Roller blading Trip is up)

Mrs. Thomas's Rollerblading Trip



- As I skated from Point A to point E, between which two points was I skating the fastest?
- What was I doing between points C and D?

- Why does the graph start to come back down again at point F?

- Was I skating faster between points F and G or between points G and H?

- Write a story about Mrs. Thomas's Rollerblading Trip. Be creative. Be sure to write about each segment of the graph.

(A student says #3)

Kathryn: Number three? The question says why would the graph start to come back down again at point F. Would someone like to start us off? Chrissy?

Chrissy: Well, because you are heading back toward the car.

Kathryn: Why does the graph coming down tell you we're headed back to the car?

Chrissy: Because it's the distance from the car, so when you are going away from the car and then you are heading back to the car.

Kathryn: Wait a minute, when I'm heading away from the car?

Chrissy: Well, wait, since you are going away from the car it says distance from the car on like the vertical axis and the time elapsed since you're on the trail, it's still timing you but you are going back toward the car, you're going down again, but you're still heading out this way because time's still ticking.

Kathryn: Ok, alright. So in other words when we get to this part of the graph remember that this vertical axis stands for distance, so what's happening to the distance, the numbers for distance on this part of the graph? James?

Joe: They're getting smaller not bigger.

Kathryn: Alright. Think about also when we did the experiment with the CBL, alright?, what it meant when the graph had an increasing slope and a decreasing slope . . .

Kathryn: Another big question . . . that I had was for this part of the graph, basically the graph is going up . . . , there's an upward trend. The big question I want to ask is does that mean that I'm skating up a hill? (pause) How many say yes, it means I'm skating up a hill? (Kathryn looks around, no hands) How many say no? (some hands go up) How many just aren't really sure (steven's hand goes way up) Ok. I know, I know there are a few because, like I said, it kind of came out. How about someone arguing for one of those positions? Joe?

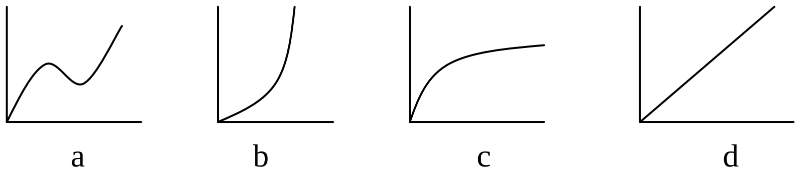
Joe: I don't think you can, I don't know if it says that you're not on a hill but it can't say that you are on a hill because (inaudible)

Kathryn: Alright. So he's not discounting the possibility that I could be on a hill but he's saying that I don't necessarily have to be. Anyone else like to add to that? (5 sec wait time). Think again about when we used the calculator-based laboratory up in the front. Were we able to make a graph that went up like this? That increased? Did the person have to walk on a hill to do that? (A couple of no's)

Kathryn: No. So that's an idea that came out in some of the stories, I was getting the impression that you felt like we had to be going up a hill here, and you had to be coming down a hill here but keep in mind what that amount on that vertical axis stands for and keep in mind what you saw when we did the CBL, also.

. . .

Kathryn: For #1 it says the population . . . increased at a rapid rate at the beginning and then leveled off as time passed. Would someone like to get us started and tell us which graph you selected? (5 sec wait time). Let's see. Rachel, what did you pick? (Kathryn nominated Rachel)



[time is on the x-axis, population on the y-axis]

Rachel: c

Kathryn: c? Did anyone select anything other than c? (Kathryn looks around the room). Rachel, you want to tell us why you picked c? Nice and loud so everyone can hear. (Kathryn nominated Rachel)

Rachel: (inaudible)

(Kathryn does a thumbs up for louder)

Rhonesia: (inaudible)

Kathryn: Alright. Uh, how about graph b, does graph b show an increase? (Kathryn is looking at Rhonesia)

Rachel (inaudible – might be "it increased but it didn't level off at the end)

Kathryn: Alright. So you didn't see the leveling off in b that you saw in c? Anyone else have any comments about this one? (pause) How about #2? Somebody want to volunteer which one they answered for that one? Adam?

Adam: b

Kathryn: b? Alright. Did anyone pick anything other than b? (no hands) Alright. Adam, you want to tell us why you picked b?

Adam: Because it says, because it starts to level off (inaudible)

Kathryn: Alright. Anyone want to add anything to that one? What about c? Why would you not pick c? (after 5 sec wait time, Beth raises her hand)

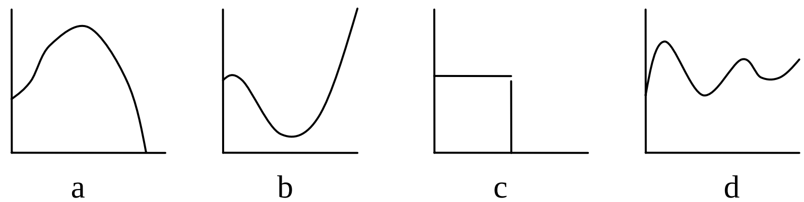
Kathryn: Beth?

Beth: Because it's decreased, er, it's a constant decrease, it became the same rate and in the thing it said it dropped rapidly and then slowed down and eventually it will deteriorate completely-

Kathryn: I'm sorry the bell rang-

Beth: Eventually it will decay completely and it doesn't say that in the words

Kathryn: Oh, ok. Alright. If eventually we extended it. Alright. How about 3#? . . . population . . . first increased then decreased until the [animals] became extinct. Selma? (Kathryn nominated Selma)



[time is on the x-axis and population in on the y-axis]

Selma: c

Kathryn: Did anyone pick anything other than c? (Some hands go up) Pat?

Pat: I picked a.

Kathryn: Anything other than a or c? Would anyone like to argue either for or against a or c? Brittany?

Brittany: Uh, I'm arguing against c because , uh, it, well first of all the graph doesn't show that it increases, in the first place, it shows that same rate but in the thing it says it increases. But also the line goes straight down (in c) but it doesn't say that all of a sudden they just disappeared, became extinct. So I think that a would be the better one because it gradually goes up and down.

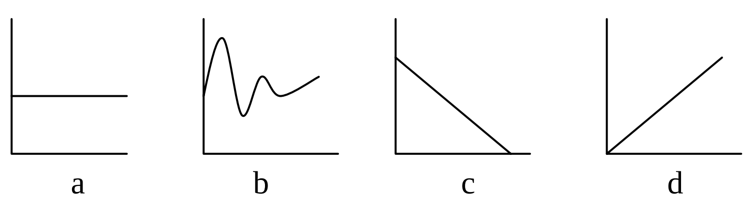
Kathryn: Ok. How many would agree with the points that Brittany made about c? (a few hands go up) Any other comments on c? Pat (nominated because he picked a earlier) why don't you tell us a little about a.

Pat: Well, I thought it would be that one because it kind of slopes up and curves for the rise in the population and it kind of peaks and goes down for the drop.

Kathryn: What about the part about becoming extinct? How is that shown on the graph? (10 sec wait time) Chrissy?

Chrissy: When they say it's extinct it goes all the way down, the population is on the bottom line which usually represents zero. (Kathryn nods several times)

Kathryn: What about #4, the population of the town remained constant? Robert? (note- Arial had her hand up)



[time is on the x-axis and population is on the y-axis]

Robert: Well I said it was a,c, and d because all of those graphs were constant.

Kathryn: Alright. Tell me what is constant about a.

Robert: Well, uh, the line didn't move, it didn't go up or down or anything, it stayed in one spot. (Kathryn nods)

Kathryn: Ok. Tell me why you say c and d are constant?

Robert: Because c decreases at a constant rate and d increases at a constant rate. (Kathryn nods)

Kathryn: Ok. Robert is right. There's something about all those graphs that's constant. Alright. Eric?

Eric: I'm arguing against c and d because if the population is staying the same that would, a represents that because it's staying level, the same amount of people, and c and d are saying that they're increasing or decreasing, meaning that the population wouldn't stay constant, because a constant would be like 50,000.

Kathryn: Alright. How many would agree with what Eric said? Any other comments about c or d? Robert, I think the difficulty comes in just reading the problem and understanding what it is that they're talking about that remains constant. Does it say that we want it to be at a constant rate?

Robert: No. It just says it remains constant.

And so on.

End of transcript

9/29/00 Period 3

Rebecca: What do you all think? I'm going to give you an equation that's not on your list but I want you to compare it to groups that you have

(Rebecca writes on the overhead $\dfrac{3(x)^2 - 9}{3}$)

Rebecca: Can you all see that? Uh, what I want you to do is look at your sheet, your pair's sheet, and figure out where you would put this equation. What category would you put that one in?

(Rebecca walks around while students look at there papers and talk)

Rebecca: Ok. Alright, let's see. Ok. What wants to start us off? Who knows where they'd put it? Who's decided where they'd put it? Raise your hand if you've decided where you put it. Ok. Who do we have in the back? Deanna?

Deanna: Uh, we had like-

Rebecca: Shh. Listen to her please. A little louder.

Deanna: Uh, we had for like our description for our category, each of these equations are formed in some kind of fraction and the denominator is always a number. So that's what we would put it in.

(Rebecca writes – fractions w/ denominator a #)

Rebecca: Does the fraction bar have to go all the way across these or could it be like 1/2 x + 2 ?

(Rebecca writes 1/2 x + 2)

Deanna: No we have it all the way across.

Rebecca: ALL the way across.

(Rebecca crosses out the 1/2 x + 2)

Rebecca: Another category. So that's one place this could go. Yeah?

Katie: The category we used were more than 2 operations.

Rebecca: Ohh. Who sees what Katie's saying? Who can explain what Katie means by more than 2 operations in this equation? Yeah?

Student: Addition, subtraction, multiplication, . . . exponents.

(Rebecca writes +, -, div symbol, exp.)

Rebecca: Ok. Who would put this in a different category than the two we've heard so far? April?

April: We put it in the equations where like it's some number to a power.

(Rebecca writes 2nd power/exp. category)

Rebecca: So you're saying, Ok. Is that right? I'll just write what our equation is over here.

(Rebecca rewrites $\frac{3(x)^2 - 9}{3}$)

Student: A new one?

Rebecca: No, I'm just writing it down so everybody can see it. What other categories could that one go in? Would any of you put that in a different category than what we have so far? The 3 times the quantity x, and then its squared minus nine over 3. Where would you put it? Tom?

Tom: We called it, the description of our equation is (inaudible)

Rebecca: Ok, division bar goes all the way across?

Tom: Yeah.

Rebecca: Ok. Deanna said that.

Tom: Ok.

Rebecca: So the question I want to ask everyone is this. What do you think is the most important characteristic of this equation? And what I mean by important is if you imagined graphing this equation, 'cause that's where we're going next, which of these things, the number of operations, the fractions, the exponent, whether the division bar goes all the way across or not, what do you think might be the most important thing in terms of what the graph is going to look like if you were to graph that equation? (15 sec wait time)

Rebecca: What do you think might be the most important characteristic if you were thinking about graphing that equation? (18 sec wait time)

Rebecca: Anybody want to put themselves out on the line and say what their feeling is? You can test it out and if you're right after we test out graphs we'll come back and give you credit for your idea. (this seems to generate some interest – i.e. noise)

Student: What is it?

Rebecca: What is the most important characteristic of that equation in terms of graphing it?

Jackie: Ok, well I-

Rebecca: Shh. Listen to her please.

Jackie: Me and Cassie thought it was division.

Rebecca: Division?

Jackie: Yeah, that's what I think it's going to be, I don't know.

Rebecca: Does anyone share Jackie's opinion?

Katie: I think it's division too (a couple of others agree)

Rebecca: Ok. Anybody have a vote for something else? (noisy) Whose got a vote for something besides division? Jeff?

Jeff: Variable

Rebecca: What do you mean?

Jeff: 'Cause you can't really do anything until you know what the variable is.

Rebecca: So, I'll add that up here.

(Rebecca writes What the variable is)

(Katie says something)

Rebecca: I think Katie needs you to say more about what you mean by that, Jeff.

Jeff: You can't do any of the other operations until you find out what the variable is because it comes 1st in the order of operations because it's in the parentheses

(17 sec wait time)

Tom: Does the variable matter? What it is? Because it can be whatever the equation asks, for, like it doesn't matter what number it is, its whatever it says . . . so it shouldn't have anything to do with how the equation goes. What are we trying to find, how it's going to turn out on the graph?

Rebecca: Yeah. What will the graph actually look like? What are the most important parts of our equation to tell us what the graph will actually look like? Not single points, the whole entire graph.

April: Uh, I think the minus 9

(Rebecca circles this part of the equation on the overhead)

Rebecca: Why do you think that?

April: Because like since it's subtracting something like even though there's division and multiplication in it, you're still subtracting something so the graph would be like, it would be going down (Shows down slope (negative) with her hand) more because there's subtracting in the equation

(bell rings)

End of transcript

9/29/00 Period 4A

Early in the class period

Rebecca: I'm goin to put an equation up here that's not on your list but you're going to recognize, it has some similarities to what's on your list.

$$y = \frac{3(x)^2 - 9}{3}$$

Rebecca: So my first question to all of you is what are some characteristics of that equation?

Student: It has exponents.

Rebecca: Mindy?

Mindy: The variables it uses x and y are the same as the ones on our sheet.

Rebecca: (gestures to Anna)

Anna: It has fractions, you divide.

Rebecca: Who can add to what Anna said about the fraction issue?

Jessica: There's a division bar all the way across.

Rebecca: Cathy, did you want to say something?

Cathy: No, not about the fractions.

Rebecca: It's Ok.

Cathy: The parentheses.

Rebecca: Who can build off Cathy's idea and say something more about parentheses?

Jessica: There's a variable in it.

Rebecca: Who can say some more about that?

Student: There's an exponent outside the parentheses.

Rebecca: Anything else we can say about parentheses in this situation?

Jessica: There's not a problem in it.

Rebecca: Who thinks they can explain what Jessica means? I see a few hands.

Anna: There is no multiplication or adding in there, just a variable.

Rebecca: Ok. Let's hear from some people who haven't talked yet. There's some other things that you can say about this equation that I don't have listed yet.

Pat: It's giving you the value of y.

Rebecca: Can you say some more about that, what you mean?

Pat: Like it's y equals.

Rebecca: Have you heard the expression y in terms of x? That means you have y equals on one side and on the other a bunch of terms with an x in it. So in this case you have y in terms of x (wait time) Ok. My next question to you, now that we've layed out some of the different pieces of this equation, which is what you kind of did on your paper, is what do you think, this is kind of an opinion question at this point because we don't have a definitive answer, but what characteristics uh do you think are important for the graph? Like what do you think is going to be important in terms of what the graph is going to look like? A big part of this remember is if I look at an equation,, what might the equation be telling me about the graph? So what characteristic do you think might be important?

Chrissy: Well I think it's the way the parentheses are set up and how the exponent is on the outside of it and how the x is alone (inaudible)

Rebecca: writes below the other equation:

$$y = \frac{3x^2 - 9}{3}$$

Rebecca: Do you think the top equation and the bottom one would have different graphs?

(a couple mm hmms and one no)

Rebecca: How many people think that? (a couple mm hmms) How many people think they would have different graphs? (a couple hands go up) Does anyone think the equations wouldn't have different graphs? (a hand or tow) That will be something interesting for you to check out, I think. Brittany, you want to add something?

Brittany: Uh, I think that, I don't know, it does make a, oh actually I guess it does influence, because if inside the parentheses with another like problem, like if there was another problem inside, it's saying that you figure out that problem and then the answer you do it to the second power its not just whatever the first one is, so it makes a different if the parentheses are there.

Rebecca: Yeah?

Adam: I change my answer to it will make a difference because without the parentheses it says three x squared and with the parentheses it just says x squared, the three x isn't squared, just the x is.

Rebecca writes:

$$y = \frac{(3x)^2 - 9}{3}$$

Rebecca: I know that's not what you said Adam, but I'm putting it up there. Now we have three different equations up there. We have x with parentheses around it, we have no parentheses, and we have three x in parentheses. Which are the same and which are different? Anna?

Anna: I kind of think that the first one and the second one are more the same because you would have to multiply them together anyways because since there, like the answer, x to the whatever power times three so that you can get whatever that is minus nine, but in the third one you don't do that, well you could but it's different.

Rebecca: Lisa?

Lisa: I think the second two are the same because you would still have to multiply three times x and then square it just like (inaudible). So whether you put parentheses around it or not it's still, you're still going to have the same thing.

Rebecca: So which are you saying?

Lisa: The second and third.

Rebecca: Angel, did you want to say?

Angel: That's what I'm saying.

Rebecca: Lindsay?

Jessica: Well it's kind of like what she's saying, the second two are alike because in the second two with the order of operations you're going to do three x squared but in the first one you are going to do x squared times three instead of three x squared.

Rebecca: Would it make any difference if I wrote it like this?

Rebecca writes:

$$y = \frac{3 \bullet x^2 - 9}{3}$$

Rebecca: How does that one fit in with the others? What do you all think?

Eric: It's the same as the first one.

Student: Uh huh

Rebecca: Adam?

Adam: I say its like the second one just without the little multiplication sign.

Student: Yeah me too.

Anna: I think they're all the same.

Brittany: Well if you replace them with numbers

Lisa: Some you are squaring the number and others the whole thing.

Rebecca: What I really like about this question is that you all will get the opportunity to get a calculator, put the equation in, punch a button, get your graph,, and see how that works out.

Eric: It'll be different if the x is a negative.

(a couple of yeahs)

Rebecca: Explain what you mean to all of us.

Eric: If the x is negative and you have the parentheses around there, I'm not sure which it is but you take away the negative and do the exponent and then add the negative back in with one of them, and the other, I think its without the parentheses, then you keep the negative in and do that to that power or whatever. There's a difference. . . .

. . . .

Rebecca: Let's move on to a different question. Unless anyone has any parting remarks on this one . . . How about this.

Rebecca writes:

$$y = \frac{3(\mathrm{x})^2}{3}$$

Rebecca: What do you think about graphing the first equation and then this one? I know they look different. Any thoughts? (wait time) I'll throw up another one and see if that sends your mind working.

Rebecca writes two equations:

$$y = \frac{3(\mathrm{x}) - 9}{3}$$

$$y = 3(x)^2 - 9$$

Rebecca: Which of those bottom three do you think will look most like the top [original] one?

Brittany: All you're really doing is taking a step out. Like in the second one down you're just taking out the step of subtracting nine and then on the other one you're taking out the step of the exponent and on the other one you're taking out the step of dividing by three.

(Rebecca writes beside the graphs A, B, C)

Rebecca: So which of those bottom three do you think might look a lot like the top one? (Wait time) Adam?

Adam: Number three because all you are doing is not dividing by three so it will be parallel to the first one, the line will.

Pat: I think number one might look the most like it, because it's like, at high numbers multiplication would affect the line of the graph more and minus nine at a high number like 300 and something wouldn't affect the line as much.

Brittany: Well what if it was a low number?

Pat: Then it would. I'm just saying.

Rebecca: Anyone want to respond to Adam or Pat or throw in their own idea? (long wait) OK.

Rebecca: What I want to say to everyone to focus you for us starting graphing is what we are going to try to do is get at this kind of family of equations issue. We started out with this one and I made some that are a little bit different but are kind of related, have some similarities. Maybe this one belongs in a family with this one, or this one, or two of them or all three of them. And so a big piece of this is getting some ideas in you own minds about equations and their graphs . . .

End of transcript.

<u>9/29/00 Period 4B</u>

Near the end of the period

Task: Mystery Equations

Rebecca: You put y = x times 2 + 2?

. . . .

Rebecca: Who put something down besides a and c? Anybody? Ok. What I need if for somebody to just start off by telling a little bit about why they put down one of these two. See if you can argue your case and get your credit here for one of these two. (long wait time). Why'd you put down a? Why'd you put down c? (long, long wait time)

Student: Not a or c.

Rebecca: Ok, why do we think it's not a, may not be a? Why do we think- yeah?

Brittany: Well if you put the numbers, the variables, say number four, if you put that in for x and minus one is three put three times three is not-

Rebecca: Ok, so basically Brittany's telling us, let me see if I write what you mean. (I Write) Right? And then

Student: You know like for the second shape, they're triangles and then if you go two plus two times two minus one and then the two minus one would be one times the (inaudible) which is eight and that's eight sides.

Anna: Maybe the formula only works for one shape then.

Student: But it didn't work for the first one, it worked for the second.

Anna: Yeah and it doesn't work for certain-

Student: Forever

Student: And the formulas not right.

Rebecca: Does anybody have anything to say about this? Adam, did you want to say something?

Adam: I changed my mind.

Rebecca: Ok, that's cool. (wait time) Does anybody have anything to say about this process that was just described here. Eric?

Eric: You wouldn't do the six plus two, you'd do two times three and then add the six and then it works.

Rebecca: Alright. We've got two methods on the board that give us two different answers.

Anna: Well, the number, it seems like for number one, it would work, but that wouldn't be (?), you'd have to count every single line.

Rebecca: So if we did what? Six plus two times one minus one?

Anna: Yeah. One minus one is zero, then there'd be eight but there's only six on the outside, so you'd have to count the ones in the middle too which (inaudible)

Rebecca: Ok, so I'm thinking that Anna's method . . . we have an issue here people that we need to think about. Anna's method and Brittany's method are somehow different from Eric's method. See what I'm saying? Does anyone have anything to say about that? Yeah, Brittany?

Brittany: I think Eric's is right and mine and Anna's is wrong because like he did the order of operations, well first he did parentheses and then he, you multiply before you add so like he said it's two times three and you do that first before you add in six so that would equal twelve and that's the perimeter of the outside.

Anna: I'd agree with that same thing but one minus one is zero so you'd-

Brittany: So two times zero is zero and then plus six is six and that's the perimeter.

Rebecca: Would that help with shape two or not? Would Eric's method-

Anna: But where's the two then? You're not doing six plus two which would be eight and then you have to multiply that by-

Brittany: No you don't.

Eric: Because order of operations you multiply before you add, therefore it works.

Anna: How would-

Student : Multiply becomes before addition so you'd do two times zero first so therefore it would be zero. So six plus zero is six.

Anna: Well how do you multiply by the two and not the six?

Student: Because (overlapping comments)

Anna: What if you wrote it the other way around, what if there was two plus six then would you multiply by six?

Rebecca: If it was like that you mean, Anna?

Anna: That's what I was saying, if it was the other way around.

Eric: But it's not. (laughter)

Rebecca: If it was, I think Eric and now Brittany would be arguing if it was this way, that they would do six times zero is zero plus two is two. I mean that's how THEY are arguing it and you're arguing that –

(a couple other comments)

Rebecca: Does anybody over here have any opinions about whether or not you should do this multiplication first or whether or not you should do this addition first?

Student: Well like Anna says it depends how you write it but according to order of operations (inaudible – but I think basically saying that order of ops backs up Eric and Brittany's argument)

Rebecca: I'll say this, order of operations is kind of like an arbitrary, I mean it's rules that are layed out so everybody's doing the same thing. There's not some grand underlying mathematical meaning to one or the other.

Jessica: Uh, well for uh, if like Anna said, if you look around or whatever that means you'd have to change the shape because the formula, on the first one, which is, whatever, that uh matches the shape. It also depends on the shape of the shape.

Rebecca: So this would be something else?

Jessica: Yeah, that would be some kind of other, but if you changed the shape, then

Rebecca: So if we use Eric's method, will this equation work for those four shapes, if we use Eric's method of this first and then add six?

(a few yeahs)

Rebecca: Ok. What about c? Some of you put c though. Is c a better choice? Will c work somehow for us on that one? Here's what c was. Let's make sure we make a decision about this one before we give a the gold star. Yeah, Adam?

Adam: C doesn't work.

Rebecca: Can you explain why?

Adam: If you plug in one for x, (inaudible – he explains how to multiply and that that doesn't give you the right perimeter)

Rebecca: Did this formula work for any other, if we happen to put in a different number and didn't test one. Like if we didn't test all of them, could we be fooled? Does it work for any of the others? (wait time). OK. Adam has given us, what's called in math, I don't know if you've seen this word before, a counterexample. Ok? Adam has proven to us that this equation can't be right because he's given us an example where it's not. And as Brittany said earlier if it doesn't always work then it doesn't work. So one example disproved the equation. Ok, so have we reached a consensus on what graph four is?

(several a's)

End of transcript.

10/31/00 Period 3

Task: Right now I am 2 miles from home. In 1 hour, I will be 5 miles from home. In 2 hours I will be 8 miles from home. Graph these points (with time from now on the horizontal axis and distance from home on the vertical axis). If the trend continues, plot a few more points that would be on this line. Write the equation of this line, define your variables.

Kathryn: So let's make a list of our equation conjectures right now because we're going to try to show in various ways which of these equations actually work and which ones of them maybe don't. And I don't know because I haven't looked at all of your equations but we're just kind of making conjectures right now. So let's just get a list of equations here. Katie?

Katie: m times three plus two and m equals the hour.

Kathryn: Ok. M times, I'm sorry- say it again.

Katie: Three plus two.

Kathryn: m times three plus two. And m stands for?

(Kathryn writes Katie's equation on the overhead and also what the variables stand for)

Katie: The hours.

Kathryn: The hours? Ok. And did you have something here equaling that?

Katie: Equals like, the distance, d.

Kathryn: d. Alright? Brad?

Brad: h three

Kathryn: Shh. Let's listen now please.

Brad: h three plus two equals d. It's better because –

(some noises from other students) (Kathryn writes Brad's equation on the overhead plus h=hours)

Kathryn: And I assume h is hours. Alright, another equation. Mia?

Mia: Never mind.

Kathryn: If you look at these, are these (pause for quiet) are these equations fundamentally the same as each other or are they different from each other?

(some students say they are the same)

Kathryn: Basically the same, right? There's different variables, here, she, and I don't remember if you said that there was a dot or not but you said times and Brad just said h three but this would mean you were multiplying, alright? So basically the same equation. Does someone have an equation that's different than that one? Cassie?

Cassie: y equals x plus three.

Kathryn: Alright (writes in on the overhead). And y and x are?

Cassie: the distance from (inaudible)

Kathryn: x is the distance? And y is the?

(Cassie – inaudible)

Kathryn: You didn't say what y was equal to? Ok. Did anyone have a different equation? (10 sec wait time) Alright. We're going to have to get calculators passed out quickly, what I'd like – let me ask this first, before we do that. Would anyone like to say anything about any of these equations? Either why you think that it would be correct or why you think that it wouldn't be correct? Brad, let's start with you and then Sarah.

Brad: Ok, well I know the first two are correct because they're the same and I know mine's correct, so, and then the-

Kathryn: Well why? Tell me why.

Brad: Well because h three would be, cause each time it goes up it goes up by three and then plus two because that's what it started from. And the third one is wrong because the equation says they're at two miles from home like right now so that equation would be wrong because it would be two miles off each time.

Cassie: If two miles is x then you add three to get five.

Brad: but that's only (inaudible)

Kathryn: Ok, say that again Cassie.

Cassie: Because if you put, ok, two in the place of x

Kathryn: Alright (writes in on the overhead)

Cassie: Start off with two and add three and that's five.

Kathryn: y is equal to five.

Cassie: Yeah. And keep on adding three –

Kathryn: Ok, what you're looking at, ok, go ahead, Sarah.

Sarah: Ok, well like, that y=x+3 I think that's not correct and uh because it's going by every hour not every half an hour

Kathryn: Now why do you think this y=x+3 represents every half an hour?

Sarah: Because you go up three every, go up, well, I don't know how they got three but on my graph, I uh, counted like six, like, no wait, six spaces between every two miles so and like the half number is the three.

Kathryn: What was your scale though? When you set your scale up did you set it up so that you were counting on here by ones?

Sarah: No.

Kathryn: So you made this two, four, six, eight, like that (draws it on the overhead)

Sarah: Yeah and between that number was four squares so.

Kathryn: So your first one, now did you number this one by one down here?

Sarah: Correct

Kathryn: One, two, so your first one was here at zero two, and then your next one was at one, five? So you were saying you went up how many spaces?

Sarah: I went up every six spaces per hour and that's how I thought

Kathryn: I'm not sure where your, I'm not sure where your, I want to see your graph. (Kathryn walks over to Sarah's desk)

Kathryn: Oh, ok. What she actually did, is, she went up one, two, three, four spaces and then put a two, right? And then one, two, three, four spaces and then put a four, and then one, two, three, four spaces and put a six. Remember though Sarah that you need to think about, uh-

Sarah: Yeah, that's what I-

Kathryn: You need to pay attention to your scale I guess not just the numbers basically, does that make sense? Ok. So, does that change your argument over here?

Sarah: Well you like, you still have to think about what it is down there

Kathryn: Now you've confused yourself, ok. Anyone-

Sarah: I think the two top ones are correct (inaudible)

Kathryn: And you were trying to argue against the bottom one ok. Alright. Anyone else like to add to what Sarah was trying to say or-

Brad: I lost it.

Kathryn: Huh?

Katie: Well I don't know, but like when they said x+3 I think they thought well they have to remember that it starts at two, starts off at two, you have to add two. And then it changes.

Cassie: Ok, I-

Jeff: What they were doing is you have to get the one before it in order to add the three

Brad: Leave it to the men to tell the girls

Kathryn: Brad. Do you remember what we called that kind of thinking where you had to know a certain item in the list before you can get the next one? (short pause) It's been a while since we've talked about this.

Brad: You said it before but I forget it.

Kathryn: Yeah, that's what's called recursive.

Brad: Recursive, that's right. I was getting ready to say that too.

Kathryn: When you need to use one answer to get the next one, alright? And I think he's correct. I think that is kind of what you were thinking was that you needed to take the two that you started with, add three to it, that gives you the next number in the list doesn't it? So I think you were kind of thinking recursively there, alright?

(class moves on to entering the graph into the graphing calculator)

End of transcript.

<u>10/31/00 Period 4</u>

Task: Right now I am 2 miles from home. In 1 hour, I will be 5 miles from home. In 2 hours I will be 8 miles from home. Graph these points (with time from now on the horizontal axis and distance from home on the vertical axis). If the trend continues, plot a few more points that would be on this line. Write the equation of this line, define your variables.

Rebecca: These are what you came up with yesterday as possible equations for this situation. Y = something times x plus two, three x + 2, (inaudible), x+2 and x+3 and we need to come to some consensus on which of these works and which of these we think don't work. (Pause). What do you think? Is there anything that you would take off that list or anything you would argue to keep on?

(somebody says "what's the question")

Rebecca: The question is, those are our possible equations for the mile situation and I'm asking you all which ones we should leave up there and which we should take off?

Brittany: I have a question

Rebecca: Yeah, Brittany?

Brittany: What does the x stand for the graph?

Rebecca: That's a good question. What did different people make x stand for? Anybody, what did they use? What did their x stand for? Different people might have done different things, so what did anybody do? Yeah, Ryanne?

Ryanne: Time from home

Rebecca: Did anybody make x to be something else? (pause)What did y stand for?

Brittany: I don't know about y but I don't think that the uh last one works.

(long pause)

Rebecca: Yeah Angel and then Brittany.

Angel: I don't think that the second to last one works

(7 sec pause)

Rebecca: How come?

Angel: Because it goes by, it's three miles away from home for every hour and x was the time from home, I don't know but I don't think it's that.

Rebecca: Yeah?

Brittany: Uh, I don't think that the one above that one or the very last one. Like the 3x-1 and the x+3, I don't think (it's those either?)

Rebecca: Anybody help us build some arguments for any of these that we have people who are doubting these last three.

Brittany: Uh, the 3x+2 one works. (3 sec wait)

Rebecca: What makes you say that?

Brittany: Because if you substitute the x which is the time from home, say it's at zero, you substitute the x in and it's three times zero is zero and then if you add two on the point of the zero it's at two, like it works, it follows the pattern

Rebecca: Can I write it like this? (RM writes 3 (0) + 2 = 2)

Rebecca: Somebody said the first one might work. What's that underline for? I wasn't here yesterday.

(several people (including Anna) say any number)

Rebecca: Any number? Ok. Thank you.

Jessica: It doesn't work.

Rebecca: What doesn't?

Jessica: The first one. Because you could substitute the three into that but it would be the same exact one as the second one but if you put like four or five in there it doesn't work.

Anna: Yeah it does, I tried it, I tried all the numbers.

Rebecca: Ok, so give us an example Anna. You said I could put like four in there. Ok. So how did you test it?

Anna: I put it in on the graph,

Rebecca: And it made a line across

Anna: Yeah.

Rebecca: Oh, ok. So you graphed 4x+2 and it went across the two, something like that (Rebecca draws a picture) and then you said you put some different numbers here and it still kept going through like this maybe (Rebecca draws some lines with different slopes going through 0,2)?

(some people start talking)

Rebecca: Yeah, speak up, what's the problem?

Brittany: I don't understand that because, yes, if you substitute x for zero, but, I don't know, if you put-

Rebecca: So if you put zero in here you get two-

Brittany: If x is like two hours, or if you substitute x for like two hours, four times two is eight plus two is ten-

Rebecca: Is that right, two hours and ten miles?

(a couple of no's)

Brittany: It doesn't work, and it has to work for every single equation

Anna: I don't know how it worked but I did one through ten I think and they all worked.

Rebecca: And if, if it was one hour it would be six, so that's what you get for that and then for this one, the 3x+2, what did we get? One, five? And two, eight? So do you see my arrows? Here's our table of values for this one.

Rebecca: Does anybody else want to talk about taking off any of these? I'd love to be able to cross of a couple of these off here, IF we can or else decide that they're correct if they are. Yeah, Ryanne?

Ryanne: Well like the last one, x plus three, if you take the first point, zero two, and you take zero plus three is going to make a point going through three(?) (inaudible)

(pause)

Rebecca: Anybody agree or disagree with Ryanne? Should I cross this one off based on what Ryanne said or do we want to leave it on?

(a few people say cross it off)

Rebecca: Who came up with y=x+3? (Pause) What if I put two in for x, what if x is two? Watch this. (RM starts to make a x,y table) What if x is two, what's y? (Some say five). Actually I shouldn't write it like that. If we take two and plug it in we get five. What if we take five and plug it in? We get eight. What if we take eight and plug it in? Eleven?

(Angel and Selma laugh in the back)

Rebecca: I got 2, 5, 8 out of y=x+3, somebody else got 2,5,8 out of y=3x+2. Who did y=x+3?

(Angel and Selma laugh and raise their hands)

Rebecca: Now you all raise your hands and admit it.

(pause)

Rebecca: Any comment on this? How did you all come up with this? Do you remember?

(pause)

Selma: Cause it goes up three.

Rebecca: Cause it goes up three (Rebecca writes this on the overhead). This is actually a recursive formula, if you know the value you can plug it in and you can generate the pattern you have here. Don't let me forget when you have your calculator, I'll show you, at least I'll show Angel and Selma, I don't know if I'll have time to show everybody, but I'll show you how to plug this in in order to generate this pattern of numbers. And the way they'll usually show this is instead of using y they'll use x n, this is going to look a little funny but, instead of using x and y they'll use x sub n and instead of y they'll use x sub n+1 and that just means like if n is five, the value, you take the fifth value and add three you get the sixth value. You take the tenth value of x and add three, you get the 11^{th} value, so every time you add three, like Selma said.. But you have to enter it into your calculator a different way and you really can't make an x/y table with this. Because as some of the rest of you pointed out if you do make an x, y table you're not going to get the right points are you because you're going to get like zero, three. And zero three isn't the point, you all said zero, two is the point you need. So to use this equation you have to be in a separate category of stuff. So this is kind of a different thing here, alright, it's recursive. What about 3x-1 or x+2? Either of those work or not work and why? Who thinks they could explain why 3x-1 doesn't work or does work? Yeah, Pat?

Pat: It doesn't work because if you put in 1 it's one times three which is 3 minus 1 which is two and the point if five, so it doesn't work.

Rebecca: What about y=x+2, will that one work? (long pause). You can test that one out with your calculator.

End of transcript

<u>11/2/00 Period 3</u>

Task: The temperature two days ago was -9 degrees, one day ago it was -5 degrees, today it was -1 degree. The assignment for homework the previous night was to come up with an equation to represent this trend.

Kathryn: your assignment last night was to come up with an equation so that's the point that we're at, so you need to have that out so we can make a list of equations, possible equations, and then get out the graphing calculators and check some.

(students get papers out)

Kathryn: Alright. Let's start by making a list of equations you came up with and then we'll kind of go from there, talking about them and trying to test them. April?

April: y = x - 4

Kathryn: Another? Deanna? Same thing?

Katie: I had y= -4x+1

Kathryn: Are there any other different equations?

(long pause)

Kathryn: Everyone had the same or one similar? How many- oh, go ahead, Kevin.

Kevin: I had the first one.

Kathryn: You had the first – yeah let me just kind of see. How many of you had an equation that looked similar to the first one (some hands). How many had an equation that looks similar to the second one? Alright. Would anyone like to say anything about either of these equations? Either arguing for or against?

Katie: Well at first I did have x-4 but then like I wanted to see if I was right, and so I took like day one and day two and for day one it worked but for two it didn't work.

Kathryn: Ok. So we had day and then we had temperature. Ok. So how did you test day one?

Katie: Like two minus four is negative two.

Kathryn: Alright. So that would be day two?

Katie: Yeah. But then on my graph if you keep subtracting by four then day two should negative seven. So.

Kathryn: Ok, yeah and I think we had, here's the table that we had, let me recopy it here. Two days ago it was nine, one day ago it was five, today it was one. One day from now, I think we came up with this yesterday, and then two days ago it supposed to be seven degrees below. So testing that particular equation, number two, alright? Would anyone like to say anything else about either of those? Yeah, Sarah?

Sarah: Now that I think about it, it's the second one, because yesterday's was the miles and the hours, we had to add two to it, yeah, uh, and so I think we should add one to it.

Kathryn: Ok, yeah, I'm trying to, let's see, the problem yesterday that Sarah's bringing up was the one, the travel problem, and that was the one where again we had days from now and we had distance and it started out saying right now, in other words day zero, I'm two miles from home and in one hour I'll be five, I guess this was hours not days, in one hour I'll be five miles in two hours I'll be eight miles, ok? And you point was on this one that we had a y=, we had y= and somewhere we had a plus two in it and then on this one then why did you think you needed the plus one?

Sarah: Well as I think about it, the now is up one, so.

Kathryn: Ok, when we, if I kind of squeeze that one in, the now is day zero and that was at one, alright? So that's why you wanted to kind of put that in there? Alright? Deanna?

Deanna: I don't understand how it doesn't work 'cause mine went like that.

Kathryn: How this one doesn't work (y=x-4)?

Deanna: Yeah

Kathryn: What were you thinking when you came up with that equation, x-4?

Deanna: Well I just like took, uh (inaudible but can't really explain here reason well)

Kathryn: Cassie?

Cassie: Because on the graph each term is going down by four.

Kathryn: Alright? If we look at either the graph or the table, if we look at these values, the temperature is going down four and then down four again and down four again and down four again. And same thing on the graph, alright? And I don't remember if we talked about this yesterday or not because I've done this with different classes, did we talk about the idea of the rate of change for this? (apparently not). What would be the rate of change of the temperature compared to the days for this particular situation? Brad?

Brad: It would be negative four degrees because (inaudible) , negative because it's going down.

Kathryn: Negative four? Alright. So in other words it's going down, let me write it down here, so it's going down four degrees Fahrenheit? Now remember that when you're talking about a rate of change, it's always a comparison between two values, it's a ratio, with a rate the two things you're comparing don't have the same unit so we need another piece to this, what are we comparing it to? Brad? Nice and loud so everybody can hear you.

Brad: Per one day.

Kathryn: Per one day. Alright? So it's going down four degrees per one day. Or Brad was offering that maybe instead of putting the word down we could say it's negative four degrees Fahrenheit for each day, alright? So that's the rate of change in that, so if we go back and take a look at this, it's the same idea, every time this goes up a day we now the temperature's going down four? So in other words if I wanted to get the next day, Deanna, if I wanted to get the next one, what would the next day be, the third day?

(long pause)

Kathryn: Here's the information that we have so far.

Deanna: I can't read it.

Kathryn: You can't see it? Do you have it on your paper that you had from yesterday, on your paper?

Deanna: Negative eleven.

Kathryn: And how did you get negative eleven?

Deanna: Substracted four.

Kathryn: Subtracted another four? Alright. But in order to get that negative eleven you needed to know the one that came before, didn't you? Alright. And do you remember what we called that kind of a formula where if you want to get a particular answer you need to know the one that comes before it? (a couple students try to say it) Recursive? That idea is recursive? And I think what Katie was pointing out was with this particular equation if you put a specific value into this equation, we end up with two minus four or negative two and that's not the value we were looking for. We're looking for two, negative seven. And if you wanted to write this as a recursive formula, and that's good thinking, that's a good way to think about the problem but the equation would have to look a little bit different. Basically it would be written something like this. X sub n plus one is equal to x sub n minus four. And then you would let n be some value of some kind, like if you wanted to know the fifth day, you'd let n be five and you'd take whatever that answer is and you'd substitute that in and then you'd come up with the sixth value. So you'd have to write it a little bit differently. And I'm not sure how to make that graph on the calculator but Ms. McGraw knows how to do that. So it anybody's interested in how you'd put in an equation like that to come up with a graph then she can show you how to do that. Ok, where's the rate of change showing up in this other equation? (pause) Do you remember what we said the rate of change was? (short pause) We said the rate of change was down four degrees Fahrenheit per day. If you look at this equation (y= -4x+1) are we seeing that idea of the rate of change reflected in the equation anywhere? Sarah, where?

Sarah: The negative four.

Kathryn: Kind of seeing it right here? Ok? Ok. Uh, we have two equations to check here and we have the values that are in this table. Some of you said you couldn't see so let me make this a little bit larger just in case you don't have it on your paper. This was the data that we had both from the problem and then when we extended the values a couple more days. So what I want you to do when you get your graphing calculator is first of all plot these five points, and remember you do that by going into the stat menu, you'll have set up the stat plot and check your window . . .

End of transcript.

<u>11/2/00 Period 4</u>

Writing Equations

Problem: Negative 4x + 1 problem. Temperature, -2, -6; -1, -5; 0, -4. This discussion followed students coming up with equations (I listed their ideas on the overhead but we did not have discussion at that time) <u>and</u> using the graphing calculators to check whether their equations did indeed produce the correct graph.

Rebecca: Ready? Alright, what I've got up here is a little additional information that's come up. I added a couple equations to this list because it's this is important kind of stuff that's going to go for other problems you'll do. I put up 4x+1 and x times 1 plus negative four and we still have the others that other people suggested. And what I have here is a list of the data and let's see, x was the day and y was the temperature, in what degrees Fahrenheit? I don't know. Alright. And we know that two days ago it was negative six, one day ago it was negative 5, and now it's negative four and that's our trend that we're looking at. Ok. I think that almost everybody in here has figured out that y= x-4 is the equation that works for this data. But how can we know that? I mean how can that information be useful to you when you get another problem and you're sitting there staring at it and you're trying to figure out what the equation is. So who can just tell us anything that they have figured out about using the table or using the equation to, using the graph to figure out the equation. What did you notice from doing this at all? Yeah, Ryanne?

Ryanne: I just looked at the table and then I looked at the y column and said how did we get the y number, how did we get the x number, I figured out how to get y by using x. No, that doesn't make sense. (pause) Never mind.

Student: What was the question?

Rebecca: The question is, uh, what did you figure out or learn or have you learned from this that can help you, how can we help each other learn to find equations?

Chrissy: Like the way I did it is when I was thinking about it, when I wrote down my equation it didn't come out exactly like the one up there but I was thinking that since it goes up it rises a degree every day, there is going to have to be a 1 in there somewhere and then there, but there isn't a 1 in that one, whatever, then I knew there had to be the number of days and that going to be x-

Rebecca: Ok, hold on one second. I want to write down Chrissy's idea so that kind of everybody can refer back to it, so this is what Chrissy's saying (I write her idea on the overhead). This is her conjecture I guess, right? That's the word we are using. (pause) Ok. Is everybody listening to Chrissy (there was no talking when I said this)? Go ahead, finish your thought.

Chrissy: Ok, and then since the first day was, or today is negative four, there has to be a negative four in there and since x is the number of days, x is going to have to be in there too, and we did one like it yesterday, one that was exactly the same yesterday and it was also, it took the number of degrees plus the number of days times the er difference in degrees so that's what I did was itook x as the number of days times 1 which would just end up being x, so you just take x and then you have to add in there the first day and then the temperature was negative four . . ? . . but then I got rid of the x times 1 so it's just x plus negative four and then you can get rid of the plus so it's just x minus four.

Rebecca: Who agrees with Chrissy about getting rid of the one there?

(several hands go up)

Rebecca: Alright! We have some strong agreement for that one so let's do it. How about combining these signs? Is that ok?

(several hands go up)

Rebecca: Ok? Alright? Those are things that, even though that's kind of pre-algebra material, that's stuff that's going to be on your semester exam actually, so there's some things like knowing that one times x is the same as x that, that it's good for you to still know. (pause) Now Chrissy looked at that table and noticed that the temperature was going up by one each time. Does anybody notice anything else from looking at that table? When they look at the x's and the y's there. Amanda I think you and I talked about something with the table and the x's and the y's there? (Amanda says something to indicate she isn't sure what I'm talking about) Oh, I just said do you notice anything in the table when you look at the x's and y's. Chrissy had said that they go up by one each time on the y's.

Amanda: You need to add negative four.

Rebecca: Where do I do that? Where do I do that?

Amanda: Anywhere. You just add negative four.

Rebecca: Brittany were you going to say something to expand to that idea?

Brittany: Uh, I just said, uh, the x column. Cause like uh you add the negative four to the days from now column, to the x column-

Rebecca: Ok.

Brittany: And that equals the temperature (Rebecca writes on the overhead)

(short pause)

Rebecca: Can you all see that? That's supposed to be a plus there. (pause) Anybody have another way of thinking about this? Any other clues that you think that everybody could use to get $y = x - 4$ for an answer on this one. 'Cause I know some of you guessed that to start with. You weren't sure if it was right but you guessed it. (long pause) Adam, did you get this one, $y = x - 4$?

Adam: Yeah.

Rebecca: How did you figure out it was $y = x - 4$?

Adam: I just looked at the y where it crosses the y-axis, minus four, and saw the line went up so it had to be a positive slope and it's going 1 degree up each time so it had to be 1x-4.

(I draw a picture of graph highlighting Adam's explanation as he talks)

Rebecca: Did you all here what Adam said over here? Say that again a little bit louder, Adam.

Adam: I got the negative four from where the line crossed the y-axis, I got the x because the slope was one so the x is one x.

Rebecca: Does my picture represent what he's talking about here? Do I need to add anything? I kind of tried to draw what he said there. Don't forget all the group work that you did when you were sorting your equations and you were looking and your conjectures that we have up on the wall, because a lot of you were thinking at that point I noticed about positive slope and negative slope and where the positive and negative numbers showed up in the equations depending on how the line was sloped. And since we're doing lines now that's real important. You can tell if your equation is right or not. How about a line like, so this is kind of like the line, does this look kind of like the line you all got for this temperature data, kind of like that? How about, some people suggested that they might do something like x=4 or y=4. What about those? Would those work for this data? Yeah, Jessica?

Jessica: I have to disagree with that because x=4 will just give you a straight line along the x-axis. It'll go up four notches and then make a straight line all the way across and that doesn't. That isn't true. And then the same for y=4 except the y axis. (Rebecca draws what she is saying – which isn't correct) (some students indicate that the lines are backwards)

Rebecca: Ok. One of them goes one way and one of them goes the other.

Jessica: Yeah.

Rebecca: Make sure that you remember that you've got to keep track in your mind which one's is horizontal, which one is vertical. Alright. Keeping everything that everyone just talked about in mind, and we've got about ten minutes to work, ok, what you're going to do is you are only going to do the first part of this, alright, so you don't even need your graphing calculators, (aside to MG) are they going to need them for anything else? (no) Alright. We'll collect them in a moment but I think I'll pass this out first . . . this is another problem, it's a different one and the graph is different so you're not doing the exact same thing again.

End of transcript

Appendix D. References for Selected Books and Articles

Kathryn, Janet, and I Discussed These Articles and Books as We Planned for and Reflected on Facilitating Discussion

Copes, L. (2000). Messy monk mathematics: An NCTM standards-inspired class. *The Mathematics Teacher, 93,* 292-298.

Forman, E., Larreamendy-Joerns, J., Stein, M., & Brown, C. (1998). “You’re going to want to find out which and prove it”: Collective argumentation in a mathematics classroom. *Learning and Instruction, 8,* 527-548.

Manouchehri, A. & Enderson, M. C. (1999). Promoting mathematical discourse: Learning from classroom examples. *Mathematics Teaching in the Middle School, 4,* 216-222.

Mendez, E. (1998). *Robust mathematical discussion*. Unpublished doctoral dissertation, Stanford University.

O’Conner, M., & Michaels, S. (1993). Aligning academic task and participation status through revoicing: Analysis of a classroom discourse strategy. *Anthropology and Education Quarterly, 24,* 318-335.

Oyler, C. (1996). *Making room for students: Sharing authority in room 104*. New York: Teachers College Press.

Reinhart, S. C. (2000). Never say anything a kid can say! *Mathematics Teaching in the Middle School, 5,* 478-483.

Robinson, H. (1994). *The ethnography of empowerment: The transformative power of classroom interaction.* Washington, D.C.: The Falmer Press.

Sherin, M., Mendez, E., & Louis, D. (2000). Talking about math talk. In M. Burke & F. Curcio (Eds.), *Learning mathematics for a new century* (pp. 188-196). Reston, VA: National Council of Teachers of Mathematics.

Silver, E. A. & Smith, M. S. (1996). Building discourse communities in mathematics classrooms: A worthwhile but challenging journey. In P. C. Elliott & M. J. Kenney (Eds.), *Communication in mathematics, k-12 and beyond* (pp. 20-28). Reston, VA: National Council of Teachers of Mathematics.

Stein, M. K., & Smith, M. S. (1998). Mathematical tasks as a framework for reflection: From research to practice. *Mathematics Teaching in the Middle School, 3*(4), 268-275.

Van Zoest, L. R. & Enyart, A. (1998). Discourse, of course: Encouraging genuine mathematical conversations. *Mathematics Teaching in the Middle School, 4,* 150-157.

Wood, T. (1998). Alternative patterns of communication in mathematics classes: Funneling or focusing? In H. Steinberg et al (Eds.), *Language and communication in the mathematics classroom* (pp.167-178). Reston, VA: National Council of Teachers of Mathematics.

Appendix E. Interview Protocols

Below are the questions and prompts I used when interviewing Kathryn before, during, and at the end of the fall semester, 2000. The interviews were semi-structured; I used the questions below as a guide but interchanged questions, altered wordings or asked follow-up questions such as "What do you mean by . . . ?" or "Can you say some more about . . . ?" as needed.

<u>Interview of Kathryn on 8/11/00</u>

Tell me a little bit about last year, with the Professional Standards Board Project, watching the tapes, thinking about classroom discourse in terms of what you liked, what you didn't like from looking at your tapes and thinking about discourse.

In terms of planning for the fall, we've looked at your textbook chapters and we've looked at lots of tasks, quite a few of which you've used before. We thought about reordering some things. Also, we've looked at articles on discourse that you or I or Janet found. So, my question is: What do you think has been more or less beneficial or helpful to you in thinking about facilitating discussion?

Let's ignore reality for just a minute, what is your vision, paint me a picture of what you wish the classroom could look like, particularly with respect to discourse? What's the ideal for you?

What are you worried about (related to this coming semester)?

Different things are suggested by different authors of articles we've looked at. What are your reactions to their suggestions? [I have articles with me to prompt Kathryn here as needed].

<u>Interview of Kathryn on 9/12/00</u>

What has pleased you the most so far related to class discussions?

What has disappointed you the most so far related to class discussions?

What has surprised you the most so far related to class discussions?

What are you most worried about in terms of the next unit related to class discussions?

<u>Interview of Kathryn on 12/15/00</u>

There were several layers in our work together. One layer was what happened in the classroom generally, another layer was our collaboration, and another was specifically the whole-class discussions. Were there any surprises that you wouldn't have thought of going into the semester?

What do you think about those things now?

Anything that surprised you in a good or bad way about the students in terms of their role during whole-class discussion or during classroom activities in general?

What was the best thing about our work together?

What is the worst thing or hardest thing about our work together?

What do you think you learned that you didn't know before, if anything?

Thinking about the whole-class discussions, when you watch yourself on video or read transcripts, what do you see that you like? That you don't like?

We've tried many different strategies in terms of leading whole-class discussions, questioning techniques, not repeating, wait time, where we stand, whether we nominate students, etc. At this point, which ideas that we've tried do you believe are most important, whether or not you feel you've been successful in implementing them?

Pretend we're going to try to let other teachers know about things they are going to have to struggle with if they want to try to facilitate whole-class discussions in their classrooms. What would you want to inform them about?

What would you say to those teachers in terms of things that you have found that support you effort to facilitate discussion?

Thinking ahead to next semester or next year, what are you excited about and what are you worried about?

Thinking about the collaboration, what do you think that teachers need after a collaboration like ours?

Rebecca H. McGraw
Vita

Education

Ph.D., Curriculum and Instruction, Mathematics Education. (2002). Indiana University, Bloomington, IN. Ph.D. thesis title: *"Facilitating Whole-Class Discussion in Secondary Mathematics Classrooms."* Directed by Dr. Frank K. Lester, Jr.

M.Ed., Curriculum and Instruction, Mathematics Education. (1997). Indiana University, Bloomington, IN. Master's thesis title: "*The Ways in Which Students Use The Geometer's Sketchpad to Make, Test, and Prove Conjectures.*" Directed by Dr. Peter Kloosterman.

Teacher Certification, Secondary Mathematics and General Science. (1992). Eastern Michigan University, Ypsilanti, MI.

B.S., Environmental Science. (1991). Michigan State University, East Lansing, MI.

Teaching

<u>M457: Methods of Teaching Mathematics in the Secondary School</u>. (2001, Fall). Indiana University, Bloomington, IN. *Assistant to Dr. Catherine Brown.*

<u>E343: Teaching Mathematics in the Elementary School</u>. (2000, Fall – 2001, Spring). Indiana University, Bloomington, IN. *Instructor of Record.*

<u>M201: Field Experience in Mathematics and Science (for Elementary Education Majors)</u>. (2000, Fall). Indiana University, Bloomington, IN. *Instructor of Record.*

<u>N716: Topical Seminar in Mathematics Education</u>. (2000, Spring). Indiana University, Bloomington, IN. *Doctoral seminar on research on student learning. Assistant to Dr. Peter Kloosterman.*

<u>Area 10 Mathematics and Technology Project</u>. (1997, Fall – 1999, Spring). Indiana University, Bloomington, IN. *Implementing technology-based mathematics lessons at local area middle and high schools.*

<u>N443: Teaching Elementary School Mathematical Problem Solving</u>. (1998, Spring). Indiana University, Bloomington, IN. *Assistant to Dr. Frank K. Lester, Jr.*

Mathematics Teacher. (1993, Fall – 1997, Spring). Bloomington High School South, Bloomington, IN.

Print Publications - Refereed

Brown, C. A., McGraw, R., Koc, Y., Lynch, R. K., Arbaugh, E. F. (in press). Lesson study in secondary mathematics. *Proceedings of the Twenty-Fourth Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education.*

Arbaugh, E. F., Brown, C. A., & McGraw, R. (in press). The messy work of studying professional development: The conversation continues. *Proceedings of the Twenty-Fourth Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education.*

Essex, K., Lambdin, D. V., & McGraw, R. (2002). Racing against time: Using technology to explore distance, rate, and time. *Teaching Children Mathematics*, 6, 322-326, 347.

Web Publications - Refereed

McGraw, R. (2000). Facilitating communication about measurement, exponents, and scientific notation. In The National Council of Teachers of Mathematics, *Illuminations: Principles and Standards for School Mathematics.* Retrieved October 5, 2001 from http://illuminations.nctm.org/reflections/912/Facilitating_Communication/index.html

Galindo, E., McGraw, R., & Treahy, D. (2000). Gathering evidence about students' understanding of volume. In The National Council of Teachers of Mathematics, *Illuminations: Principles and Standards for School Mathematics*. Retrieved October 5, 2001 from http://illuminations.nctm.org/reflections/6-8/GatheringEvidence/index.html

Galindo, H., McGraw, R., & Treahy, D. (2000). Developing geometry concepts using computer programming environments. In The National Council of Teachers of Mathematics, *Illuminations: Principles and Standards for School Mathematics.* Retrieved October 5, 2001 from http://illuminations.nctm.org/imath/prek2/GeometryConcepts/index.html

Papers and Presentations

McGraw, R., Koc, Y., Brown, C. A., & Lynch (2002, April). *Talking about mathematics teaching: An examination of the use of a multimedia case to stimulate the* *conversation.* Paper presented at the Annual Meeting of the American Educational Research Association, New Orleans, LA.

Brown, C. A., Koc, Y., McGraw, R., & Lynch, K. (2002, March). *The use of a web-based professional development forum to enhance in-service and pre-service teacher education*. Session to be held at the Thirteenth International Conference of the Society for Information Technology and Teacher Education, Nashville, TN.

Caulfield, R., & McGraw, R. (2001, November). *Mathematical tasks in the middle school classroom*. Presentation at the Annual Conference of the Indiana Council of Teachers of Mathematics, Indianapolis, IN.

Brown, C. A., Arbaugh, E. F., Lutz, M. P., McGraw, R. (2001, April). *Studying professional development is messy work: What are the research issues?*Working session conducted at the Research Presession of the Annual Meeting of the National Council of Teachers of Mathematics, Orlando, FL.

Brown, C.A., Arbaugh, E. F., McGraw, R., & Koc, Y. (2001, April). *Lesson study in secondary math: Professional development through lesson development.* Presentation at the Annual Meeting of the National Council of Supervisors of Mathematics, Orlando, FL.

McGraw, R. (2000, April). *Usability testing of the electronic version of the Principles and Standards: Users' expectations, reactions, and suggestions.* Paper presented at the Research Presession of the Annual Meeting of the National Council of Teachers of Mathematics, Chicago, IL.

McGraw, R., & Arbaugh, E. F. (2000, April). *Problem solving with The Geometer's Sketchpad.* Workshop conducted at Annual Meeting of the National Council of Teachers of Mathematics, Chicago, IL.

Smith, M., Stein, M. K., Arbaugh, F., Henningsen, M., McGraw, R., Wilson, M., & Zawojewski, J. (2000, February). *Cases in mathematics teacher education: What do teachers learn from these experiences?* Presentation at the Annual Conference of the Association of Mathematics Teacher Educators, Charlotte, NC.

Galindo, E., Galindo, H., Siebold, R., McGraw, R., & Essex, K. (1999, April). *Principles and Standards for School Mathematics in electronic format: The possibilities and what we are learning from users.* Paper presented at the Research Presession of the Annual Meeting of the National Council of Teachers of Mathematics, San Francisco, CA.

Riggle, M., Hannah-Hansen, M. B., McGraw, R., & Ban, E. (1997, March). *When change isn't enough: Looking at process, problems and solutions related to implementing a block schedule.* Presentation at the Indiana Department of Education Re:Learning Spring Forum, Indianapolis, IN.

Research and Development

Research Assistant. (2000 - 2002). Indiana University, Bloomington, IN. *Investigating Lesson Study Groups composed of secondary mathematics teachers, mathematics teacher educators, university mathematicians, and pre-service secondary mathematics teachers. Principal Investigator: Dr. Catherine Brown.*

Research Assistant. (1999 – 2000). Indiana University/Purdue University – Indianapolis, Indianapolis, IN. *Investigating the relationship between secondary teachers' conceptions about the nature of mathematics and their teaching practices. Principal Investigator: Dr. Sue Mau.*

Research Assistant. (1998 – 2000). Indiana University, Bloomington, IN. *Usability testing of the web version of NCTM's Principles and Standards for School Mathematics document*. *Principal Investigator: Dr. Enrique Galindo.*

Website Developer. (1997 – 1999). National Council of Teachers of Mathematics Principles and Standards for School Mathematics Web Development Team. Indiana University, Bloomington, IN. *Assisted Dr. Enrique Galindo with the development of the electronic version of the NCTM's Principles and Standards for School Mathematics document as well as Web-based resources that support the document.*

Research Consultant. (1999). Bloomington Montessori School, Bloomington, IN. *Examined the re-creation and enactment of gender roles in Montessori (grade) 1-6 classrooms. Co-investigator: Angela Allen. Funded by the Bloomington Montessori School.*

Curriculum Developer. (1997 – 1998). Area 10 Mathematics and Technology Project. Indiana University, Bloomington, IN. *Developed a web-based instructional unit appropriate for secondary mathematics students. See http://www.indiana.edu/~atmat/units/area_perimeter/area_intro.htm*

Curriculum Developer. (1994 – 1995). Indiana University, Bloomington, IN. *Wrote learning guides and evaluation materials for two introductory, correspondence geometry courses.*

Professional Development Facilitation

Collaboration for the Enhancement of Mathematics Instruction Project. (2000 – 2002). Indiana University, Bloomington, IN. *A multi-year professional development project focusing on the incorporation of Japanese Lesson Study methods into U.S. professional development activities. Lead Facilitator: Dr. Catherine Brown.*

The Geometer's Sketchpad: Problems and Activities for Secondary Students. (2001, August). Metropolitan School District of Decatur Township, Indianapolis, Indiana. *Two workshops designed to introduce middle and high school teachers to The Geometer's Sketchpad software and its potential as a tool for exploring mathematics from pre-algebra to calculus.*

Problem Solving in Grades 3-5. (2000, August). Clover Public Schools, Clover, SC. *A workshop focusing on the potential of non-routine problems to help children develop conceptual understanding of mathematics.*

Enhancing Students' Mathematical Learning by Developing Mathematics Teachers for the 21st Century Workshop Series. (2000). Indiana University/Purdue University - Indianapolis, Indianapolis, IN. *Five workshops focusing on supporting the development of secondary teachers' mathematical knowledge.*

Algebraic Thinking in the Middle School Workshop Series. (1999 – 2000). Indianapolis Public Schools, Indianapolis, IN. *A series of six day-long workshops for 7th and 8th grade mathematics teachers. Content focus: supporting middle school students' algebraic thinking. Co-facilitator: Fran Arbaugh.*

Rational Numbers: Grades 4-6. (1999, Summer). Indianapolis Public Schools, Indianapolis, IN. *Three day-long sessions for grades 4-6 teachers. Content focus: supporting 4-6 grade students' understanding of rational number. Lead Facilitator: Dr. Beatriz D'Ambrosio.*

Area 10 Mathematics and Technology Project Summer Workshops. (1998, Summer & 1999, Summer). Indiana University, Bloomington, IN. *An eight day workshop (each summer) for high school mathematics teachers. Content focus: integrating technology into the teaching and learning of high school mathematics. Lead Facilitators: Dr. Enrique Galindo and Dr. Peter Kloosterman.*

Service

<u>Research, Development, and Equipment Committee Member</u>. (2001 – 2002). Indiana University School of Education.

Graduate Program Committee Member. (2000 – 2002). Indiana University School of Education.

Problem Developer. (1999). *Mathematics Teacher* math calendar.

Manuscript Reviewer. (1998). Eye on Education.

Manuscript Reviewer. (1997). *Mathematics Teacher*.

Problem Developer. (1996). Indiana Council of Teacher of Mathematics State High School Mathematics Contest.

Workshop Assistant. (1996, November). National Council of Teachers of Mathematics Regional Conference, South Bend, IN.

Awards and Honors

Linebeck Fellowship. Indiana University. (2000, Fall – 2001, Spring). *A competitive fellowship for mathematics and science education doctoral students at Indiana University, reception is based on scholarship and potential contribution to the field.*

University Graduate School Fellowship. Indiana University. (1997, Fall – 1999, Spring). *A scholarship based fellowship for doctoral students*.

Apple Award for Excellence in Teaching. (1997, Spring). *Awarded by the faculty of Bloomington High School South, Bloomington, IN.*

Professional Affiliations

American Educational Research Association and Special Interest Group for Research in Mathematics Education
Association of Mathematics Teacher Educators
National Council of Teachers of Mathematics
North American Group for the Psychology of Mathematics Education

Areas of Special Professional Interest

Characteristics of classroom talk, the nature and of teachers' efforts to engage students in talking about mathematics, and the growth of mathematical ideas through small group and whole class discussions

In-service and pre-service teacher professional development, particularly the potential of lesson studies and multimedia case studies for teacher development

The ways in which technology influences students' understandings of mathematics