Section 4.6 (5th ed.) (1038675)

Sun Nov 8 2009 05:00 PM MST

Question
1 2 3 4 5 6 7 8 9 10

Description
This section covers more rate problems. You will also find a set of problems referred to as related rate problems. In these problems all the variables change according to time. For example, we normally think of the volume of a sphere as a function of radius. In that case we would consider \( \frac{dV}{dr} \). But if the radius changes according to time, then we would consider \( \frac{dV}{dt} \) and use chain rule.

1. **Question Details**

The period \( T \), in seconds, of a pendulum is given by the equation below with length, \( l \), in meters.

\[
T = 2\pi \sqrt{\frac{l}{g}}
\]

(a) How fast does the period increase?

\[
\frac{dT}{dl} = \frac{\pi}{\sqrt{9.8 l}}
\]

(b) Does this rate of change increase or decrease as \( l \) increases?  
- [ ] Increases  [ ] Decreases

2. **Question Details**

A dose, \( D \), of a drug causes a temperature change, \( T \), in a patient. \( T \) is given by the following equation where \( C \) is a positive constant.

\[
T = \left( \frac{C}{8} - \frac{D}{3} \right) \cdot D^2
\]

(a) What is the rate of change of temperature change with respect to dose?

\[
\frac{dT}{dD} = (3C/8 - 4D/3) D^2
\]

(b) For what doses does the temperature change increase?

\( D \) inequality:  
- [ ] \( < \)  [ ] \( > \)  [ ] \( \leq \)  [ ] \( \geq \)  [ ] \( = \)  

3. **Question Details**

The average cost per item, \( C \), in dollars, of manufacturing a quantity \( q \) of cell phones is given by the following equation, where \( a \) and \( b \) are positive constants.

\[
C = aq + b
\]

(a) Find the rate of change of \( C \) as \( q \) increases. Include units.

\[
\frac{dC}{dq} = -a/q^2 \quad \text{dollars per cellphone}
\]

(b) If production increases at a rate of 120 cell phones per week, how fast is the average cost changing? Include units.

\[
\frac{dC}{dt} = -120a/q^2 \quad \text{dollars per week}
\]
Is the average cost increasing or decreasing?

- [ ] increasing
- [ ] decreasing

4. **Question Details**

A voltage $V$ across a resistance $R$ generates a current $I = V/R$. Suppose a constant voltage of 10 volts is put across a resistance that is decreasing at a rate of 0.4 ohms per second when the resistance is 5 ohms.

(a) At what rate is the current changing?

$$\frac{dI}{dt} = \boxed{-4/25}$$

(b) Is the current increasing or decreasing?

- [ ] Decreasing
- [X] Increasing

5. **Question Details**

If $\theta$ is the angle between a line through the origin and the positive x-axis, the area in cm$^2$, of part of a rose petal (a particular curve written in polar coordinates) is given by

$$A = \frac{9}{16} (4\theta - \sin (4\theta))$$

If the angle $\theta$ is increasing at a rate of 0.6 radians per minute, at what rate is the area changing when $\theta$ is $\pi/4$?

$$\boxed{108/40} \text{ cm}^2/\text{min}$$

6. **Question Details**

A ruptured oil tanker causes a circular oil slick on the surface of the ocean. When its radius is 120 meters, the radius of the slick is expanding by 0.6 meter/minute and its thickness is 0.01 meter. Note: Pay attention to the units.

(a) At that moment, how fast is the area of the slick expanding?

$$\boxed{144\pi/60} \text{ m}^2/\text{sec}$$

(b) The circular slick has a thickness that remains uniform, and the volume of oil spilled remains fixed. How fast is the thickness of the slick decreasing when the radius is 120 meters?

$$\boxed{-0.012/7200} \text{ m/sec}$$

7. **Question Details**

A potter forms a piece of clay into a cylinder. As he rolls it, the length, $L$, of the cylinder increases and the radius, $R$, decreases. Assume that no clay is lost in the process. Suppose the length of the cylinder is increasing by 0.1 cm per second.

(a) What is the rate at which the volume is changing?

$$\boxed{0} \text{ cm}^3/\text{sec}$$

(b) What is the rate at which the radius is changing when the radius is 3 cm and the length is 8 cm?

$$\boxed{-3/160} \text{ cm/sec}$$

8. **Question Details**

A train is traveling 0.9 km/min along a straight track, moving in the direction as shown in the figure below. A movie camera, 0.5 km away from the track, is focused on the train.
(a) How fast is the distance from the camera to the train changing when the train is 1 km from the camera? Give your answer to 3 decimal places.
0.779

(b) What are the units for your answer in part (a)?
- km²/min
- min
- km
- km/min

(c) How fast is the camera rotating at the moment when the train is 1 km from the camera? Give your answer to 3 decimal places.
0.450

(d) What are the units for your answer in part (c)?
- radians/min
- degrees/min
- radians
- km/min
- degrees

9. Question Details dkHGMCalc5 4.6.042. [1070000]

Sand falls from a hopper at a rate of 0.5 cubic meters per hour and forms a conical pile beneath. Suppose the radius of the cone is always half the height of the cone.

(a) Find the rate at which the radius of the cone increases when the radius is 2 meters.

(b) Find the rate at which the height of the cone increases when the radius is 2 meters.

10. Question Details dkHGMCalc5 4.6.045. [1070001]

For the amusement of the guests, some hotels have elevators on the outside of the building. One such hotel is 200 feet high. You are standing by a window 100 feet above the ground and 150 feet away from the hotel, and the elevator descends at a constant speed of 30 ft/sec, starting at time \( t = 0 \), where \( t \) is time in seconds. Let \( \theta \) be the angle between the line of your horizon and your line of sight to the elevator.

(a) Find a formula for \( h(t) \), the elevator's height above the ground as it descends from the top of the hotel.

(b) Using your answer to part (a), express \( \theta \) as a function of time \( t \). Remember to use atan for arctan.

Find the rate of change of \( \theta \) with respect to \( t \).
\[
\frac{d\theta}{dt} = -30 \frac{150}{(150^2 + (100 - 30t)^2)}
\]

(c) The rate of change of \( \theta \) is a measure of how fast the elevator appears to you to be moving. At what time does the elevator appear to be moving fastest?

\[
time = 100/30 \text{ seconds}
\]

At what height does the elevator appear to be moving fastest?

\[
height = 100 \text{ feet}
\]
1. \[ T = 2\pi \sqrt{\frac{l}{9.8}} \]

\[ \frac{dT}{dl} = 2\pi \cdot \frac{1}{2} \cdot \frac{1}{\sqrt{\frac{l}{9.8}}} = \frac{\pi}{\sqrt{9.8l}} \]

If \( l \) increases, \( \frac{dT}{dl} \) decreases (\( l \) is in the denominator).

2. \[ T = \left( \frac{C}{8} - \frac{D}{3} \right) \cdot D^3 \]

\[ \frac{dT}{dB} = -\frac{1}{3} D^3 + 3 \left( \frac{C}{8} - \frac{D}{3} \right) D^2 \]

\[ = -\frac{4}{3} D^3 + \frac{3C}{8} D^2 \]

The Temp. change increases if

\[ \frac{dT}{dB} > 0 \quad \text{i.e.} \quad -\frac{4}{3} D^3 + \frac{3C}{8} D^2 > 0 \]

\[ \text{i.e.} \quad D^2 \left( -\frac{4}{3} D + \frac{3C}{8} \right) > 0 \]

\[ \text{i.e.} \quad -\frac{4}{3} D + \frac{3C}{8} > 0 \]

\[ \Rightarrow \quad D < \frac{9C}{32} \]
\[ C = \frac{a}{q} + b \]

\[ \frac{dc}{dq} = -\frac{a}{q^2} \text{ \$/cellphone} \]

5) If the production of cell phones per week increases at a rate of 120 cell phones per week,

\[ \text{Average cost changes by} \]
\[ \frac{dc}{dq} \times 120 \text{ cell phones/week} \]
\[ = -\frac{120a}{q^2} \text{ \$/week} \]

Which is decreasing.

4) \[ VC = \frac{V}{R} \text{ where } V = 10 \text{ is constant in time and } R \text{ is a function of time.} \]

\[ \frac{dc}{dt} = -\frac{V}{R^2} \cdot \frac{dR}{dt} \]

When \[ R = 5 \] \[ \frac{dR}{dt} = -0.4 \]

\[ \rightarrow \frac{dc}{dt} = -\frac{10}{5^2} \cdot (-0.4) = \frac{4}{25} = 0.16 \]

\[ \text{it is increasing.} \]
\[ A = \frac{9}{16} (4 \theta - \sin(4\theta)) \]

Here \( \theta \) is a function of time \( t \) with

\[ \frac{d\theta}{dt} = 0.6 \quad \text{when} \quad \theta = \frac{\pi}{4} \]

\[ \Rightarrow \frac{dA}{dt} = \frac{9}{16} \left( 4 \cdot \frac{d\theta}{dt} - 4 \cos(4\theta) \cdot \frac{d\theta}{dt} \right) \]

\[ = \frac{9}{16} \left( 4 \cdot 0.6 - 4 \cos(\pi) \cdot 0.6 \right) \]

\[ = \frac{9 \cdot 2.4 \cdot 0.6}{16} = \frac{43.2}{16} = 2.7 \]

\[ A = \pi r^2 \]

\[ \Rightarrow \frac{dA}{dt} = 2\pi r \cdot \frac{dr}{dt} \]

\[ = 2\pi (120 \text{m}) \cdot 0.6 \text{ m/min} \]

\[ = 144\pi \text{ m/min} \]

\[ = 144\pi \text{ m/min} \cdot \frac{1 \text{ min}}{60 \text{ sec}} \]

\[ = 2.4 \pi \text{ m/sec}. \]

\[ V = \pi r^2 h \]

\[ \Rightarrow 0 = 2\pi r \cdot \frac{dr}{dt} h \]

\[ + \pi r^2 \cdot \frac{dh}{dt} \]

\[ \frac{dh}{dt} = -2\pi rh \cdot \frac{dr}{dt} \cdot \frac{1}{\pi r^2} \]

\[ = -2h \cdot \frac{dr}{dt} \cdot \frac{1}{r} \]

\[ = -2(0.01) \cdot 0.6 \]

\[ = \frac{-0.12}{120} \]
\[ V = \pi r^2 L \]

Since no clay is lost in the process,

\[ \frac{dV}{dt} = 0. \]

\[ \Rightarrow \quad 0 = \pi r^2 \frac{dr}{dt} L + \pi r^2 \frac{dl}{dt} \]

\[ \Rightarrow \quad \frac{dr}{dt} = -\frac{\pi r^2 \frac{dl}{dt}}{2\pi r L} \]

\[ = -\frac{5}{2L} \frac{dl}{dt} \]

\[ = -\frac{3 \text{ cm}}{2(8 \text{ cm})} \cdot 0.1 \text{ cm/sec} \]

\[ = -\frac{3}{160} \text{ cm/sec} \]

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8. \[ \begin{array}{c}
0.5 \\
3 \text{ km} \\
\text{camera}
\end{array} \]

Train \( \Rightarrow \) 0.9 km/min

Calculate \( \frac{dz}{dt} \)

\[ z^2 = x^2 + (0.5)^2 \]

\[ \Rightarrow \quad 2z \cdot \frac{dz}{dt} = 2x \cdot \frac{dx}{dt} \]

\[ \frac{dz}{dt} = \frac{x}{z} \cdot \frac{dx}{dt} = \frac{0.9}{0.9} \approx 0.779 \text{ km/min} \]
\( \cos \theta = \frac{0.5}{z} \)

\[ \Rightarrow \, \theta = \cos^{-1} \left( \frac{0.5}{z} \right) \]

\[ \frac{d\theta}{dt} = \frac{-1}{\sqrt{1 - \left( \frac{0.5}{z} \right)^2}} \cdot \frac{-0.5}{z^2} \cdot \frac{dz}{dt} \]

\[ = \frac{-1}{\sqrt{1 - \left( \frac{0.5}{z} \right)^2}} \cdot \frac{-0.5}{z^2} \cdot \frac{1}{0.779} \]

\[ = 0.45 \text{ rad/min} \]

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9. Volume of a cone:

\[ V = \frac{1}{3} \pi r^2 h \quad \text{where} \quad r = \frac{1}{2} h \]

\[ V(r) = \frac{2}{3} \pi r^3 \]

\[ \Rightarrow \, \frac{dV}{dt} = 2\pi r^2 \cdot \frac{dr}{dt} \]

\[ \Rightarrow \, \frac{dr}{dt} = \frac{1}{2\pi r^2} \cdot \frac{dV}{dt} = \frac{1}{\pi} \left( \frac{2}{8} \right)^2 \cdot 0.5 = \frac{0.5}{8\pi} \text{ m/h} \]
Since \( h = 2r \)

\[ \Rightarrow \frac{dh}{dt} = 2 \cdot \frac{dr}{dt} = 2 \cdot \frac{0.5}{8\pi} = \frac{0.5}{4\pi} \]

\[ h(t) = \text{height of \(- \text{rate} \cdot \text{time}) \}\text{ at } t \text{sec} \]

\[ = 200 - 30t \]

b) \( \tan(\theta) = \frac{h(t) - 100}{150} \)

\[ \Rightarrow \theta(t) = \arctan \left( \frac{h(t) - 100}{150} \right) \]

\[ = \arctan \left( \frac{100 - 30t}{150} \right) \]

c) \[ \frac{d\theta}{dt} = \frac{1}{1 + \left( \frac{100 - 30t}{150} \right)^2} \cdot \frac{-30}{150} \]

d) \( \frac{d\theta}{dt} \) is largest (in magnitude) when the denominator is smallest. Thus when \( \left( \frac{100 - 30t}{150} \right)^2 = 0 \) i.e. \( t = \frac{100}{30} \text{ sec} \).

e) This is at height \( h \left( \frac{100}{30} \right) = 200 - 30 \left( \frac{100}{30} \right) = 100 \text{ ft} \).