MATH 413-513
HOMEWORK 4

SPRING 2012

The following assignment is to be turned in on
Friday, February 17th, 2012.

Math 413:

1) Let $\mathbb{F}$ be a field. Determine the dimension of the following subspaces of $\mathbb{F}_4$.

(a) $\{(x_1, x_2, x_3, x_4) \in \mathbb{F}_4 \,|\, x_4 = 0\}$

(b) $\{(x_1, x_2, x_3, x_4) \in \mathbb{F}_4 \,|\, x_4 = x_1 + x_2\}$

(c) $\{(x_1, x_2, x_3, x_4) \in \mathbb{F}_4 \,|\, x_4 = x_1 + x_2, x_3 = x_1 - x_2\}$

(d) $\{(x_1, x_2, x_3, x_4) \in \mathbb{F}_4 \,|\, x_4 = x_1 + x_2, x_3 = x_1 - x_2, x_3 + x_4 = 2x_1\}$

(e) $\{(x_1, x_2, x_3, x_4) \in \mathbb{F}_4 \,|\, x_1 = x_2 = x_3 = x_4\}$

2) Determine the values of $\lambda \in \mathbb{R}$ for which each list of vectors is linearly dependent.

(a) $\{(\lambda, -1, -1), (-1, \lambda, -1), (-1, -1, \lambda)\} \subset \mathbb{R}_3$

(b) $\{\sin^2(x), \cos(2x), \lambda\} \subset \mathcal{C}(\mathbb{R})$

where $\mathcal{C}(\mathbb{R})$ is the vector space of continuous, real-valued functions defined on $\mathbb{R}$.

3) Let V be a finitely generated vector space over $\mathbb{F}$. Suppose that $U \subset V$ is a subspace and that $\dim(U) = \dim(V)$. Prove that $U = V$.

Math 513: In addition to the above, also do:

1) Let V be a finitely generated vector space over $\mathbb{F}$ with $\dim(V) = n \geq 1$. Prove that there are n, one-dimensional vector spaces U_1, U_2, $\cdots$, U_n of V with

$$V = U_1 \oplus U_2 \oplus \cdots \oplus U_n \,.$$

2) Let U and V be five dimensional subspaces of $\mathbb{R}_9$. Prove that $U \cap V \neq \{0\}$.