MATH 464
HOMEWORK 3

SPRING 2016

The following assignment is to be turned in on **Thursday, February 11, 2016**.

1. Three couples are invited to a dinner party. They will independently show up with probabilities 0.9, 0.8, and 0.75 respectively. Let N be the number of couples that show up. Calculate the probability that $N = 2$

2. Statistics show that 5% of men are color blind and 0.25% of women are color blind. If a person is randomly selected from a room with 35 men and 65 women, what is the likelihood that they are color blind?

3. You have two boxes. Box 1 contains 3 white balls and 4 black balls. Box 2 contains 2 white balls and 6 black balls. Here is an experiment. Pick a ball at random from Box 1 and put it into Box 2. Afterwards, pick a ball at random from Box 2. What is the probabiity that the ball you picked from Box 2 is black?

4. On a multiple choice exam with four choices for each question, a student either knows the answer to a question or marks it at random. Suppose the student knows the answers to 60% of the exam questions. If he marks the answer to question 1 correctly, what is the probability that he knows the answer to that question?

5. In a certain city, 30% of the people are conservative, 50% are liberals, and 20% are independents. In a given election, 2/3 of the conservatives voted, 80% of the liberals voted, and 50% of the independents voted. If we pick a voter at random, what is the probability that this person is a liberal?

6. Let $(\Omega, \mathcal{F}, P)$ be a probability space and suppose that $\{A_n\}_{n=1}^{\infty}$ is an increasing sequence of events. For each integer $n \geq 1$, set

$$C_n = \begin{cases} A_1 & \text{if } n = 1 \\ A_n \setminus A_{n-1} & \text{for } n \geq 2. \end{cases}$$

Show that the C_n's are mutually disjoint and that

$$\bigcup_{n=1}^{\infty} A_n = \bigcup_{n=1}^{\infty} C_n \,.$$

7. Let $(\Omega, \mathcal{F}, P)$ be a probability space and suppose that $\{A_n\}_{n=1}^{\infty}$ is a sequence of events. Set

$$B_n = \bigcup_{m=n}^{\infty} A_m \quad \text{and} \quad C_n = \bigcap_{m=n}^{\infty} A_m$$

It is clear that B_n is a decreasing sequence of events, while C_n is an increasing sequence of events. Show that

$$B = \bigcap_{n=1}^{\infty} B_n = \{\omega \in \Omega : \omega \in A_n \text{ for infinitely many values of n}\}$$

and

$$C = \bigcup_{n=1}^{\infty} C_n = \{\omega \in \Omega : \omega \in A_n \text{ for all but finitely many values of n}\}$$

8. Let $(\Omega, \mathcal{F}, P)$ be a probability space with

$$\Omega = \{1, 2, 3, 4, 5, 6\} \quad \text{and} \quad \mathcal{F} = \{\emptyset, \{2, 4, 6\}, \{1, 3, 5\}, \Omega\} .$$

Let $U : \Omega \to \mathbb{R}$ be given by $U(\omega) = \omega$.
Let $V : \Omega \to \mathbb{R}$ be given by

$$V(\omega) = \begin{cases} 1 & \text{if } \omega \text{ is even,} \\ 0 & \text{if } \omega \text{ is odd.} \end{cases}$$

Let $W : \Omega \to \mathbb{R}$ be given by $W(\omega) = \omega^2$.
Explain whether or not these functions are discrete random variables.

9. Suppose we roll two fair 6-sided dice. Let X be a random variable corresponding to the minimum value of the two rolls. Find the probability mass function f_X corresponding to the random variable as a table of values (see below).

10. The probability mass function of a discrete random variable X is given below as a table of values. Compute the following:
a) the probability that X is even (here we regard 0 and -4 as even)
b) the probability that $1 \leq X \leq 8$
c) the probability that X is -4 given that $X \leq 0$
d) the probability that $X \geq 3$ given that $X > 0$

$\mathbf{x}$	-4	-1	0	2	4	5	6
$\mathbf{f_X(x)}$	0.15	0.2	0.1	0.1	0.2	0.2	0.05