MATH 464
HOMEWORK 5

SPRING 2016

The following assignment is to be turned in on
Thursday, February 25, 2016.

1. Let $x, y \in \mathbb{R}$ and take $n \geq 2$ an integer. Prove that

$$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^k y^{n-k}$$

using an induction argument.

2. Let X be a binomial random variable with parameters n and p. Find the mean and variance of X. **Hint:** Sometimes it is easier to calculate $E(X(X-1)) = E(X^2) - E(X)$ rather than $E(X^2)$ directly.

3. Let X be a geometric random variable with parameter $p > 0$. Find the mean and variance of X. **Hint:** By geometric series, we know that

$$\sum_{n=0}^{\infty} r^n = \frac{1}{1-r} \quad \text{for any real number r with } |r| < 1.$$

By taking derivatives, you can get some useful formulas:

$$\sum_{n=1}^{\infty} n r^{n-1} = \frac{d}{dr}\frac{1}{1-r} \quad \text{and} \quad \sum_{n=2}^{\infty} n(n-1) r^{n-2} = \frac{d^2}{dr^2}\frac{1}{1-r}$$

4. a) Let X be a discrete random variable. Show that if $E(X^2) = 0$, then $P(X = 0) = 1$.

b) Use part a) to prove that if X is a discrete random variable and $\text{var}(X) = 0$, then $P(X = \mu) = 1$ where $\mu = E(X)$.

5. Let X be a geometric random variable with parameter $p > 0$. Let m and n be non-negative integers. Show that

$$P(X > n + m \,|\, X > m) = P(X > n)\,.$$

This shows that X has the *lack of memory* property.