The following problems are due **Monday, September 18, 2017**.

1. Let \( \{ X_\alpha \}_{\alpha \in A} \) be a family of non-empty sets indexed by a non-empty set \( A \). Suppose that for each \( \alpha \in A \), \( \mathcal{M}_\alpha \) is a \( \sigma \)-algebra on \( X_\alpha \) which is generated by a set \( \mathcal{E}_\alpha \). Denote by \( \mathcal{F} \subset X = \prod_{\alpha \in A} X_\alpha \) the set
   \[ \mathcal{F} = \{ \pi_\alpha^{-1}(E_\alpha) : E_\alpha \in \mathcal{E}_\alpha, \alpha \in A \} \]
   Show that for each \( \alpha \in A \), the set
   \[ \hat{\mathcal{E}}_\alpha = \{ E \subset X_\alpha : \pi_\alpha^{-1}(E) \in \mathcal{M}(\mathcal{F}) \} \]
   is a \( \sigma \)-algebra on \( X_\alpha \).

2. For each \( 1 \leq j \leq n \), let \( (X_j, \rho_j) \) be separable metric spaces. Show that
   \[ \bigotimes_{j=1}^{n} \mathcal{B}X_j = \mathcal{B}X \quad \text{where} \quad X = \prod_{j=1}^{n} X_j \]
   Recall: We proved half of this in class. You need only show to other containment.

3. Do problems 8 and 10 on page 27.

4. Let \( X \) be a non-empty set and take \( \mathcal{M} = \mathcal{P}(X) \). For any \( f : X \to [0, \infty] \), define \( \mu_f : \mathcal{M} \to [0, \infty] \) by setting
   \[ \mu_f(E) = \sum_{x \in E} f(x) \]
   We showed in class that for each such \( f \), \( \mu_f \) is a measure. Show that \( \mu_f \) is semi-finite if and only if \( f(x) < \infty \) for all \( x \in X \).