

CHALLENGE BOARD, VOLUME 27 ISSUE 1

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1. PROBLEMS

C99. Find all collections of polynomials $p_{11}, p_{12}, p_{21}, p_{22}$ with complex coefficients satisfying the relation

$$\begin{pmatrix} p_{11}(XY) & p_{12}(XY) \\ p_{21}(XY) & p_{22}(XY) \end{pmatrix} = \begin{pmatrix} p_{11}(X) & p_{12}(X) \\ p_{21}(X) & p_{22}(X) \end{pmatrix} \cdot \begin{pmatrix} p_{11}(Y) & p_{12}(Y) \\ p_{21}(Y) & p_{22}(Y) \end{pmatrix}.$$

C100. (Proposed by José Luis Diaz, Universitat Politecnica de Catalunya, Spain.) Let x_1, x_2, \dots, x_n be positive real numbers, let $S = \sum_{k=1}^n x_k$, and suppose that $(n-1)x_k < S$ for all k . Prove that

$$\prod_{j=1}^n (S - (n-1)x_k) \leq \prod_{j=1}^n x_j.$$

When does equality occur?