MATH 511A, HOMEWORK 8

1. Compute the order of the group G with the presentation

$$G = \langle a, b, c, d \mid bab^{-1} = a^2,\ bdb^{-1} = d^2,\ c^{-1}ac = b^2,\ dcd^{-1} = c^2,\ bd = db \rangle\,.$$

2. Prove that every finite group is finitely presented.

3. Let F be a free group with basis X, and let $A \subset X$. Prove that if N is the normal subgroup of F generated by A, then F/N is a free group.

4. Let F be a free group.

(i) Prove that F has no elements (other than 1) of finite order.
(ii) Prove that F is abelian if and only if the rank of F is at most 1.
(iii) Prove that if the rank of F is at least 2, then $Z(F) = 1$, where $Z(F)$ is the center of F.

5. Let T be the group

$$\langle a, b \mid a^6 = 1,\ b^2 = a^3 = (ab)^2 \rangle\,.$$

(i) Use the presentation to prove that $\#T \leq 12$.
(ii) Let G be the subgroup of $\mathrm{GL}_2(\mathbb{C})$ generated by the two matrices

$$C = \begin{pmatrix} \omega & 0 \\ 0 & \omega^2 \end{pmatrix}, \qquad B = \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix}.$$

Prove that G has order 12 and is isomorphic to T.
(iii) Prove that G (equivalently, T) is not isomorphic to either A_4 or D_{12}.