

MATH 111: HW 2 SOLUTIONS AND COMMENTS

2.2 #1. The first fifteen Fibonacci numbers are:

$$1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610.$$

2.2 #4. Working upwards from the bottom of each fraction, we have:

$$2 + \frac{2}{2 + \frac{2}{2}} = 2 + \frac{2}{2 + 1} = 2 + \frac{2}{3} = \frac{6}{3} + \frac{2}{3} = \frac{8}{3}.$$

Similarly,

$$3 + \frac{3}{3 + \frac{3}{3}} = 3 + \frac{3}{3 + 1} = 3 + \frac{3}{4} + \frac{12}{4} + \frac{3}{4} = \frac{15}{4}.$$

2.2 #6. One finds the Fibonacci sequence:

Time in Months	Start	1	2	3	4	5	6	7
Number of Pairs	1	1	2	3	5	8	13	21

2.2 #8. First let's make a small table of Fibonacci numbers, for reference:

n	1	2	3	4	5	6	7	8	9	10
F_n	1	1	2	3	5	8	13	21	34	55

Now let's look at a few examples:

n	F_{n+1}	F_{n-1}	F_{n+1}^2	F_{n-1}^2	$F_{n+1}^2 - F_{n-1}^2$
2	2	1	4	1	3
3	3	1	9	1	8
4	5	2	25	4	21
5	8	3	64	9	55

When $n = 2$, the result is 3, which we observe is equal F_4 . When $n = 3$, the result is 8, which is F_6 ; when $n = 4, 5$ we get F_8 and F_{10} , respectively. It seems reasonable to guess that this pattern continues, i.e., that we have the formula

$$F_{n+1}^2 - F_{n-1}^2 = F_{2n}.$$

Two notes. First, take care that F_{n+1} does not mean the same thing as $F_n + 1$. For example, when $n = 7$, $F_{n+1} = F_{7+1} = F_8 = 21$; whereas $F_n + 1 = F_7 + 1 = 13 + 1 = 14$.

Second, the table starts at $n = 2$ instead of $n = 1$ because we haven't decided what we might mean by F_0 . In fact, if we say that $F_0 = 0$ — which makes sense, as then $F_0 + F_1 = 0 + 1 =$

$1 = F_2$, so it satisfies the rule for generating Fibonacci numbers —then we could start at $n = 1$.

2.2 #18. We use the procedure described on page 55 to write the number 50 as a sum of non-consecutive Fibonacci numbers. The largest Fibonacci number less than or equal to 50 is 34, and $50 = 34 + 16$. The largest Fibonacci number less than or equal to 16 is 13, and $16 = 13 + 3$, so $50 = 34 + 13 + 3$. Since 3 is a Fibonacci number, we stop here. We have obtained

$$50 = 34 + 16 + 3.$$

The smallest Fibonacci number in this sum is 3, so our first move is to take 3 sticks.

3.1 #2. Ed corresponds to the Saab. The Trail-a-Bike corresponds to Julia.

3.1 #3. There are many possible answers to this question, but there is one thing that I would like to emphasize to you about this question: it is important to say something along the lines of “There is no such thing as a biggest number”, rather than “No one knows whether there is a biggest number”. The latter is false, and badly misleading, because it distracts from the point: the issue is not that we don’t understand large numbers very well, it’s that *we never run out of numbers*.

One thing you might do is ask your nephew to tell you his largest number, then ask him what happens if you add 1 to it (or if you add an extra digit to it). A few people suggested illustrating this point in a very nice way, namely that you can say something concrete along the lines of “No matter how many cookies you have had, you can always have one more”.

3.1 #5. No, at least not if we assume that these are typical cans of tennis balls: a can usually contains 3 balls (though some brands contain more). There are many more balls than cans, hence no one-to-one correspondence.

3.1 #8. There are exactly as many @s as ©s. Since the @s and the ©s are perfectly lined up, with a © underneath each @, we are provided with a one-to-one correspondence between the @s and the ©s: namely, each @ corresponds to the © underneath it. Since the two collections are in one-to-one correspondence, there are exactly as many of the one as of the other.

3.1 #18. At the start of the game, there is no one-to-one correspondence between people and chairs. However, after the music stops and one chairless person moves to the sidelines, there is a one-to-one correspondence between the chairs and the remaining players. (In fact, there is even a specific correspondence: each remaining player corresponds to the chair they are sitting in.) Then the players stand up, one chair is removed, and there is not a one-to-one correspondence any more. This pattern continues until the end of the game.