MATH 111: HW 4 SOLUTIONS AND COMMENTS

4.1 #3. The Pythagorean theorem states that in any right triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the other two sides. In the right triangle of this problem, we therefore have

$$\text{hypotenuse}^2 = 1^2 + x^2$$

and so

$$\text{hypotenuse} = \sqrt{1 + x^2}\,.$$

4.1 #4. We know one side of the rectangle has length 4 inches; to find the area of the rectangle, we want find the length of the other side of the rectangle.

The diagonal divides the rectangle into two right triangles. Both of those triangles have hypotenuse 5 inches and one side of length 4 inches. By the Pythagorean theorem,

$$5^2 = 4^2 + (\text{other side})^2\,,$$

so

$$(\text{other side})^2 = 25 - 16 = 9$$

and therefore (other side) = 3 inches.

Therefore, the rectangle has sides of length 3 inches and 4 inches, and the area of the rectangle is 3 inches times 4 inches: that is, the area is 12 square inches.

4.1 #6. The angles sum to 180 degrees (a straight angle).

4.1 #8. No, there cannot be a right triangle with sides of length 1, 2, 3. If there were such a triangle, the hypotenuse (always the longest side) would have length 3. Then the Pythagorean theorem would require that the equation

$$1^2 + 2^2 = 3^2$$

be correct.

But, we know that $1^2 + 2^2$ does not equal 3^2 (because $1 + 4$ does not equal 9!!) So there cannot be a right triangle with those side lengths.

4.1 #15. The first step is to note that 1 mile is 5280 feet. (If you didn't know this already, this is easy to look up on the internet.) In the problem, the track expands to $5280+2 = 5282$ feet.

The two halves of the track are each right triangles, with base $5280/2 = 2640$ feet and hypotenuse $5282/2 = 2641$ feet. The height of the midpoint of the track is therefore

$$\text{height}^2 = 2641^2 - 2640^2 = 5281$$

and so

$$\text{height} = \sqrt{5281} \approx 72.67\,\text{feet}\,.$$

4.3 #3. The ratio of two sides of a Golden rectangle is

$$\phi = \frac{1+\sqrt{5}}{2} \approx 1.618\,..$$

The ratios of the sides of the rectangles in the problem are: $5/3 \approx 1.666$; $11/8.5 \approx 1.294$; $14/11 \approx 1.273$; $17/11 \approx 1.545$. The closest of these to 1.618 is the first, so the index card most closely resembles a Golden rectangle.

4.3 #4. The two equations have the same solutions because they are essentially the same equation: the second equation is gotten from the first equation by taking the reciprocal of both sides. Therefore, any number which solves the first equation will solve the second, and vice-versa.

4.3 #5. Recall the quadratic formula: if we want to solve the equation $ax^2+bx+c=0$ for x, then there are two solutions:

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}\,.$$

Now let's look at the particular equations in this problem.

(1) Cross-multiplying yields $2x(x-1)=1$, which expands to $2x^2-2x=1$. Bring the 1 to the other side of the equation: $2x^2-2x-1=0$. Now apply the quadratic formula (in this case $a=2$, $b=-2$, $c=-1$) to see that the two solutions are

$$x = \frac{2\pm\sqrt{12}}{4} = \frac{1\pm\sqrt{3}}{2}\,.$$

Caution! Notice that $\sqrt{12}=\sqrt{4\cdot 3}=\sqrt{4}\cdot\sqrt{3}=2\sqrt{3}$; so it is NOT the case that $\sqrt{12}/4=\sqrt{3}$.

(2) Cross-multiplying yields $x(x-4)=2\cdot 3$, which expands to $x^2-4x=6$. Bring the 6 to the other side of the equation: $x^2-4x-6=0$. Now apply the quadratic formula (in this case $a=1$, $b=-4$, $c=-6$) to see that the two solutions are

$$x = \frac{4\pm\sqrt{40}}{2} = 2\pm\sqrt{10}\,.$$

(3) Cross-multiplying yields $3x(x+1)=2$, which expands to $3x^2+3x=2$. Bring the 2 to the other side of the equation: $3x^2+3x-2=0$. Now apply the quadratic formula (in this case $a=3$, $b=3$, $c=-2$) to see that the two solutions are

$$x = \frac{-3\pm\sqrt{33}}{6}\,.$$

4.3 #9. Yes, the new smaller rectangle is a Golden rectangle. The sides of the new rectangle are half the length of the sides of original rectangle; but this means that the ratio of the sides of the new rectangle is exactly the same as the ratio of the sides of the original rectangle.

Since the original rectangle has the right ratio to be a Golden Rectangle, so does the new rectangle.

4.3 #20. Take two sides of your triangle, find the midpoints of these sides, and draw the line segment joining these midpoints. Repeat with the other two pairs of sides. You will have divided your original triangle into four pieces, each identical in shape and proportion to the original triangle, but with one-fourth the area.