MATH 111: HW 5 SOLUTIONS AND COMMENTS

4.4 #3.

(1) This pattern has one flip symmetry (the line of symmetry is horizontal).
(2) This pattern has six flip symmetries.
(3) The number of flip symmetries in this pattern depends on whether you are only flipping the picture in the book, or whether you view the patterns as being extended to cover the entire plane. If you're just talking about the picture in the book, there is just one flip symmetry, through the vertical line down the middle of the picture. If you are looking at the pattern extended to the entire plane, there are many: flips across the vertical lines down the middle of each square, and flips across the horizontal lines which run through the middle of each square.

4.4 #4.

(1) This pattern has no non-trivial rotations.
(2) This pattern has six 60-degree rotations (i.e. five non-trivial rotations).
(3) Again it depends on whether you are only flipping the picture in the book, or whether you view the patterns as being extended to cover the entire plane. If the former, there are no non-trivial rotations. If the latter, there are 180-degree rotations around the center of each square.

4.4 #5. In these patterns, a "small tile" is just a single one of the nine small trapezoids (in the first picture) and one of the twelve triangles (in the second picture). In both cases, there are 4 tiles in a super-tile and, therefore 16 in a super-super-tile (a group of 4 super-tiles). I accepted 4 and 9 as an alternate answer.

4.4 #16. See the solution in the book.

4.4 #20. No, as we demonstrated in class, they are not the same. See the solution in the book. It is important to be careful to use the same diagonal line both times (e.g. use the northeast-to-southwest line twice, or the northwest-to-southeast line twice, but don't mix and match).

4.5 #1. A regular polygon is a polygon whose edges are all of the same length, and whose angles are all equal. (Note that it is possible to have polygons whose edges are all of the same length, but whose angles are not all equal. Rhombi are examples of these.)

Remember that all the edges of a polygon must be straight, so, for example, a circle is not a polygon.

4.5 #2. A regular solid is a solid in which all faces are identical regular polygons, and the same number of polygons meet at each vertex (i.e. the same number of edges meet at each vertex).

4.5 #9. (I'll leave the pictures to your imaginations.)

4.5 #14. The tetrahedron, cube, and icosahedron all yield triangular cuts, since three edges meet at each vertex. The octahedron yields a square cut, since four edges meet at each vertex. The icosahedron yields a pentagonal cut, since five edges meet at each vertex.

4.5 #20. I will explain carefully how to do this for the example of an icosahedron, and you can extend the method to the other solids. We saw in #14 that each time we slice, one of the original vertices of the icosahedron is removed, and a pentagonal face is created. This pentagonal face has 5 vertices. The original icosahedron has 12 vertices; since each vertex is removed and replaced by 5 vertices, the number of new vertices is $12 \times 5 = 60$.

What about edges? Each of the 30 edges of the original icosahedron is shortened a little bit by the slicing. There are also many edges added: the 5 edges of each of the 12 pentagons that we've added. So the number of edges is $30 + 5 \times 12 = 30 + 60 = 90$.

What about faces? Each of the 20 original faces is trimmed a little bit, and we have added 12 pentagonal faces (one for each original vertex). So there are $20 + 12 = 32$ faces.

Therefore $V = 60$, $E = 90$, and $F = 32$. Notice that $V - E + F = 60 - 90 + 32 = 2$, as we would expect.