MATH 111: HW 6 SOLUTIONS AND COMMENTS

4.7 #3. Every point on the plane can be found by giving two coordinates — that's what it means when we say that the plane is 2-dimensional! No point in the plane requires three coordinates.

4.7 #6.

(1) A diamond
(2) A triangle
(3) A square (or a rectangle)
(4) A vertical line

4.7 #9. The third and fourth pictures require tearing the wall to get at the gold.

4.7 #10. Using the third dimension, one need only reach inside the barrier and lift the gold out of the plane (as though one were removing a sticker from the page).

Here's another way to think about it, which might be more useful for the next problem. You can grab one edge of the barrier and lift it out of the page (into the third dimension), then slide the gold out underneath the barrier. Notice what this looks like from the point of view of someone living in the plane, who can't see out of the plane into the third dimension: the barrier appears to break, and the gold slides out.

4.7 #11. Take one of the rings, and lift part of it into the fourth dimension. From the point of view of someone living in the third dimension, this ring appears to break. (Remember, it isn't actually broken, it just appears broken to someone who can't see the part that has been lifted into the 4th dimension.) Now (in 3-d) slide the other ring off of the one that appears to be broken, and they are unlinked.

5.1 #4. If you remove the red dot from the X, there will be 4 pieces. If you remove the red dot from the Y, there will be 3 pieces. If you remove the red dot from the Z, there will be 2 pieces.

There is no point that you can remove from either the Y or the Z which breaks them into 4 pieces. So, the Y and the Z are not equivalent to the X by distortion. Also, there is no point that you can remove from the Z to break it into 3 pieces, so it is not equivalent to the Y by distortion. Therefore, none of the three original letters are equivalent to any of the others by distortion.

5.1 #7. (Caution: this problem was to be solved using the letters as printed in the book. The font I am using here is slightly different, so you may want to refer back to the font in the book when reading this solution.)

The letters CGIJLMNSUVWZ are all equivalent to a straight line segment.

The letters EFTY are all equivalent to three line segments joining at a point.

The letters KX are equivalent to four line segments joining at a point.

The letter H has two points where three segments meet up. I think it's reasonable to interpret the K as falling into this category as well — it's hard to tell. So, depending on how you think the K looks, you could group K with H instead of with X.

The letters DO are equivalent to a simple loop.

The letters PQ have a simple loop with a tail attached.

The letters AR have a simple loop with two tails attached.

The letter B has two loops.

5.1 #9. Yes, the straw and the coin with the whole through it are both equivalent by distortion to a cylinder.

5.1 #22. Yes, the two spheres pictured are equivalent by distortion. It is important to notice that one thing you can do, when distorting the sphere, is moving around the holes where the ropes enter the sphere.

Here's what we can do: move the bottom loop up towards the middle of the sphere (moving the holes as well as the rope). Then move the bottom hole of the upper loop down close to the bottom of the sphere, leaving the upper hole of the upper loop where it is. (This step involves stretching the upper rope.) After these moves, the first sphere has been distorted to look like the second sphere.

5.1 #23. **Solution 1:** No, the sequence does not describe an equivalence by distortion, because the step between the third and fourth pictures involves gluing, which isn't permitted in equivalence by distortion.

Solution 2: The first picture is a sphere, and the last one is a torus. Since a sphere and a torus are not equivalent by distortion, this sequence of drawings cannot represent an equivalence by distortion.