MATH 111: HW 7 SOLUTIONS AND COMMENTS

5.2 #3. The one side of the Möbius band will be both blue and white. The two colors will meet at the place where the ends of the strip of paper are taped together.

5.2 #8. What you get is: two strips, each with a full twist (i.e. 2 half-twists), linked together. You can see this using edge identification diagrams (the kind of diagram we drew on the blackboard in class, and that are used on page 349 and 350 of the textbook).

Here's another way to deduce that this is what you'll get: the strip with two half-twists has two sides and two edges. Cutting the strip down the center cuts each side in half, so the result has to have four sides; therefore there will be two pieces, each with two sides. (Since the pieces each have two sides, they can't be Möbius strips.) The two edges of the original strip are two linked circles; they stay linked when the strip is cut down the middle, so the two resulting strips have to be linked.

5.2 #9. Using an edge identification diagram, you can see that the result will be exactly the same as in #8 — just, one of the two pieces will be wider, and the other will be thinner.

5.2 #12. The original Möbius band is cut into two pieces, one consisting of the outer part of the original strip, and the other consisting of the inner part. The outer strip is twice as long as the original strip; the inner strip has the same length as the original strip.

5.2 #28. The pieces are interlocked because the centerlines of the two objects form linked rings; that is, the centerline of the Möbius strip is a circle which is linked with the edge circle of the Möbius strip.

Here is one way to see this. If you hold the strip so that the centerline forms a perfect circle, then the edge of the strip intersects the plane containing that circle twice: one inside the centerline circle, and once outside, so the two loops are indeed linked.

5.3 #4. There are 5 vertices, 8 edges, and 5 faces; so $V - E + F = 5 - 8 + 5 = 2$.

5.3 #7. The original graph has 7 vertices, 8 edges, and 3 faces. Here are the numbers for the altered graphs:

(1) $V = 7$, $E = 9$, $F = 4$; the number of vertices is unchanged from the original, and the edges and faces have each increased by 1.
(2) $V = 8$, $E = 9$, $F = 3$; the number of faces is unchanged from the original, and the vertices and edges have each increased by 1.
(3) $V = 7$, $E = 9$, $F = 4$; the number of vertices is unchanged from the original, and the edges and faces have each increased by 1.
(4) $V = 8$, $E = 9$, $F = 3$; the number of faces is unchanged from the original, and the vertices and edges have each increased by 1.

5.3 #15. In any connected graph in the plane, we have $V - E + F = 2$, so $V = E - F + 2$. Here $E = 36$ and $F = 18$. Therefore

$$V = E - F + 2 = 36 - 18 + 2 = 20\,.$$

5.3 #23. Let's take the third picture as an example. It's a stellated octahedron: that is (as the problem explains) it was built by starting with an octahedron, then attaching a pyramid to each of its faces.

As we know, an octahedron originally has 6 vertices, 12 edges, and 8 faces.

Each original vertex is still part of the new polyhedron, but there some new vertices: the top vertex of each pyramid, so there's one new vertex for each of the eight faces. Therefore there are $6 + 8 = 14$ vertices.

Each original edge is still part of the new polyhedron, but there are some new edges: the three edges leading to the top of each of the eight pyramids. So there are $12+3\times 8 = 12+24 = 36$ edges.

The original eight faces are gone, but each face has been replaced by the three faces of each pyramid. So there are $8 \times 3 = 24$ faces. So $V = 14$, $E = 36$, $F = 24$, and $14 - 36 + 24 = 2$.

Note that for the stellated cube (the second picture) each pyramid will have 4 edges leading to the top vertex, and for the stallated dodecahedron (the fourth picture) there are 5. I'll leave it to you to work out the details for the other four pictures.

5.3 #29. There are 9 vertices, 18 edges, and 9 faces on this torus-shaped figure. The Euler characteristic is $V - E + F = 9 - 18 + 9 = 0$ for this figure. (Notice that the Euler characteristic is 0, not 2, because the shape of the figure — in the topological sense — is a torus, not a sphere.)