

## MATH 111: HW 9 SOLUTIONS AND COMMENTS

**7.3, #1.** If 52 million people die every year, the average number of people who die each day can be computed by dividing 52,000,000 by 365, the number of days in a year. We have  $52,000,000/365 \approx 142465$ .

Note that since 52,000,000 is obviously not the exact number of people who die year each, there is no point in giving an answer that is so precise as 142465. It is much better to round off and say that approximately 140,000 people die each day, on average.

**7.3, #2.** The chance that the event will not happen that day is  $\frac{999}{1000}$ .

The chance that the event will not happen two days in a row is

$$\frac{999}{1000} \times \frac{999}{1000} = 0.998001.$$

The chance that the event will not happen in an entire year is

$$\left(\frac{999}{1000}\right)^{365} \approx 0.694.$$

**7.3 #5.** If you flip a fair coin three times, there are eight equally likely possibilities: HHH, HHT, HTH, HTT, THH, THT, TTH, TTT. One of these possibilities is all heads, so the probability is  $1/8$ .

**7.3 #12.** A reasonable approximation for the fraction of people who die each year is 0.01, or 1%; then the probability that a randomly chosen individual does *not* die is 0.99. Under this estimate, the probability that none of 1000 randomly chosen individuals will die is

$$.99^{1000} \approx 0.000043.$$

Therefore, under this estimate, the probability that at least one of the randomly chosen people will die is approximately  $1 - .000043 \approx 0.99996$ .

Note that a different approximation for the fraction of people who die each year will lead to a different final answer. For example, a  $1/75$  probability that a randomly chosen individual will die leads to a final answer of approximately 0.999998.

Note that the question was NOT: on average, how many of these people do you expect to die within the next year? That's a related but different question.

**7.3 #20.** In case you're curious, the probability of ending up at your starting point after 50 pairs of flips is approximately 0.0126, or approximately 1 in 79.

**7.3 #22.** The probability that the first person you ask says yes is  $1/2$ . The probability that the first five people you ask say no is  $(1/2)^5 = 1/32$ , or 0.03125.

**7.5 #1.** Out of the 100 students, we would expect (on average) 50 of the students to flip heads and answer yes no matter what. In that case, of the other 50 students (the ones who flip tails and answer truthfully),  $54 - 50 = 4$  have cheated. Since we estimate that 4 students have cheated in the half of the class that flipped tails, we estimate that a total of 8 students in the entire class have cheated.

**7.5 #7.** 380 students out of 1000 is 38%. We would expect 500 students to flip heads and answer yes. We would expect 500 students to flip tails and answer truthfully, and we expect that 38% of these still like to watch cartoons; 38% of 500 is  $500 \times 0.38 = 190$ . Therefore we expect a total of  $500 + 190 = 690$  students to answer “yes”.

**7.5 #9.** Let  $D$  denote the number of people who flipped  $HH$  and who have used drugs. Let  $N$  denote the number of people who flipped  $HH$  and who have not used drugs.

We expect three times more people not to flip  $HH$  than flip  $HH$ , and we expect (on average) that the proportion among them who have used drugs is the same as among those who flipped  $HH$ . Therefore, we expect that a total of  $3D$  people used drugs and didn't flip  $HH$ ; and a total of  $3N$  people haven't used drugs and didn't flip  $HH$ . Notice that the total number of people in the survey is given by  $D + N + 3D + 3N = 4D + 4N$ , and the total number who actually have used drugs — the number we want to calculate — is  $4D$ .

The people who will *say* they used drugs are the  $3D$  people who have used drugs and didn't flip  $HH$  (they are telling the truth), and the  $N$  people who have not used drugs and did flip  $HH$  (they are lying). The total of  $3D + N$ , so we have the equation

$$3D + N = 78.$$

Similarly, we have this equation for the people who say they did not use drugs:

$$D + 3N = 172.$$

There are many ways to solve these equations. Here's one: the total number of people in the survey is  $4D + 4N = 250$ , so  $D + N = 250/4 = 62.5$ . It follows that

$$2D = (3D + N) - (D + N) = 78 - 62.5 = 15.5.$$

Therefore  $4D = 2 \times 2D = 2 \times 15.5 = 31$ . We expect that around 31 of the 250 students have used drugs (or 12.4%).

**7.5 #16.** For example, people often lied and said that they watched much more educational TV than they actually did!