

Introduction to Number Theory

McGill 189-346/377B

David Savitt

Winter 2002

Number theory, loosely speaking, is motivated as an attempt to solve *Diophantine equations*: systems of equations with integer coefficients for which we'd like to find integer or rational solutions. For example, it is quite easy to find all the integer solutions to the equation $x^2 + y^2 = 1$, but it is not so obvious how to find all rational solutions.

The most fundamental notion in number theory is that of a prime number; the most fundamental technique is that we can study a Diophantine equation by 'isolating' the behaviour the equation at any particular prime number using *modular arithmetic*. This is how we will begin the course.

With several key results on modular arithmetic (Fermat's little theorem, the Chinese Remainder theorem, primitive roots, and quadratic reciprocity) in hand, we will turn some particular Diophantine problems. For example, we will see how the theory of continued fractions finds the integer solutions to the equation $x^2 - Ny^2 = \pm 1$, we will see what we can say about which primes p can be written in the form $x^2 + Ny^2 = p$ for x, y integers, and we will show that every integer can be written as a sum of four squares.

Texts: The textbook for the course will be H. Davenport, "The Higher Arithmetic" (required), supplemented at times by Kato, Kurakawa, and Saito "Number Theory I" (not required). Both books can be found in the bookstore, and are in the \$30-\$40 range. The classic introductory text by Niven, Zuckerman, and Montgomery, and the wonderful "Primes of the form $x^2 + ny^2$ " by David Cox will also be useful.

Contact information: my office is Burnside 1127, though I am rarely there. The most efficient way to reach me is by email: dsavitt@math.mcgill.ca.

Course website: <http://www.math.mcgill.ca/~dsavitt/nt.html>

Office hours: I will hold office hours on Mondays and Wednesdays at 2:30 pm, although this means that in practise, if you would like to meet, I would like you to find me in the classroom at the end of class. On the other hand, I am always happy to meet with you by appointment at times other than my official office hours.

Homework: Homework assignments will be handed out and collected on Fridays. Late assignments will be accepted if and only if I have not yet graded what everyone else has handed in for that assignment. The assignments will be the same for both sections of the course, but there will be some problems which for 189-377B are counted as usual and for 189-346B are counted as bonus problems.

Grades: For students in 189-346B, homework will count for 60% of the grade, and a take-home final exam will count for 40%. For students in 189-377B, homework will count for 50% of the grade, the take-home final will count for 30%, and a written project will count for 20%. *This will be subject to change if we are able to get a grader for this course.*