Exam 3

Math 124, Section 25, Fall 2004

November 8, 2004

For full credit, show all your work/sketch any graphs that you produce on your calculators.

Problem 1 Short Computations 40 points

(a) Find the equation of the tangent to the curve $y^3 + \sin(\pi xy) + 2xy^2 - x^3 = 15$ at the point $(1, 2)$.

(b) Calculate

$$\lim_{x \to 0} \frac{\ln(1+ax)}{\sin(2x)}, \qquad \text{where } a \text{ is a constant.}$$

(c) Find a and b such that the function $f(x) = a\cosh(bx)$ satisfies $f(0) = 5$ and $f''(x) - 9f(x) = 0$. What is $f(2)$?

(d) $x > a$. Show that $x^2 > a^2 + 2a(x - a)$. (Hint: mean value theorem on $[a, x]$ for $f(x) = x^2$.)

Problem 2 20 points

The acceleration due to gravity g, is given by

$$g = \frac{GM}{r^2}$$

where M is the mass of the earth, r is the distance from the center of the earth and G is the universal gravitational constant.

(a) Show that, when r changes by Δr, the change in the acceleration due to gravity Δg is given by

$$\Delta g \approx -2g\frac{\Delta r}{r}.$$

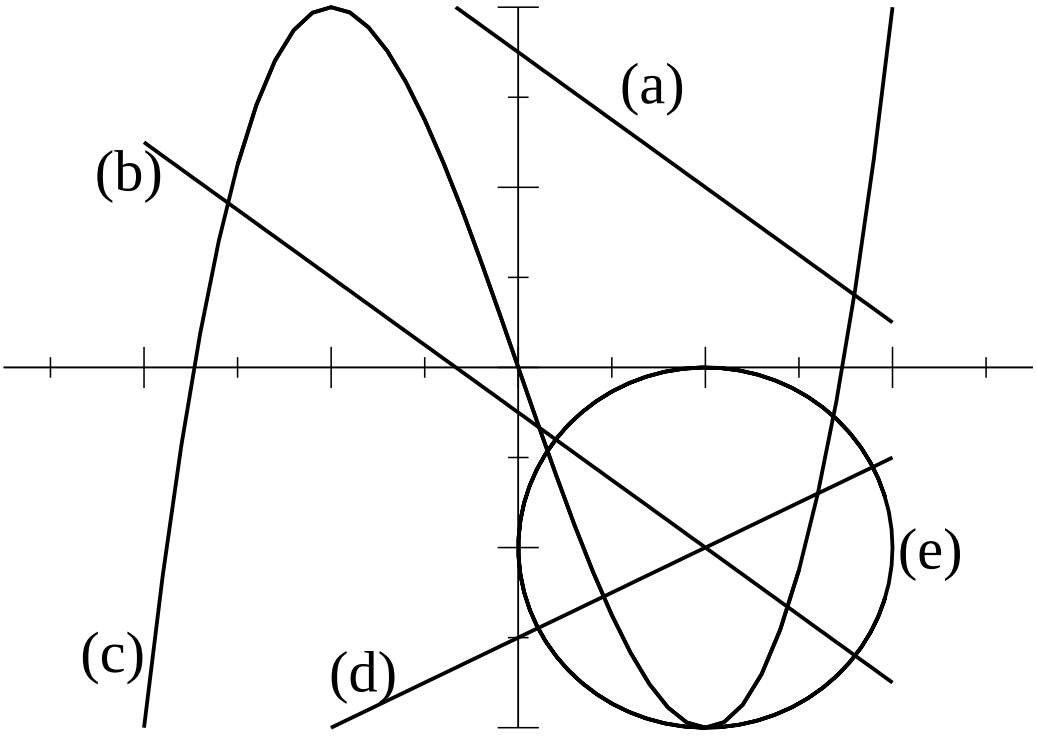
(b) What is the (approximate) value of g 100 km above sea level? Assume that at sea level, the radius of the earth is 6400 km and $g = 9.8m/s^2$.

(c) What is the percent change in g when moving from sea level to the top of Pike's peak (4.315 km)?

Problem 3 Suppose $a, b, c, d, m, n, p, q > 0$. Match the following parametric equations with the graphs in the figure. 20 points

$$I: \begin{cases} x = a + ct \\ y = -b + dt \end{cases} \qquad II: \begin{cases} x = m + pt \\ y = n - qt \end{cases}$$

$$III: \begin{cases} x = a + \cos(t) \\ y = -b - \sin(t) \end{cases} \qquad IV: \begin{cases} x = t \\ y = t^3 - 3t \end{cases}$$



Identify the curves corresponding to the various parameterizations:

I :

II :

III :

IV :

Problem 4 (20 points)

A rectangular beam is cut from a semi-circular log of wood of radius a. The strength of the beam of width w and height h is proportional to wh^2. (See figure) Find the width and the height of the beam with maximum strength.