

Exam 4
Math 124, Section 25, Fall 2004
December 2, 2004

For full credit, show all your work/sketch any graphs that you produce on your calculators.

Problem 1 Short Computations 40 points

(a) Find upper and lower estimates that are accurate to one decimal place for the area enclosed by the curve $y = \sin(x^2)$, the x -axis and the line $x = 1$.

(b) Calculate the average value of the function $y = e^x - x^{-1}$ in the interval $1 \leq x \leq 4$.

(c) Find

$$\int \left[\cos(t) + \frac{(t+1)^2}{2t} \right] dt.$$

(d) An object is dropped from a 400 ft tall tower. What is its velocity when it reaches the ground? Assume that the acceleration due to gravity is 32 ft/sec^2

Problem 2

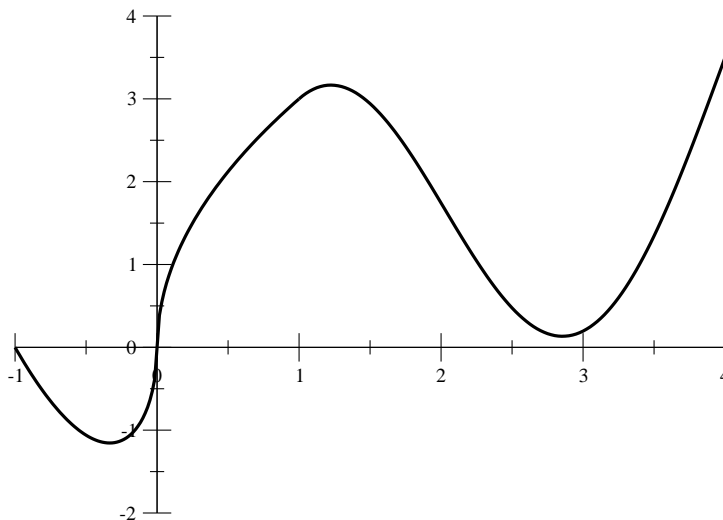
(20 points)

A news broadcast in early 1993 said that the average American's annual income is changing at the rate of $r(t) = 40(1.002)^t$ dollars per month, where t is in months from January 1, 1993. How much did the average American's income change in 1993? (Useful formula: If $F(x) = a^x$, then $F'(x) = \ln(a)a^x$.)

Problem 3

20 points

Consider the function whose graph is shown below.



- (a) Give an expression (in terms of an integral) for a function $F(x)$ that is an anti-derivative of $f(x)$ and also satisfies $F(2) = 1$.

(b) Sketch the graph of the anti-derivative $F(x)$ from part (a).

(c) Order the following quantities from the smallest to the largest:

$$\begin{array}{lll} A = \int_{-1}^0 f(x)dx & B = \int_{-1}^1 f(x)dx & C = \int_0^2 f(x)dx \\ D = \int_2^0 f(x)dx & E = \int_1^3 f(x)dx & F = \int_4^0 f(x)dx \end{array}$$

Problem 4

20 points

The rate at which the layer of ice on a lake thickens is inversely proportional to the square-root of the time since the layer starts to form.

- (a) Write down a differential equation that models this process.
- (b) What is the thickness of the of the layer of ice at time t ? (Note that, at $t = 0$, the layer just begins to form)
- (c) If the layer is 4 inches thick after one hour, what is it's thickness after 1 day?