Math 527B – Spring 09
Homework 1 : Due Jan. 28

2.1 Assume that ϕ is continuous and bounded on $\mathbb{R}$. Prove that

$$\lim_{a\to 0} \frac{1}{\pi} \int \frac{a}{x^2 + a^2}\, \phi(x)\, dx = \phi(0)$$

(This is a restatement of problem 4.0.7 Pg. I-176 in the book)

2.2 Let

$$\theta(x) = \begin{cases} \frac{1}{2} & |x| < 1 \\ 0 & \text{otherwise} \end{cases}$$

(a) Assume that ϕ is continuous and bounded on $\mathbb{R}$. Prove that

$$\lim_{a\to 0} \frac{1}{a} \int \theta\left(\frac{x}{a}\right) \phi(x)\, dx = \phi(0)$$

(This is problem 4.0.8 Pg. I-176 in the book)

(b) Assume that ϕ is bounded and ϕ' is continuous. Prove that the following limit exists, and evaluate the limit:

$$\lim_{a\to 0} \frac{1}{a^2} \int \left[\theta\left(\frac{x-a}{a}\right) - \theta\left(\frac{x+a}{a}\right)\right] \phi(x)\, dx$$

2.3 Find appropriate hypothesis on ϕ that ensure that the following limit exists:

$$\lim_{a\to 0} \int \frac{x}{a^2 + x^2}\, \phi(x)\, dx$$

Can you identify this limit (in terms of objects you know)?

2.4 Problems 4.1.32, 4.1.33 and 4.1.34, Pg. I-186 of the class notes.