Quantum Chaology

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- Introduce the stadium billiard problem and investigate the related dynamics.
- Show a method to find the trace of the quantum operator for the stadium billiard.
- Explain some open research problems in the field of quantum chaology.

Hamiltonian

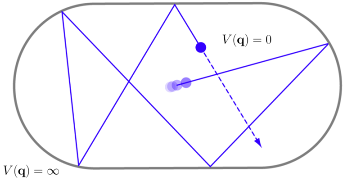

Figure: http://www.aimath.org/news/que/stadium1.png

- Energy of the system is given by the Hamiltonian

$$H(\mathbf{q}, \mathbf{p}) = \frac{1}{2m}\mathbf{p}^2 + V(\mathbf{q}) = E.$$

- Equations of motion (Newton's Second Law) are given by

$$\frac{\partial q_i}{\partial t} = \frac{\partial H}{\partial p_i}, \qquad \frac{\partial p_i}{\partial t} = -\frac{\partial H}{\partial q_i} \qquad \forall i = 1, 2, ..., n.$$

Action Integral

- Define the Φ integral for some curve C that connects $\mathbf{q}(t_0) = \mathbf{q}_0$ and $\mathbf{q}(t_1) = \mathbf{q}_1$ as,

$$\Phi(C) = \int_{t_0}^{t_1} L(\dot{\mathbf{q}}, \mathbf{q}, t)\, dt.$$

- Hamilton's principle of least action says that the extremal of the Φ integral is satisfied with

$$\Phi(C) = \int_{t_0}^{t_1} L\, dt, \quad \text{where } L = T - U.$$

- Define the action integral $S(\mathbf{q}(t_0), \mathbf{q}(t_1), E) = \Phi(C) + Et$.

Mapping the Stadium

- Given initial position and momentum of particle $\mathbf{q}_0, \mathbf{p}_0$, can give the position and velocity at some later time t, $\mathbf{q}_t$ (may take some work).

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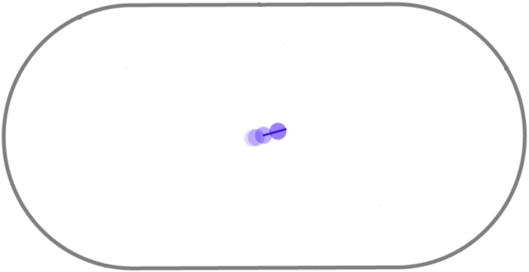

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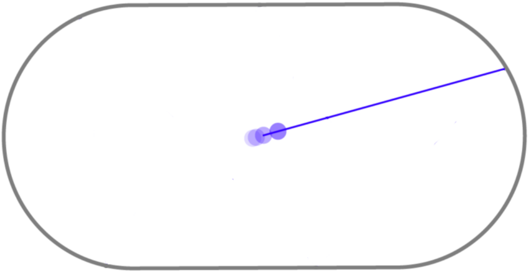

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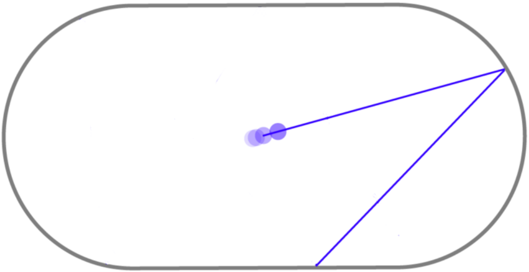

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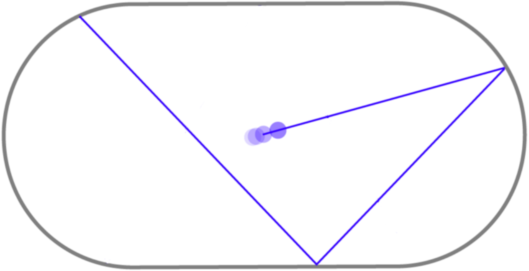

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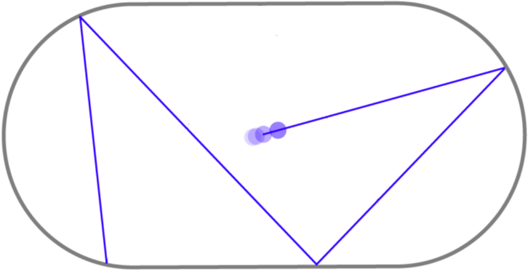

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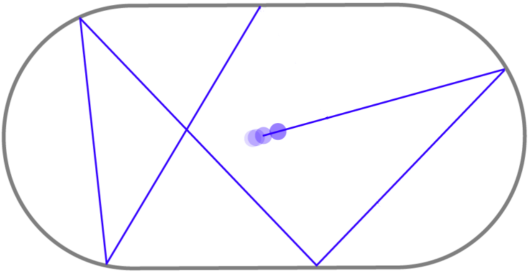

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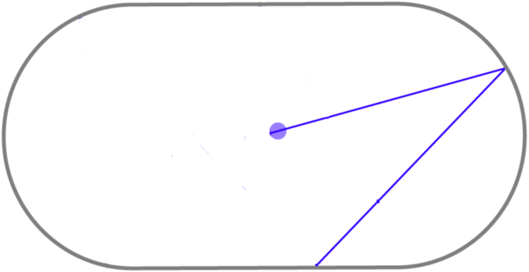

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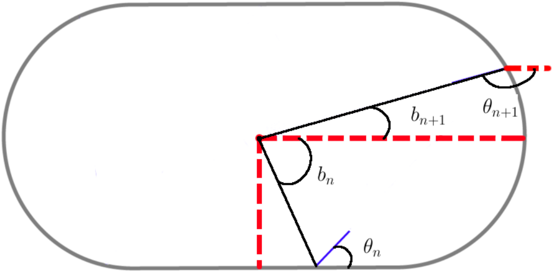

Billiard Poincaré map

- The map for a reflection off the line segment,

$$\begin{pmatrix} b_{n+1} \\ \theta_{n+1} \end{pmatrix} = \Psi_l \begin{pmatrix} b_n \\ \theta_n \end{pmatrix} = \begin{pmatrix} \arcsin\left(\frac{\sin(\pi - b_n - \theta_n)}{s_3}\right) s_2 \\ \theta_n + \pi/2 \end{pmatrix}.$$

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- and for the arc,

$$\begin{pmatrix} b_{n+1} \\ \theta_{n+1} \end{pmatrix} = \Psi_a \begin{pmatrix} b_n \\ \theta_n \end{pmatrix} = \begin{pmatrix} \arccos\left(\frac{s_1^2 + s_3^2 - s_2^2}{2 s_1 s_2}\right) \\ 2r \frac{\sin(\arcsin((d - s_3)\sin(\pi - \theta_n)/r)}{\sin(\theta_n)} + (\pi - \theta_n) \end{pmatrix}.$$

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- The linear stability of a trajectory is defined as the Jacobian of the discreet mapping.

$$J\left(\psi_{l,a}\begin{pmatrix} b_n \\ \theta_n \end{pmatrix}\right) = \begin{pmatrix} \frac{\partial b_{n+1}}{\partial b_n} & \frac{\partial b_{n+1}}{\partial \theta_n} \\ \frac{\partial \theta_{n+1}}{\partial b_n} & \frac{\partial \theta_{n+1}}{\partial b_n} \end{pmatrix}.$$

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- e.g. $\mathbf{x}_0 = (0, 0, 1/\sqrt{2}, 1/\sqrt{2})^T$

$$\Psi_4 \mathbf{x}_0 = \Psi_l(\Psi_a(\Psi_l(\Psi_a(\mathbf{x}_0)))) = \mathbf{x}_0.$$

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$$\Psi_4 \mathbf{x}_0 = \Psi_l(\Psi_a(\Psi_l(\Psi_a(\mathbf{x}_0)))) = \mathbf{x}_0.$$

- Now compute linear stability matrix

$$D\Psi^{(k)} = J(\Psi_4)|_{\mathbf{x}_0}.$$

Quantum Mechanics

Postulates of QM

- Postulate I: an observable A can be assigned an operator, $\hat{A}$, such that when the observable is measured with value a the corresponding equation is satisfied,

$$\hat{A}\phi = a\phi.$$

- Example: momentum $\mathbf{p}$ is an observable and

$$\hat{\mathbf{p}}\phi = i\hbar \frac{\partial}{\partial \mathbf{q}} \phi = i\hbar \nabla \phi.$$

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- Postulate II: The measurement a of the observable A leaves the system in the state ϕ_a, where ϕ_a is the eigenvector of $\hat{A}$ with corresponding eigenvalue a.

Postulates of QM

- Postulate III: The state of a systems can be represented by a continuous and differentiable function called the wave function and denoted as ψ. The properties of ψ is that for any observable A, relevant to the system in the state $\psi(\mathbf{q}_0, t)$ at time t, has the average or expected value

$$\langle A \rangle = \int \psi^* \hat{A} \psi d\mathbf{q} = \langle \psi | \hat{A} | \psi \rangle.$$

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- The scale of the problem where both classical and quantum mechanics is important and will be called the *semi classical domain* and can be thought of $\hbar \to 0$.

Semi classical domain

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- Now state of the system develops in time by

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- Now the stationary Schrödinger equation is satisfied

$$-\left(\frac{\hbar^2}{2m}\right)\nabla^2\phi(\mathbf{q}) = E\phi(\mathbf{q}).$$

Dilemma

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- Stadium billiard problem displays hard chaos and the mapping is nonlinear.
- The Schrödinger equation for the quantum analogue is linear and thus can not have chaos.
- If there is chaos in the system, it is to be found in

$$\psi(\mathbf{q}, t) = \phi(\mathbf{q}) e^{-i\omega t}.$$

- Chaos will only show up in high energy states.
  - Fits with limit $\hbar \to 0$.

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- Define the Green's function G_c as the propagator in the energy domain.

$$G_c(\mathbf{q}_1, \mathbf{q}_0, E) = (i\hbar)^{-1} \int_0^\infty K(\mathbf{q}_1, \mathbf{q}_0, t) e^{iEt/\hbar} \, dt.$$

$$G_c(\mathbf{q}_1, \mathbf{q}_0, E) = \int_0^\infty C(\mathbf{q}_1, \mathbf{q}_0, t) e^{(\Phi(\mathbf{q}_1, \mathbf{q}_0, t) + Et)i/\hbar} \, dt.$$

- By construction, this Green's function satisfies

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- Want a simpler expression for G_c.

Stationary Phase

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$$\int e^{iS/\hbar}\, d\mathbf{q} \approx \int_{\mathbf{q}_0-\epsilon}^{\mathbf{q}_0+\epsilon} \exp\left(i(S(\mathbf{q}_0) + S''(\mathbf{q}_0)/2(\mathbf{q}-\mathbf{q}_0)\right)\, d\mathbf{q}.$$

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$$\int \exp\left(\frac{iS''(\mathbf{q}_0)(\mathbf{q}-\mathbf{q}_0)^2}{2\hbar}\right)\, dq = \sqrt{\left(\frac{2\pi\hbar}{|S''(\mathbf{q}_0)|}\right)} \exp(\pm i\pi/4),$$

Stationary Phase

- As $\hbar \to 0$, the previous integrand converges to the delta function in the sense of distributions.
- Thus

$$I \leq \exp(iS(\mathbf{q}_0)/\hbar) \int_{-\infty}^{\infty} f(\mathbf{q}) \exp\left(\frac{-i|S''(\mathbf{q}_0)|(\mathbf{q}-\mathbf{q}_0)^2}{2\hbar} \right) d\mathbf{q}$$

$$= \sqrt{\left(\frac{2\pi\hbar}{|S''(\mathbf{q}_0)|} \right)} \left| f(\mathbf{q}_0) \exp(iS(\mathbf{q}_0)/\hbar) \exp(\pm i\pi/2) \right|,$$

- $$\int f(\mathbf{q}) e^{iS/\hbar} \, d\mathbf{q} = f(\mathbf{q}_0) \exp(iS(\mathbf{q}_0)/\hbar) O(\hbar^{1/2}),$$

Green's Function

Figure: http://www.csicop.org/si/2006-04/Quincey Fig 2.jpg

$$G_c(\mathbf{q}_1, \mathbf{q}_0, E) = \frac{2\pi}{(2\pi i\hbar)^{(n+1)/2}} \sum_{a\in\sigma} \sqrt{(-1)^{n+1} D_a}$$
$$\exp(iS_a(\mathbf{q}_1, \mathbf{q}_0, E)/\hbar - i\mu\pi/2).$$

- $\sigma = \{$All classical trajectories of the system from $\mathbf{q}_0$ to $\mathbf{q}_1\}$ and

$$D(\mathbf{q}_1, \mathbf{q}_0, E) = \frac{1}{|\mathbf{p}_0||\mathbf{p}_1|} \left| \frac{-\partial^2 S_a}{\partial q_i(t_0)\partial q_j(t_1)} \right|,$$

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- Very hard to find all the classical trajectories.
  - Next best thing is to find trace.
- The Gutzwiller trace formula is given by

$$\int G_c(\mathbf{q}_1, \mathbf{q}_0, E)dq = \hbar^{-1} \sum_{k \in \{\text{p.o.}\}} \frac{T_k}{(\det |D\Psi^k - I|)^{1/2}} \exp(iS_k/\hbar - \ell i\pi/2).$$

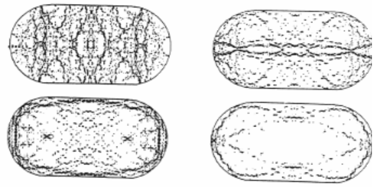

Figure: Gutzwiller pg 251

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- Given a classically chaotic dynamical system, the Bohigas-Giannoni-Schmit (BGS) conjecture claims that this sequence of energies is statistically equivalent to the eigenvalues of a random matrix belonging to one of the three standard Gaussian ensembles.

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- Given a classically chaotic dynamical system, the Bohigas-Giannoni-Schmit (BGS) conjecture claims that this sequence of energies is statistically equivalent to the eigenvalues of a random matrix belonging to one of the three standard Gaussian ensembles.
- Suggests that the linear quantum operator associated with a chaotic system has the properties of a random hermitian matrix.

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- Riemann Hypothesis: All zeros in this critical strip are $z = 1/2 + ti$.
- Question: Could a certain classically chaotic system have an analogous quantum operator such that t is in the spectrum of the operator?

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- The numbers t appear to be distributed like eigenvalues for random matrices.
- The t's could be distributed according to eigenvalue of a linear quantum operator with a classically chaotic counterpart.
  - If so then the properties of the dynamical system would have periodic orbits labeled by primes p, and all periodic orbits are unstable.
- Question: Is there a system responsible for producing the same t's?
  - If so, what is this system?