Decay of the Singlet Conversion Probability in One Dimensional Quantum Networks

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Networks

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Figure: A one dimensional lattice

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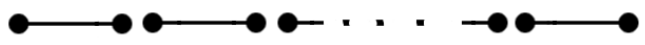

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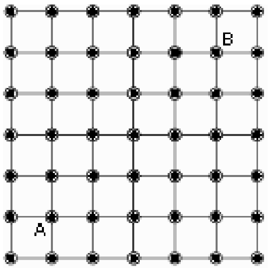

Figure: A two dimensional lattice

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  - Give a brief introduction to quantum mechanics in the quantum computation setting.
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  - Show that this method is not reliable in large 1-D quantum networks.

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  - So the inner product of v and w is $\langle w|v\rangle$, a bracket.
  - An operator (matrix), $A \in \mathbb{C}^{n\times m}$ acting on a ket is written $A|v\rangle$.
  - and the inner product of the result with w is $\langle w|A|v\rangle$.

Quantum Mechanics

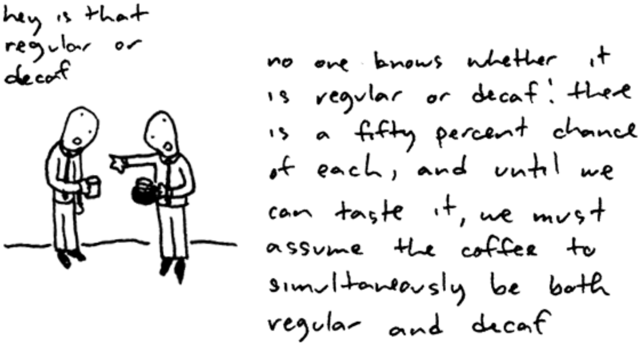

Figure:
http://www.toothpastefordinner.com/041205/schroedingers-decaf.gif

Postulates of QM

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Example: A quantum bit, called a *qubit*, is a physical system with state space $\mathbb{C}^2$ and is described by the state vector

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle.$$

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- Postulate III: Let $\{M_m\}$ be measurement operations. The $\{M_m\}$ must satisfy $\sum_m M_m^\dagger M_m = I$. If the quantum system is in state $|\psi\rangle$ immediately before the measurement is taken, then the probability that the result of the measurement is m is given by

$$p(m) = \langle\psi|M_m^\dagger M_m|\psi\rangle,$$

and the state of the system immediately after the measurement is given by

$$\frac{M_m|\psi\rangle}{\sqrt{p(m)}}.$$

Measurement Example

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$$|\psi_0\rangle = \frac{|0\rangle\langle 0|\psi\rangle}{\sqrt{p(0)}} = \frac{\alpha}{|\alpha|}|0\rangle.$$
$$|\psi_1\rangle = \frac{\beta}{|\beta|}|1\rangle.$$

Postulates of Quantum Mechanics

- Postulate IV: The state space of a composite physical system is the tensor product of the state spaces of the component physical systems.

Example: The joint state of two qubits $|\psi_1\rangle = \alpha_1|0\rangle + \beta_1|1\rangle$ and $|\psi_2\rangle = \alpha_2|0\rangle + \beta_2\rangle$ is

$$\begin{aligned}|\psi_1\rangle \otimes |\psi_2\rangle &= \alpha_1\alpha_2|0\rangle \otimes |0\rangle + \alpha_1\beta_2|0\rangle \otimes |1\rangle \\ &\quad + \beta_1\alpha_2|1\rangle \otimes |0\rangle + \beta_1\beta_2|1\rangle \otimes |1\rangle \\ |\psi_1\psi_2\rangle &= \alpha_1\alpha_2|00\rangle + \alpha_1\beta_2|01\rangle + \beta_1\alpha_2|10\rangle + \beta_1\beta_2|11\rangle.\end{aligned}$$

- $|\psi_1\psi_2\rangle \in \mathbb{C}^2 \otimes \mathbb{C}^2$ or $\mathbb{C}^4$.

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- A change of basis gives the joint system as,

$$|\psi_1\psi_2\rangle = \sqrt{\lambda_0}|\downarrow\downarrow\rangle + \sqrt{\lambda_1}|\uparrow\uparrow\rangle.$$

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- Where $\lambda_0 \geq \lambda_1$ and $\lambda_0 + \lambda_1 = 1$.
- Without loss of generality, any two qubit system is of the form

$$|\psi\rangle = \sqrt{\lambda_0}|00\rangle + \sqrt{\lambda_1}|11\rangle.$$

Entangled Pairs

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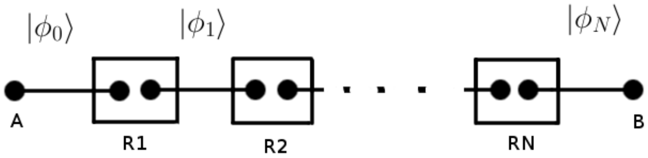
- A joint system of two qubits are referred to as an *entangled pair*.
- An important basis in two qubit systems ($\mathbb{C}^4$) is the Bell basis,

$$|\Phi^{\pm}\rangle = \frac{|00\rangle \pm |11\rangle}{\sqrt{2}}, \qquad |\Psi^{\pm}\rangle = \frac{|01\rangle \pm |10\rangle}{\sqrt{2}}.$$

1-D Quantum Networks



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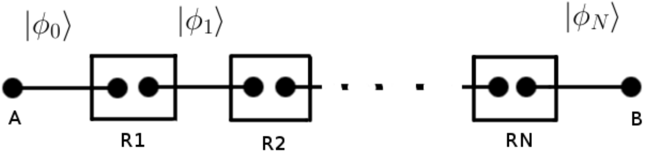

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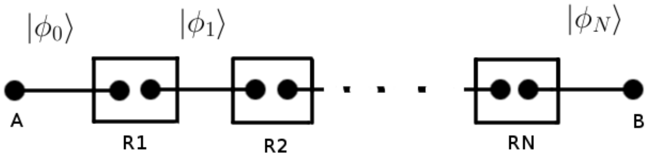

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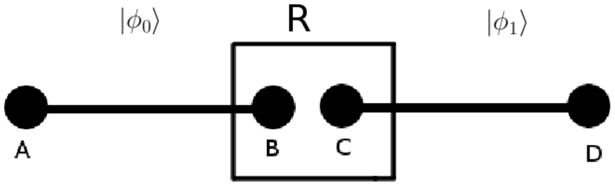
- Want to send information from A to B. Need singlets.
- Probability of converting an entangled pair to a singlet, or Singlet Conversion Probability (SCP), is $2\lambda_1$ (Vidal).
- Is there a better way?

Entanglement Swapping



- Perform the Bell measurements on qubits BC. The initial state is,

$$|\phi_{AB}\phi_{BC}\rangle = \lambda_0|0000\rangle + \sqrt{\lambda_0\lambda_1}|0011\rangle + \sqrt{\lambda_0\lambda_1}|1100\rangle + \lambda_1|1111\rangle.$$

(Acín, Cirac, Lewenstein)

Entanglement Swapping

- Four different outcomes for states BC including,

$$|\psi_{ABCD}\rangle = \frac{I_1 \otimes (|\Phi^+\rangle\langle\Phi^+|)_{23} \otimes I_4|\phi\rangle}{p(\Phi^+)^{1/2}}$$

$$= p(\Phi^+)^{-1/2}\left(\frac{\lambda_1}{\sqrt{2}}(|0\rangle \otimes |\Phi^+\rangle \otimes |0\rangle) + \frac{\lambda_2}{\sqrt{2}}(|1\rangle \otimes |\Phi^+\rangle \otimes |1\rangle)\right).$$

- Where $p(\Phi^+) = (\lambda_0^2 + \lambda_1^2)/2$ is the probability of getting the result $|\Phi^+\rangle$ on qubits BC.

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- Where $p(\Phi^+) = (\lambda_0^2 + \lambda_1^2)/2$ is the probability of getting the result $|\Phi^+\rangle$ on qubits BC.
- The advantage: Using the density language, and the partial trace,

$$\rho^{AD} = \mathrm{tr}_{BC}(\rho^{ABCD}) = |\psi_{AD}\rangle\langle\psi_{AD}|,$$

$$|\psi_{AD}\rangle = \frac{\lambda_0}{\sqrt{\lambda_0^2 + \lambda_1^2}}|00\rangle + \frac{\lambda_1}{\sqrt{\lambda_0^2 + \lambda_1^2}}|11\rangle.$$

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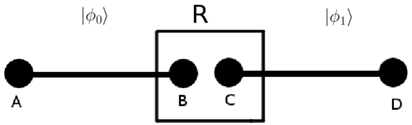
$$\sum_{k=1}^{4} p_k \lambda_1^k = 2\lambda_1.$$

- No loss in mean SCP!
- Bad News: Same trick doesn't work with 2 repeaters.
- Mean SCP exponentially decays for Bell measurements (Perseguers et al.).

One Repeater



- Ensemble of states,

$$|\psi^k_{AB}\rangle = \sqrt{\lambda^k_0}|00\rangle + \sqrt{\lambda^k_1}|11\rangle, \qquad \text{with probability } p_k,$$
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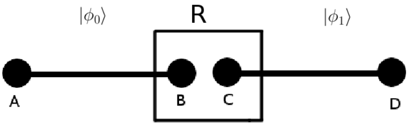

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- Perform a measurement, $\{|u_m\rangle\langle u_m|\}^4_{m=1}$, on BC where

$$|u_m\rangle = \sum_{i,j=0}^{1} a_{i,j,m}|ij\rangle, \quad \sum_{i,j=0}^{1} |a_{i,j,m}|^2 = 1,$$

- and $\langle u_n|u_m\rangle = \delta_{nm}$.

One Repeater

- The resulting state is

$$|\psi_{ABCD}\rangle = \frac{I_1 \otimes (|u_m\rangle\langle u_m|)_{23} \otimes I_4 |\psi^k_{AB}\psi^\ell_{CD}\rangle}{\sqrt{p(u_m)}}$$

$$= \left(\frac{1}{\sqrt{p(u_m)}}\right) (a_{0,0,m}\sqrt{\lambda^k_0\mu^\ell_0}|0\rangle \otimes |u_m\rangle \otimes |0\rangle$$

$$+ a_{0,1,m}\sqrt{\lambda^k_0\mu^\ell_1}|0\rangle \otimes |u_m\rangle \otimes |1\rangle + a_{1,0,m}\sqrt{\lambda^k_1\mu^\ell_0}|1\rangle \otimes |u_m\rangle \otimes |0\rangle$$

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$$+ a_{1,1,m}\sqrt{\lambda^k_1\mu^\ell_1}|1\rangle \otimes |u_m\rangle \otimes |1\rangle).$$

$$|\psi_{AD}\rangle = \left(\frac{1}{\sqrt{p(u_m)}}\right)\Big(a_{0,0,m}\sqrt{\lambda_0\mu_0}|00\rangle + a_{0,1,m}\sqrt{\lambda_0\mu_1}|01\rangle$$

$$+a_{1,0,m}\sqrt{\lambda_0\mu_1}|10\rangle + a_{1,1,m}\sqrt{\lambda_1\mu_1}|11\rangle\Big).$$

One Repeater

- Using the Schmidt Decomposition Theorem,

$$|\psi_{AD}\rangle = \frac{s_0(A(\lambda^k, \mu^\ell, m))}{\sqrt{p(u_m)}}|\downarrow\downarrow\rangle + \frac{s_1(A(\lambda^k, \mu^\ell, m))}{\sqrt{p(u_m)}}|\uparrow\uparrow\rangle.$$

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- Now we compute the mean SCP between AD,

$$2\sum_{k,\ell} p_k q_\ell \sum_{m=1}^{4} p(u_m)\left(\frac{s_1(A(\lambda^k, \mu^\ell, m))}{\sqrt{p(u_m)}}\right)^2.$$

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- The smallest singular value of a matrix $A \in \mathbb{C}^{2\times 2}$ has the property,

$$s_1(A)^2 = \min_{\|x\|_2=1, x\neq 0} \|Ax\|_2^2.$$

Mean SCP

$$
\begin{aligned}
s_1(A(\lambda^k, \mu^\ell, m))^2 &\leq \|A(\lambda^k, \mu^\ell, m)(0,1)^T\|_2^2 \\
&= \left\| \begin{bmatrix} a_{0,1,m}\sqrt{\lambda_0^k \mu_1^\ell} \\ a_{1,1,m}\sqrt{\lambda_1^k \mu_1^\ell} \end{bmatrix} \right\|_2^2 = |a_{0,1,m}|^2 \lambda_0^k \mu_1^\ell + |a_{1,1,m}|^2 \lambda_1^k \mu_1^\ell.
\end{aligned}
$$

- Note: u_m are an orthonormal basis in $\mathbb{C}^4$.

$$
|\mathbf{a_m}\rangle = \begin{bmatrix} a_{0,0,m} \\ a_{0,1,m} \\ a_{1,0,m} \\ a_{1,1,m} \end{bmatrix}.
$$

- Then the matrix $\mathbf{A} = [\mathbf{a_1}|\mathbf{a_2}|\mathbf{a_3}|\mathbf{a_4}]$ is orthogonal $AA^\dagger = A^\dagger A = I$.

Mean SCP

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$$\begin{aligned}\overline{\mathrm{SCP}}_{AD} &\le 2\sum_{k,\ell} p_k q_\ell \sum_{m=1}^{4} |a_{0,1,m}|^2 \lambda_0^k \mu_1^\ell + |a_{1,1,m}|^2 \lambda_1^k \mu_1^\ell \\ &= 2\sum_{k,\ell} p_k q_\ell \mu_1^\ell (\lambda_0^k + \lambda_1^k) = 2\sum_{k,\ell} p_k q_\ell \mu_1^\ell .\end{aligned}$$

Mean SCP

- Also, the singular values of $A(\lambda_1^k, \mu_1^\ell, m)$ are the singular values of $A(\lambda_1^k, \mu_1^\ell, m)^\dagger$. So

$$s_1(A^\dagger(\lambda^k, \mu^\ell, m))^2 \leq \|A(\lambda^k, \mu^\ell, m)^\dagger(0,1)^T\|_2^2 = \left\| \begin{bmatrix} \overline{a_{1,0,m}}\sqrt{\lambda_1^k \mu_0^\ell} \\ \overline{a_{1,1,m}}\sqrt{\lambda_1^k \mu_1^\ell} \end{bmatrix} \right\|_2^2$$

$$= |a_{1,0,m}|^2 \lambda_1^k \mu_0^\ell + |a_{1,1,m}|^2 \lambda_1^k \mu_1^\ell.$$

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$$= |a_{1,0,m}|^2 \lambda_1^k \mu_0^\ell + |a_{1,1,m}|^2 \lambda_1^k \mu_1^\ell.$$

- So the mean SCP is also bounded by,

$$\overline{\text{SCP}}_{AD} \le 2 \sum_{k,\ell} p_k q_\ell \lambda_1^k \sum_{m=1}^{4} \mu_0^\ell |a_{1,0,m}|^2 + \mu_1^\ell |a_{1,1,m}|^2$$
$$= 2 \sum_{k,\ell} p_k q_\ell \lambda_1^k (\mu_0^\ell + \mu_1^\ell) = 2 \sum_{k,\ell} p_k q_\ell \lambda_1^k.$$

Mean SCP

So the mean SCP is,

$$\overline{\mathrm{SCP}}_{AD} \leq 2 \sum_{k,\ell} p_k q_\ell \min\{\lambda_1^k, \mu_1^\ell\}.$$

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- The splitting of states that occurs results in a decay of Schmidt coefficients.
- Therefore, the mean SCP decays to zero as the number of repeaters grows.
- But what is the rate of the decay?

Future Work

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- Look at geometrical structure in 2-dimensional quantum networks.

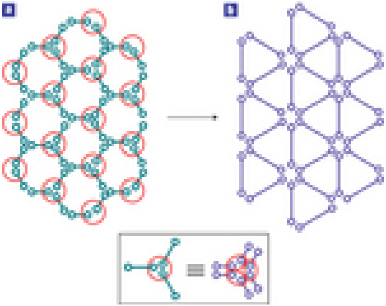

Figure: Figure 3, Nature Physics, VOL 3, 25 Feb. 2007

Questions?