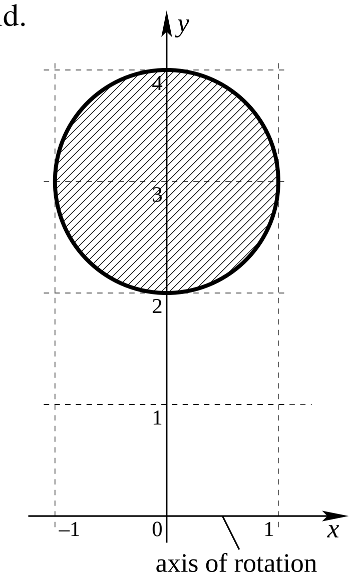
Math 129 — Quiz 2 Print Name ____________________

Sec 019

Fall 2009

The toroid (object whose surface is a torus) is bounded by a circle of radius 1 with center at the point with coordinates $(0,3)$ rotated around x-axis. Find the volume of the toroid.



Let us slice the object by planes perpendicular to the x-axis. Then slices would be rings of thickness Δx, of external radius R_{big} and of internal radius R_{small}.

The external radius of the ring is $R_{\text{big}} = 3 + \sqrt{1-x^2}$, and the internal radius is $R_{\text{small}} = 3 - \sqrt{1-x^2}$. The volume of the ring is $\pi\left(R_{\text{big}}^2 - R_{\text{small}}^2\right)\Delta x$. The total volume is the sum of volumes of all individual rings, so it is equal to

$$\text{volume} = \int_{-1}^{1} \mathrm{d}x\, \pi\left(R_{\text{big}}^2 - R_{\text{small}}^2\right) =$$

$$= \int_{-1}^{1} \mathrm{d}x\, \pi\left(\left(3+\sqrt{1-x^2}\right)^2 - \left(3-\sqrt{1-x^2}\right)^2\right) =$$

$$= \pi \int_{-1}^{1} \mathrm{d}x\, \left(3^2 + 6\sqrt{1-x^2} + 1 - x^2 - 3^2 + 6\sqrt{1-x^2} - (1-x^2)\right) =$$

$$= 12\pi \int_{-1}^{1} \mathrm{d}x\, \sqrt{1-x^2} = 12\pi \cdot \frac{\pi}{2} = 6\pi^2.$$

The integral $\int_{-1}^{1} \mathrm{d}x\, \sqrt{1-x^2} = \pi/2$ could be calculated using the Table of Integrals, or just by noticing that it is the area of a half-disk.