

MATH 583A

Principles and Methods of Applied Mathematics

Section 001, Fall 2018 (instructor: Misha Stepanov)

due September 6, 2018

Homework 1

1. Find all z such that $\sin z = i$. Sketch them on the complex plane.
2. Find all z such that $\tan z = i$. Sketch them on the complex plane.
3. Find all z such that $\exp(z^3) = 1$. Sketch them on the complex plane.
4. Consider a branch of $z^{1/3}$ with the cut being a ray from the origin passing through $-i$, with $(-1)^{1/3} = -1$. Find $((1 - i\sqrt{3})/2)^{1/3}$.
5. Consider linear fractional transformations as symmetries of the Riemann sphere. Show that modulo them $\arctan(z)$ has the same structure as $\log(z)$.
6. Find and sketch the range of the branch of $\arctan(z)$ with the cut going over the semi-circle $z = e^{i\theta}$, $-\pi/2 \leq \theta \leq \pi/2$, with $\arctan(0) := 0$.