MATH 583A

due October 9, 2018

Principles and Methods of Applied Mathematics

Section 001, Fall 2018 (instructor: Misha Stepanov)

Homework 3

1. Find the inverse Fourier transform of

$$\hat{f}(k) = \begin{cases} 1 + \cos(\pi k), & -1 < k < 1; \\ 0, & k < -1 \text{ or } k > 1. \end{cases}$$

At which values of x the function $f(x) = \mathscr{F}^{-1}\big[\hat{f}(k)\big](x)$ is singular?

2. Find the Fourier transform of

$$f_\beta(x) = \frac{(\beta/\mathrm{e})^{\beta^2}}{\Gamma(\beta^2)} \begin{cases} (x+\beta)^{\beta^2-1}\mathrm{e}^{-\beta x}, & x > -\beta; \\ 0, & x < -\beta. \end{cases}$$

Assume $\beta > 0$. Find the limit $\hat{f}(k) = \lim\limits_{\beta\to\infty} \hat{f}_\beta(k)$.

3. Let $f(x) = \exp(-x^2/2)/\sqrt{2\pi}$. Find and sketch the Fourier transforms of $x^2\, f(x)$ and $x^3\, f(x)$.

4. Let $h(x) = \exp\big(-\sqrt{|x|}\big)$. For what values of α the limit $\lim\limits_{k\to\infty} |k|^\alpha \hat{h}(k)$ is finite and non-zero?

5. The Fourier transform of $\exp\big(-\sqrt{x^2+a^2}\big)$ (here $a > 0$) decays exponentially with k, similar to $\exp(-\gamma|k|)$ for large k. What is the value of γ?