

## Homework 4

1. Find how the distribution  $x^n \delta''(x)$ , with  $n = 0, 1, 2, 3$ , acts on a test function.

2. Consider  $F(x) = \delta(x) + \delta''(x)$ . How the distribution  $F$  acts on test functions? Find its Fourier transform. What is  $\hat{F}(\pm 1)$ ? Of what operator  $\hat{L}$  the function  $F(x - y)$  is the integral kernel? Solve the equation  $\hat{L}\varphi = 0$ .

3. Consider the function (actually a distribution)

$$g(x) = \frac{1}{N} \sum_{n=1}^N \delta\left(x + 1 - \frac{2n-1}{N}\right).$$

Draw the “graph” of  $g(x)$ . Find its Fourier transform  $\hat{g}(k)$ . Compare it with the one of  $H(1 - x^2)/2 = H(1 - x)H(1 + x)/2$ .

4. Find the Fourier transform of  $\sum_{n=-\infty}^{\infty} \delta(x - n)$ . Explain the similarity of it at small  $k$  with the Fourier transform of 1.

5. Find the Fourier transform of the floor function  $[x]$ . (Pay attention to the vicinity of  $k = 0$ .)