## MATH 583A

Principles and Methods of Applied Mathematics
Section 001, Fall 2018 (instructor: Misha Stepanov)

## Homework 5

1. Consider a periodic with period $2 \pi$ function $f(x)=\sum_{n=-\infty}^{\infty} f_{n} \mathrm{e}^{\mathrm{i} n x}$. Consider also the "rectangular window" function $g(x)=H\left(\pi^{2}-x^{2}\right) / 2 \pi$. Calculate the convolution $f * g$ in both $x$-space (i.e., without doing the Fourier transform) and in $k$-space (i.e., what multiplication by $\hat{g}(k)$ does?).
2. Let $b(x)=H(x) H(1-x)$. Calculate $(b * b * b)(x)$.
3. For any $a>0$ let $f_{a}(x)=H(x) \cdot x^{a-1} / \Gamma(a)$. Calculate $\left(f_{a} * f_{b}\right)(x)$.
4. Let $f(x)=-x \exp \left(-x^{2} / 2\right) / \sqrt{2 \pi}$. Calculate $\delta^{\prime} * \delta^{\prime}$ and $f * f$ and sketch them (together with $\delta^{\prime}$ and $f$ ).
5. Solve the equation $f+\varepsilon f * f+\varepsilon^{2} f * f * f+\varepsilon^{3} f * f * f * f+\ldots=\mathrm{e}^{-|x|}$.
