

Homework 5

1. Consider a periodic with period 2π function $f(x) = \sum_{n=-\infty}^{\infty} f_n e^{inx}$. Consider also the “rectangular window” function $g(x) = H(\pi^2 - x^2)/2\pi$. Calculate the convolution $f * g$ in both x -space (*i.e.*, without doing the Fourier transform) and in k -space (*i.e.*, what multiplication by $\hat{g}(k)$ does?).
2. Let $b(x) = H(x)H(1 - x)$. Calculate $(b * b * b)(x)$.
3. For any $a > 0$ let $f_a(x) = H(x) \cdot x^{a-1}/\Gamma(a)$. Calculate $(f_a * f_b)(x)$.
4. Let $f(x) = -x \exp(-x^2/2)/\sqrt{2\pi}$. Calculate $\delta' * \delta'$ and $f * f$ and sketch them (together with δ' and f).
5. Solve the equation $f + \varepsilon f * f + \varepsilon^2 f * f * f + \varepsilon^3 f * f * f * f + \dots = e^{-|x|}$.