due November 20, 2018

## MATH 583A

Principles and Methods of Applied Mathematics

Section 001, Fall 2018 (instructor: Misha Stepanov)

## Homework 5

**1.** Consider a periodic with period  $2\pi$  function  $f(x) = \sum_{n=-\infty}^{\infty} f_n e^{inx}$ . Consider also the "rectangular window" function  $g(x) = H(\pi^2 - x^2)/2\pi$ . Calculate the convolution f \* g in both *x*-space (*i.e.*, without doing the Fourier transform) and in *k*-space (*i.e.*, what multiplication by  $\hat{g}(k)$  does?).

**2.** Let b(x) = H(x)H(1 - x). Calculate (b \* b \* b)(x).

**3.** For any a > 0 let  $f_a(x) = H(x) \cdot x^{a-1} / \Gamma(a)$ . Calculate  $(f_a * f_b)(x)$ .

**4.** Let  $f(x) = -x \exp(-x^2/2)/\sqrt{2\pi}$ . Calculate  $\delta' * \delta'$  and f \* f and sketch them (together with  $\delta'$  and f).

5. Solve the equation  $f + \varepsilon f * f + \varepsilon^2 f * f * f + \varepsilon^3 f * f * f * f + \dots = e^{-|x|}$ .