(a) "x-space"

$$(f*g)(x) = \int_{-\infty}^{\infty} dy\, f(y) \cdot \underbrace{\left(\begin{cases} 1/2\pi, & |x-y|<\pi \\ 0, & \text{otherwise} \end{cases} \right)}_{\substack{\parallel \\ g(x-y)}} =$$

$$= \frac{1}{2\pi} \underbrace{\int_{x-\pi}^{x+\pi}}_{\text{integration over the whole period}} dy\, f(y) = \frac{1}{2\pi}\int_0^{2\pi} dy\, f(y) = \boxed{\langle f(x)\rangle = f_0}$$

can shift due to periodicity

i.e., average value of $f(x)$ over the period

The answer doesn't depend on x, it is constant.

(b) "k-space"

$$\widehat{(f*g)}(k) = \hat{f}(k)\cdot\hat{g}(k) = \hat{f}(k)\cdot \underbrace{\frac{\sin(\pi k)}{\pi k}}_{\hat{g}(k)}$$

$f(x)$ is periodic with period 2π,

thus
$$\left.\begin{aligned} \hat{f}(k) &= 2\pi \sum_{m=-\infty}^{\infty} f_m\, \delta(k-m) \\ f(x) &= \sum_{m=-\infty}^{\infty} f_m e^{imx} \end{aligned}\right] \text{Fourier series}$$

$$\widehat{(f*g)}(k) = 2\pi \sum_{m=-\infty}^{\infty} f_m\, \delta(k-m) \underbrace{\frac{\sin(\pi k)}{\pi k}}_{\substack{\parallel \\ 0 \text{ at all } k=m \text{ except } m=0}} = 2\pi f_0 \delta(k)$$

$$(f*g)(x) = \mathcal{F}^{-1}[2\pi f_0 \delta(k)] = \boxed{f_0}$$