$$\widehat{A} = \begin{bmatrix} 34 & 21 \\ 21 & 13 \end{bmatrix}, \quad \widehat{Q} = \frac{1}{\sqrt{1597}} \begin{bmatrix} 34 & -21 \\ 21 & 34 \end{bmatrix}, \quad \widehat{R} = \frac{1}{\sqrt{1597}} \begin{bmatrix} 1597 & 987 \\ 0 & 1 \end{bmatrix}$$

Modified Gram-Schmidt

$$\mathbf{v}_1 = \begin{bmatrix} 34 \\ 21 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} 21 \\ 13 \end{bmatrix}$$

$$r_{11} = \|\mathbf{v}_1\| = \sqrt{34^2 + 21^2} = \sqrt{1597}, \quad \mathbf{q}_1 = \mathbf{v}_1 / r_{11} = \frac{1}{\sqrt{1597}} \begin{bmatrix} 34 \\ 21 \end{bmatrix}$$

$$r_{12} = \mathbf{q}_1^\dagger \mathbf{v}_2 = \frac{34 \cdot 21 + 21 \cdot 13}{\sqrt{1597}} = \frac{987}{\sqrt{1597}}$$

$$\mathbf{v}_2 = \mathbf{v}_2 - r_{12}\mathbf{q}_1 = \begin{bmatrix} 21 \\ 13 \end{bmatrix} - \frac{987}{1597} \begin{bmatrix} 34 \\ 21 \end{bmatrix} = \frac{1}{1597} \begin{bmatrix} -21 \\ 34 \end{bmatrix}$$

$$r_{22} = \|\mathbf{v}_2\| = \frac{1}{\sqrt{1597}}, \quad \mathbf{q}_2 = \mathbf{v}_2 / r_{22} = \frac{1}{\sqrt{1597}} \begin{bmatrix} -21 \\ 34 \end{bmatrix}$$

Householder triangularization

$$\mathbf{x} = \begin{bmatrix} 34 \\ 21 \end{bmatrix}, \quad \mathbf{v}_1 = \operatorname{sign}(x_1)\|\mathbf{x}\|\mathbf{e}_1 + \mathbf{x} = \sqrt{1597} \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 34 \\ 21 \end{bmatrix}$$

$$\mathbf{v}_1 = \frac{\mathbf{v}_1}{\|\mathbf{v}_1\|} = \begin{bmatrix} \sqrt{\frac{\sqrt{1597}+34}{2 \cdot 1597}} \\ \frac{21}{\sqrt{2\sqrt{1597}(\sqrt{1597}+34)}} \end{bmatrix} = \begin{bmatrix} \sqrt{\frac{1}{2} + \frac{17}{\sqrt{1597}}} \\ \sqrt{\frac{1}{2} - \frac{17}{\sqrt{1597}}} \end{bmatrix}$$

$$\widehat{A} = \widehat{A} - 2\mathbf{v}_1(\mathbf{v}_1^\dagger \widehat{A}) = \begin{bmatrix} 34 & 21 \\ 21 & 13 \end{bmatrix} - \begin{bmatrix} 34 + \sqrt{1597} & 21 + \frac{987}{\sqrt{1597}} \\ 21 & 13 - \frac{1}{\sqrt{1597}} \end{bmatrix} = -\frac{1}{\sqrt{1597}} \begin{bmatrix} 1597 & 987 \\ 0 & -1 \end{bmatrix}$$

$$\mathbf{x} = \begin{bmatrix} 1/\sqrt{1597} \end{bmatrix}, \quad \mathbf{v}_2 = \operatorname{sign}(x_1)\|\mathbf{x}\|\mathbf{e}_1 + \mathbf{x} = \begin{bmatrix} 2/\sqrt{1597} \end{bmatrix}$$

$$\mathbf{v}_2 = \frac{\mathbf{v}_2}{\|\mathbf{v}_2\|} = \begin{bmatrix} 1 \end{bmatrix}, \quad \text{corner of } \widehat{A} = \text{corner of } \widehat{A} - 2\mathbf{v}_2(\mathbf{v}_2^\dagger \text{ corner of } \widehat{A}) = -\frac{1}{\sqrt{1597}}$$