1. (14 points) Consider the area enclosed by the y-axis, the x-axis, the vertical line \( x = 1 \) and the curve \( y = e^{-x} \). It is rotated about the y-axis. Find the volume of the resulting solid. For full credit you must do the integral analytically. However, a numerical answer is better than nothing.

2. (14 points) A water truck weighs 10,000 lbs when it is full of water. The truck starts up a mountain road full of water. The truck travels at a constant speed and the road has a constant incline. At the start of the trip the truck springs a leak. Water leaks out at a constant rate and at the top the truck only weighs 6,000 lbs. The top of the road is 5,000 feet higher than the bottom.

(a) Find a formula for the weight of the truck as a function of its elevation \( h \) assuming that \( h = 0 \) where the truck starts its trip.

(b) Find the total work done by the truck.

3. (8 points) Find the arc length of the graph of \( y = 2x^{3/2} \) for \( 0 \leq x \leq 3 \).

4. (14 points) A dam is 200 feet across the top and 100 feet tall at its midpoint. Its shape is approximately given by the parabola \( y = x^2/100 \) with \(-100 \leq x \leq 100\). The water behind the dam goes up to the very top of the dam. Find the total force on the dam.

(Recall that at a depth of \( h \) feet below the surface the water pressure is \( 62.4h \) lbs per ft\(^2\))

5. (14 points) A cylindrical barrel is 5 ft tall has a radius of 1.5 ft. It it filled to a depth of 4 ft with a mysterious liquid whose density depends on the depth in the liquid. The density \( d \) depends on the distance \( h \) below the surface according to \( d(h) = 40(1 + h/10) \). (The density is in lbs/ft\(^3\) and the distance \( h \) is in feet.) Find the total work needed to pump the liquid to the top rim of the barrel.

6. (12 points) Find the sums of the following series

\[
1 + \frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} \cdots \frac{1}{2^{20}}
\]

\[-3 + 1 - \frac{1}{3} + \frac{1}{9} - \frac{1}{27} + \frac{1}{81} \cdots \]

\[
\sum_{n=0}^{\infty} x^{2n}
\]
7. (14 points) A tetrahedron has vertices at \((0,0,0), (2,0,0), (0,1,0)\) and \((0,0,1)\). Find its volume. (A tetrahedron has four faces, each of which is a triangle. Moreover, any slice through a tetrahedron is a triangle.)

8. (10 points) Determine whether the following improper integral converges and explain your reasoning.

\[
\int_0^{\infty} e^{-x} (1 + \cos x) \, dx
\]

There are lots of ways to do this. Grading will be based on how well you explain your reasoning.