Math 250a (Fall ‘07) - Homework 10 extra problems

1. This problem is about the definite integral

\[ \int_0^1 \sin^2(\pi x) \, dx \]

(a) Show the exact value of this integral is \( \frac{1}{2} \).
(b) Use the program to compute the trapezoid approximation for \( N = 10, 20 \) and 40. Find the error in each of the three cases.
(c) Use extrapolation with \( N = 10, 20 \) to find a better approximation and compute its error. Now use extrapolation with \( N = 20, 40 \) and find its error. What do you think is the order of the extrapolation method using the trapezoid rule?

2. The left, right and trapezoid rules for approximating a definite integral all have the form

\[ Af(x_0) + \sum_{i=1}^{n-1} f(x_i) + B f(x_n) \]

where the constants \( A \) and \( B \) depend on the method. For the left rule, \( A = 1, B = 0 \). For the right rule, \( A = 0, B = 1 \). What are \( A \) and \( B \) for the trapezoid rule?