

Example

10/11
①

$$\frac{dy}{dx} = y(1-y^2) = y - y^3$$

Equilibria: $y = 0, 1, -1$

$$g(y) = y(1-y^2)$$

$$g'(y) = (1-y^2) + y(-2y)$$

$$= 1 - y^2 - 2y^2$$

$$= 1 - 3y^2$$

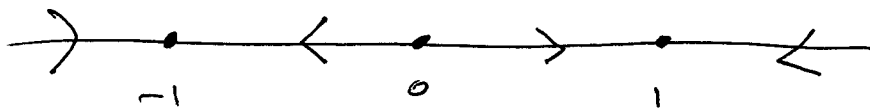
$$g'(0) = 1$$

$$g'(1) = 1 - 3 = -2$$

$$g'(-1) = -2$$

0 unstable

1, -1 stable



$$\frac{d^2 y}{dx^2} = ?$$

$$\underbrace{y = y(x)}_{(10/1) \text{ (2)}}$$

$$\frac{d^2 y}{dx^2} = 1 \cdot \frac{dy}{dx} - 3y^2 \frac{dy}{dx}$$

$$= (1 - 3y^2) \frac{dy}{dx}$$

$$= (1 - 3y^2) y (1 - y^2)$$

§ 2.6 Bifurcation Diagrams

Example

$$y' = y(a - y)$$

a is parameter

Eq. solutions : $y = 0, a$

$$g(y) = ay - y^2$$

$$g'(y) = a - 2y$$

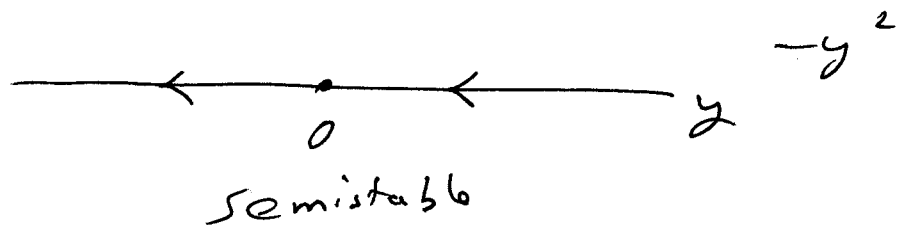
$$g'(0) = a$$

$$g'(a) = a - 2a = -a$$

$a > 0$ 0 unstable
 a stable


$a < 0$ 0 stable
 a unstable

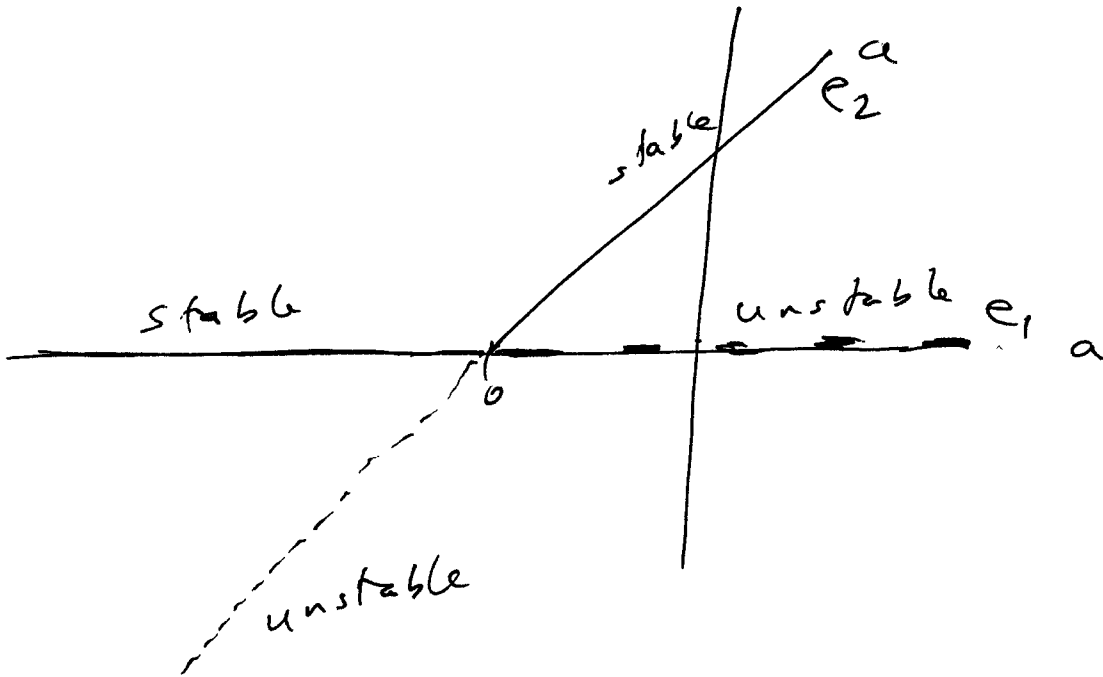
$a = 0$



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③

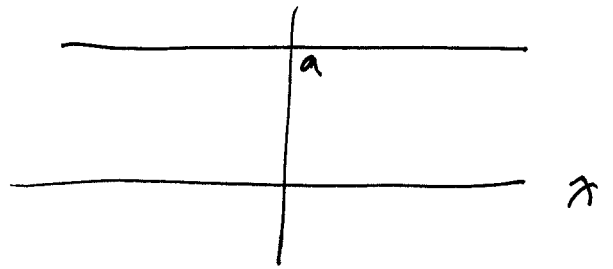
Bifurcation diagram

— stable 
 - - - unstable



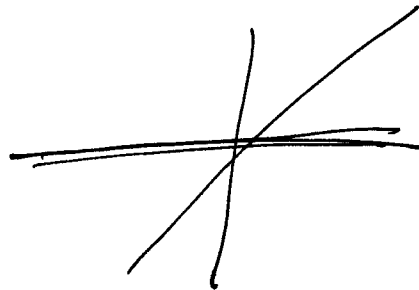
$$\left. \begin{aligned} e_1(a) &= 0 \\ e_2(a) &= a \end{aligned} \right\} \text{equilibria}$$

$$y(x) = a$$



$$r_1(x) = 0$$

$$r_2(x) = x$$



$$\frac{dx}{dt} = x(e^{x^2} - a)$$

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⑤

Equilibria: $x = 0$ always

$$e^{x^2} - a = 0$$

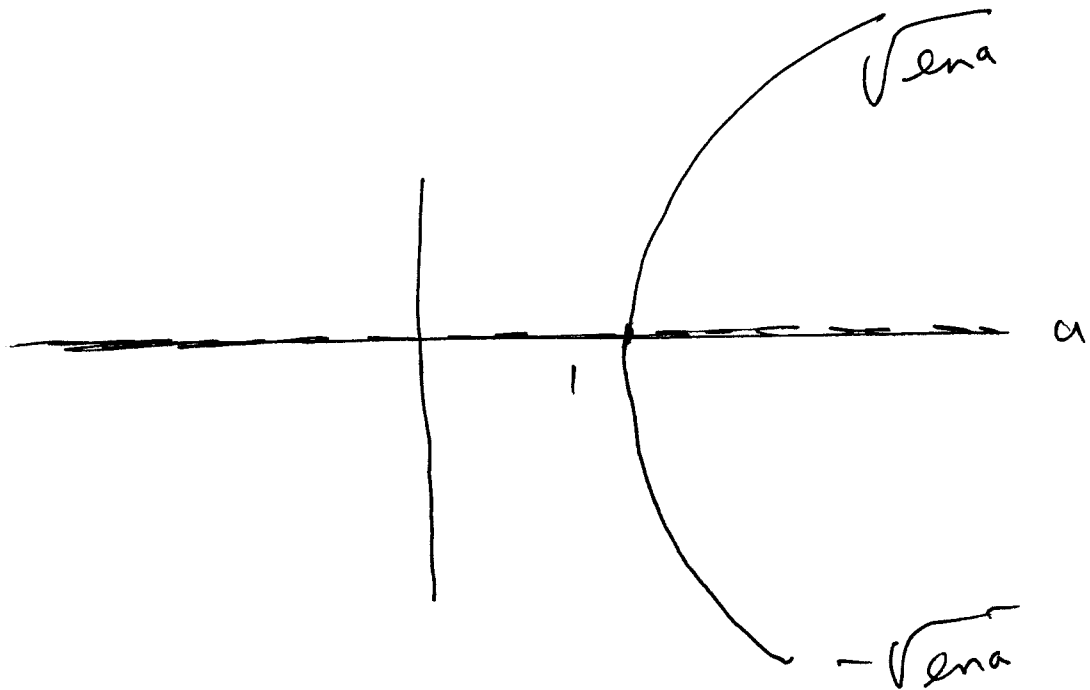
$$e^{x^2} = a$$

$$x^2 = \ln a \quad \text{if } a > 0$$

$$x = \pm \sqrt{\ln a} \quad \text{if } a > 1$$

$a < 1$ 1 eq. at 0

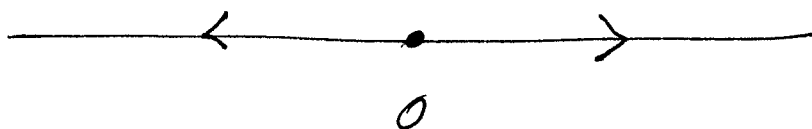
$a > 1$ 3 eq. : 0, $\pm \sqrt{\ln a}$



$$a < 1$$

$$\frac{dy}{dx} = x(e^{x^2} - a)$$

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⑥



unstable

$$a > 1$$

