\[
\text{Error} = \left| \int_a^b f(x) \, dx - \text{approximation} \right|
\]

Usually, Error \to 0 as \( N \to \infty \).

If Error goes as \( \frac{c}{N^p} \)

we say the method has order \( p \).

\underline{Example}

\[
\int_1^2 (x^2 - 2x^3) \ln x \, dx
\]

\[= \frac{9}{8} - 6 \ln 2\]

\underline{LEFT} \quad p = 1 \quad \underline{RIGHT} \quad p = 1

\[N = 10, 20, 40, 80\]

\[\frac{c}{N} = \frac{c}{10}, \frac{c}{20}, \frac{c}{40}, \frac{c}{80}\]
If $p = 2$

$N = 10, 20, 40$

$err = \frac{c}{(10)^2}, \frac{c}{(20)^2}, \frac{c}{(40)^2}$

Error goes down by factor of 4 when we double $N$.

Midpoint: $p = 2$

[Diagram showing a decreasing trend in error with increasing $N$, with $c/N$ on the y-axis and $N$ on the x-axis.]
\[ \text{Suppose} \quad \rho = 4 \]

Bounds

\[ \frac{1}{N^4} \leq 1 \]

- \( f \) decreases: LEFT is over estimate, RIGHT is under
- \( f \) increases: LEFT is under, RIGHT is over
- \( f \) concave up: TRAP is over, concave down: TRAP is under
Midpoint

If concave up

Red area = midpoint

If concave up, mid is under
concave down

oven