

Extrapolation

$$\text{error} = \int_a^b f(x) dx - \text{approximation}$$

$$\text{error} \approx \frac{C}{N^p}$$

know p , C unknown

Look at $N = 10, 20$

approximation = MID

$$\int_a^b f(x) dx = \text{MID}(10) + \frac{C}{10^2}$$

$$\int_a^b f(x) dx = \text{MID}(20) + \frac{C}{(20)^2}$$

Two eqs, two unknowns C, \int_a^b
~~top~~ 4x (bottom eq.) - top eq.:

$$4 \int_a^b f(x) dx - \int_a^b f(x) dx =$$

$$4 \text{MID}(20) - \text{MID}(10)$$

$$+ 4 \frac{C}{400} - \frac{C}{100}$$

$$3 \int_a^b f(x) dx = 4 \text{ MID}(20) - \text{MID}(10)$$

$$\int_a^b f(x) dx = \frac{4 \text{ MID}(20) - \text{MID}(10)}{3}$$

Test case

$$\int_1^2 (x - 2x^3) \ln x dx = \frac{9}{8} - 6 \ln 2$$

$$\int_1^2 [(x - 2x^3) \ln x - \frac{9}{8} + 6 \ln 2] dx$$

$$\frac{4 \text{ MID}(20) - \text{MID}(10)}{3}$$

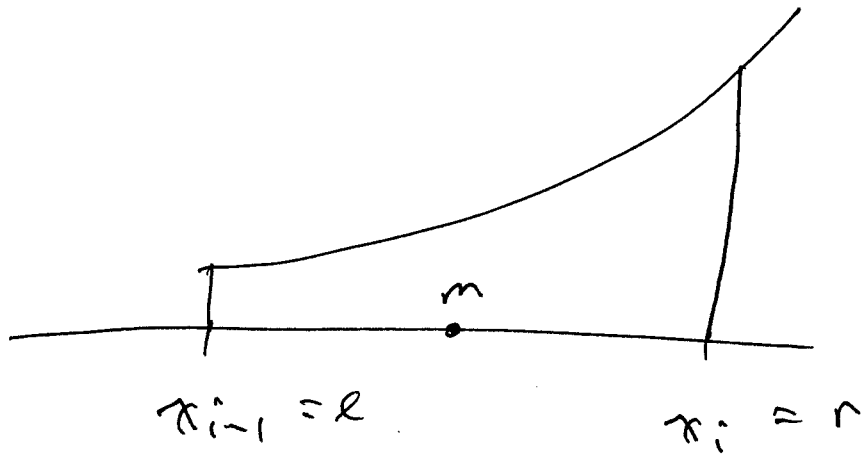
$$= \frac{6.6 \times 10^{-7}}{2.78 \times 10^{-5}}$$

error is a lot
too times smaller

$$\text{Approx} = -3.033882852$$

Simpson

Look at one interval



$$\text{TRAP} = \Delta x \frac{1}{2} (f(x_{i-1}) + f(x_i))$$

$$\text{MID} = \Delta x f\left(\frac{x_{i-1} + x_i}{2}\right)$$

Both have the form

$$\Delta x \left[A f(x_{i-1}) + B f\left(\frac{x_{i-1} + x_i}{2}\right) + C f(x_i) \right]$$

$$\text{TRAP} \quad A = \frac{1}{2}, \quad B = 0, \quad C = \frac{1}{2}$$

$$\text{MID} \quad A = 0, \quad B = 1, \quad C = 0$$

Is there a "better" choice of A, B, C ?

$$\text{Let } l = x_{i-1}, \quad m = \frac{x_{i-1} + x_i}{2}, \quad r = x_i$$

$$\rightarrow \Delta x [A f(l) + B f(m) + C f(r)] = \star$$

$$\boxed{f = 1}$$

$$\int_e^r 1 dx = r - e$$

$$\star = (r - e) [A + B + C]$$

would
like

$$\underline{A + B + C = 1}$$

$$\boxed{f = x}$$

$$\int_e^r x dx = \frac{1}{2} x^2 \Big|_e^r$$

$$= \frac{1}{2} (r^2 - e^2) = \frac{1}{2} (r - e)(r + e)$$

$$\star = (r - e) \left[A e + B \frac{e+r}{2} + C r \right]$$

So \star is exactly right if

$$A e + B \frac{e+r}{2} + C r = \frac{1}{2} (r + e)$$

$$(A + \frac{B}{2}) e + (\frac{B}{2} + C) r = \frac{1}{2} r + \frac{1}{2} e$$

would like $\underline{A + \frac{B}{2} = \frac{1}{2}}, \quad \underline{\frac{B}{2} + C = \frac{1}{2}}$

$$\underline{f = x^2} \quad \dots$$

$$A + \frac{B}{4} = \frac{1}{3}$$

$$\frac{B}{2} = \frac{1}{3}$$

$$C + \frac{B}{4} = \frac{1}{3}$$

Solution

$$A = C = \frac{1}{6}, \quad B = \frac{2}{3}$$

MID

$A = 0$

$B = 1$

$C = 0$

TRAP

$A = \frac{1}{2}$

$B = 0$

$C = \frac{1}{2}$

Solution $\approx \frac{2}{3}$ MID $\mp \frac{1}{3}$ TRAP