

### 3.3 Continued

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①

#### Order

Solve the diff eq. over  
some interval  $a \leq x \leq b$ .

error =  $y(b)$  - approximation  
 $y(a)$  ← initial condition

Typically

$$\text{error} \approx ch^p$$

Euler

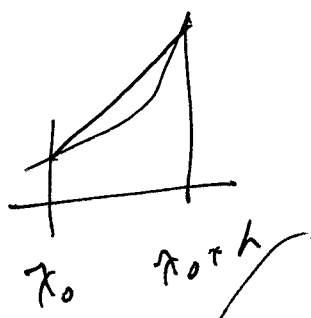
$$p = 1$$

first order

Better methods

FTC

$$y(x_0 + h) = \int_{x_0}^{x_0 + h} y'(x) dx + y(x_0)$$



$$= \int_{x_0}^{x_0 + h} g(x, y) dx + y(x_0)$$

$$\approx g(x_0, y(x_0)) h + y(x_0)$$

LEFT corresponds to Euler

What about trapezoid

$$\rightarrow \approx \frac{h}{2} [g(x_0, y(x_0)) + g(x_0 + h, y(x_0 + h))] + y(x_0) \rightarrow ?$$

Approximate  $y(x_0 + h) \approx y(x_0) + g(x_0, y(x_0)) h$

Let  $m_0 = g(x_0, y_0)$

$k_1 = g(x_1, y_0 + g(x_0, y_0) h)$

$y(x_1) = y(x_0) + \frac{1}{2} h (m_0 + k_1)$

Modified Euler, Heun's method  $p=2$

Even better method

Runge - Kutta fourth order

$p = 4$

Numerical problems

If  $g(x, y)$  is nice then solution curves ~~doesn't~~ don't cross



Computer doesn't know this.

$\frac{dy}{dx} = y^2$        $y(0) = -12$

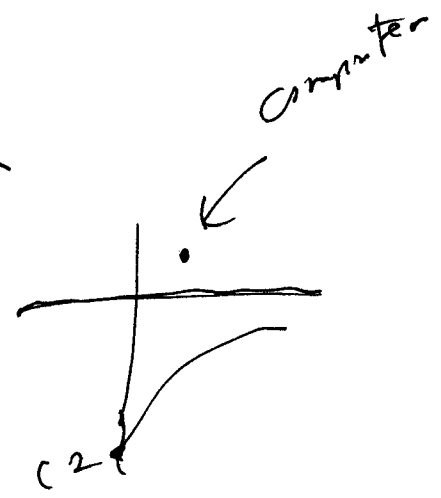
$h = 0.1$       Euler

$y = 0$  is a solution

$y(0.1) = y(0) + g(0, -12) \cdot 0.1$

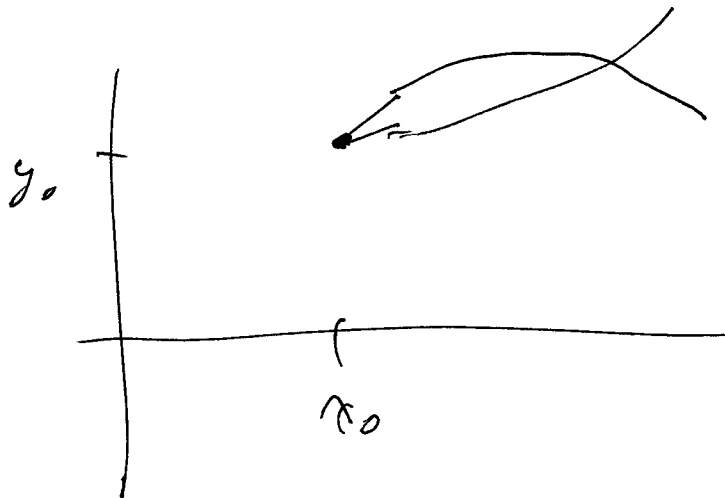
$= -12 + 144 \cdot 0.1$

$= 2.4$



### 3.4 Comparing Solutions 10/26 ⑨

Bral's statement of theorem is slightly wrong.



not allowed

Example

$$y' = y^2$$

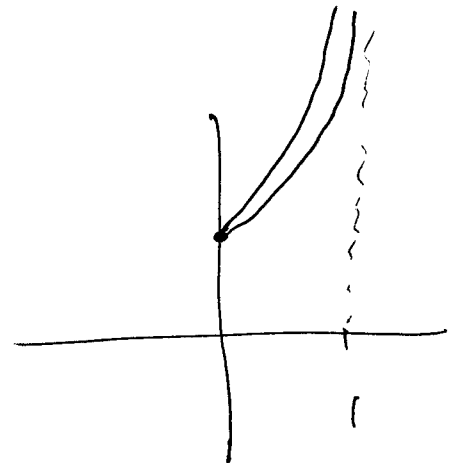
$$y(0) = 1$$

$$y = \frac{1}{1-x}$$

by sep. of vars

Look at

$$y' = y^2 + x$$



$$f(x, y) = y^2 + x$$

$$g(x, y) = y^2$$

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$$f(x, y) > g(x, y) \quad \text{for } x > 0$$

So the new say solution to

$$\frac{dy}{dx} = f(x, y)$$

with initial cond.  $y(0) = 1$

is above solution to

$$\frac{dy}{dx} = g(x, y) \quad \text{with same I.C.}$$

So solution to  $\frac{dy}{dx} = y^2 + x$   
must have a vertical asymptote  
at  $x = a$ ,  $a \leq 1$ .