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①

Chap 1

$$\frac{dy}{dx} = g(x)$$

$$y(x) = \int g(x)\,dx$$

Chap 2

$$\frac{dy}{dx} = g(y)$$

$$\int \frac{dy}{g(y)} = \int dx$$

Chap 3

$$\frac{dy}{dx} = g(x,y)$$

No analytic solutions in general

$$\int \frac{dy}{h(y)} = \int \frac{dx}{f(x)}$$

if $g(x,y) = f(x)\,h(y)$

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$$\frac{dy}{dx} = g(x,y)$$

Isocline: $g(x,y) = m$

$g(x,y) = 0$ $\leftarrow$ possible local max's, min's of solution

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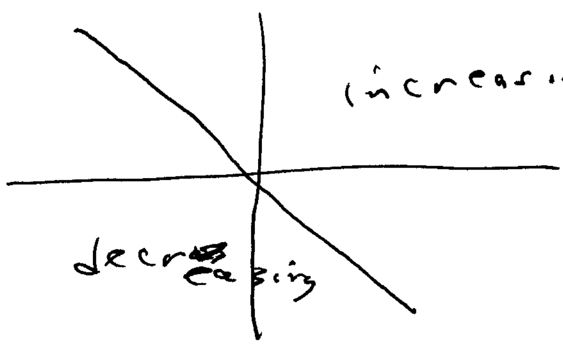
$y' = x + y$ $\qquad$ $y(x) = c$

increasing vs decreasing

increase if $x + y > 0$



$x + y = 0$

$y = -x$

$(0,1)$ satisfies

$x + y > 0$

Concavity $\qquad y'' = 1 + y' = 1 + x + y$

Example

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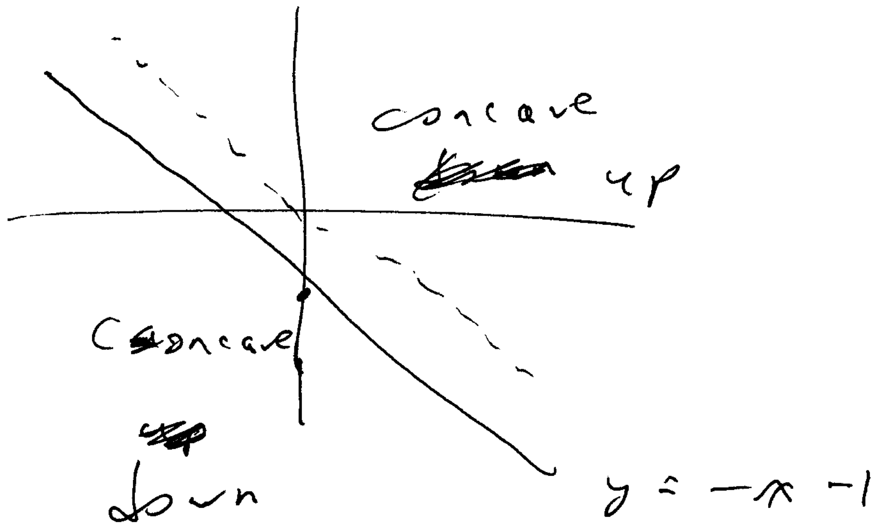
$$y'' = 1 + x + y$$

$$1 + x + y = 0 \qquad y = -x - 1$$



Looks like $\boxed{y = -x - 1}$ is a solution. Try it:

$$y' = -1$$

$$x + y = x + (-x - 1) = -1$$

So yes it satisfies

$$y' = x + y$$

$$\frac{dy}{dx} = g(x, y) \qquad y(x)$$

Example

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$$\frac{dy}{dx} = x(1-y^2)$$

Equilibrium $\quad y(x) = c$

$$0 \stackrel{?}{=} x(1-c^2)$$

$$c = \pm 1$$

So $y(x) = +1$, $y(x) = -1$

are equilibria.

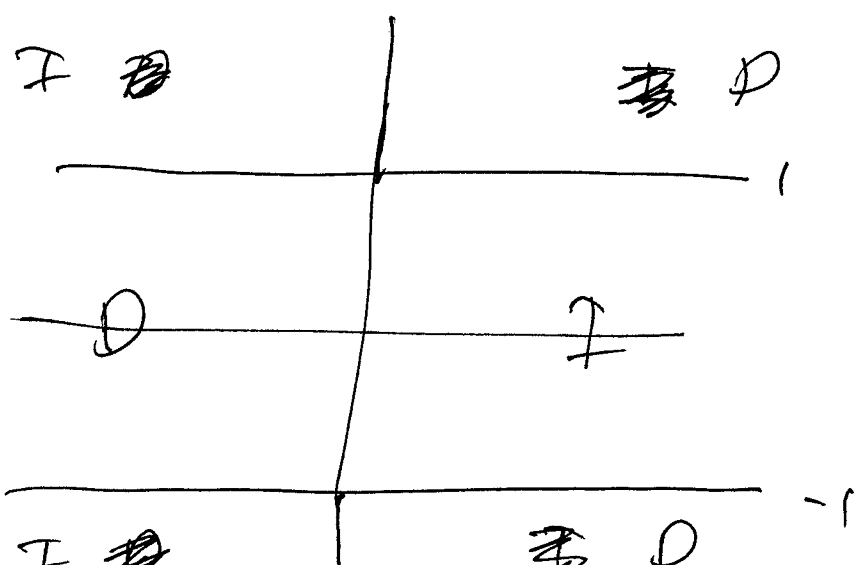
Monotonicity

$x > 0$, $-1 < y < 1$ increasing

$x < 0$, $-1 < y < 1$ decrease

$x > 0$, $|y| > 1$ decrease

$x < 0$, $|y| > 1$ increase



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$$\frac{dy}{dx} = x(1-y^2)$$

$$\frac{d^2y}{dx^2} = (1-y^2) + x \frac{d}{dx}(1-y^2)$$

$$= (1-y^2) + x(-2)y\frac{dy}{dx}$$

$$= 1-y^2 - 2xy\,x(1-y^2)$$

$$= (1-y^2)\left[1 - 2x^2y\right]$$

$$\frac{d^2y}{dx^2} = 0 \qquad y = \pm 1$$

$$1 = 2x^2y$$

$$y = \frac{1}{2x^2}$$

$y = 1$, $y = -1$, $y = \frac{1}{2x^2}$

are boundaries of regions where it is concave up, concave down