3.1 Cont

\[(x+y)^2 + (x-y)^2 = 1\]

Graph this curve.

Idea: \(\frac{dy}{dx}\) of and get a diff. eq. to study.

\[3(x+y)^2 (1 + \frac{dy}{dx}) + 3(x-y)^2 (1 - \frac{dy}{dx}) = 0\]

\[\left[(x+y)^2 - (x-y)^2\right] \frac{dy}{dx}\]

\[+ (x+y)^2 + (x-y)^2 = 0\]

\[4xy \frac{dy}{dx} + 2x^2 + 2y^2 = 0\]

\[\frac{dy}{dx} = -\left(\frac{x^2 + y^2}{2xy}\right)\]
\[ \begin{array}{c|c|c}
\text{inc} & \text{dec} \\
\hline
\text{dec} & \text{inc} \\
\end{array} \]

\[ \begin{align*}
\kappa = 0 & \quad y^3 - y^3 = 1 \\
y = 0 & \quad \kappa^3 + \kappa^3 = 1 \\
\kappa^3 & = \frac{1}{2} \\
\kappa & = \left( \frac{1}{2} \right)^{\frac{1}{3}} , \quad y = 0
\end{align*} \]

\[ y \left( \frac{1}{2}^{\frac{1}{3}} \right) \quad \geq \quad 0 \]
Suppose \( g(-x, y) = -g(x, y) \)

Let \( y(x) \) be a solution.

Define \( \overline{y}(x) = y(-x) \)

Then \( \frac{d}{dx} \overline{y}(x) = \frac{1}{dx} y(-x) \)

\[ = y'(-x)(-1) = -y'(-x) \]

But \( y'(x) = g(x, y) \)

So \( \overline{y}'(x) = -g(-x, y_{\overline{y}}(-x)) \)

\[ = g(x, y_{\overline{y}}(-x)) = g(x, \overline{y}) \]
So \( \tilde{y}(x) = y(-x) \) is another solution.

So \( \tilde{y}(0) = y(0) \)

So \( \tilde{y}, y \) have same initial condition at \( x = 0 \).

So by uniqueness then

\[ \tilde{y}(x) = y(x) \]

i.e. \( y(x) = y(x) \)

i.e. \( y(x) \) is even.

Example

\[ \frac{dy}{dx} = x \cos y = g(x, y) \]

\( g(-x, y) = -x \cos y = -g(x, y) \)

It fails even condition.

\[ \frac{dy}{dx} = \sin (xy) \]

\( \sin (-xy) = -\sin (xy) \)
Now suppose
\[ g(-x, -y) = g(x, y) \] (A)

\[ \frac{dy}{dx} = g(x, y) \]

Let \( y(x) \) be a solution.

Let \( \bar{y}(x) = -y(-x) \)

\[ \frac{d}{dx} \bar{y}(x) = -y'(-x)(-1) = y'(-x) = g(-x, y(-x)) \]

So \( \frac{dy}{dx} \) \( \bar{y}(x) = g(x, -y) \)

So \( \frac{1}{a} \frac{d}{dx} \bar{y}(x) = g(x, -y(-x)) = g(x, \bar{y}(x)) \)

\( \bar{y}(x) \) is another solution.

\( \bar{y}(0) = -y(0) \)
Suppose \( Y(x) \) is a solution of
\[ Y' = Y(1 - Y) \] \( Y'(x) = Y(x)(1 - Y(x)) \)

Let \( y(x) = c Y(dx) \), \( c, d \) constants

Let \( y(x) = a Y(bx) \), \( a, b \) constants

\[ y'(x) = a Y'(bx) b \]
\[ = ab Y(bx) (1 - Y(bx)) \]
\[ = b y(x) (1 - \frac{1}{a} y(x)) \]
\[ = \frac{b}{a} y(x) (a - y(x)) \]

\[ \frac{dy}{dx} = Aa y (B - y) \]

\( a \) = population \( \frac{dy}{dx} \) = time

\( B \) = number of people

Change units: thousands of people

Note: The text appears to be a mathematical derivation related to a differential equation, possibly involving population dynamics.
Given
\[ y' = g(y + ax + b) \]

Look at
\[ u(x) = y(x) + ax + b = g(x) \]

\[ u'(x) = y'(x) + a \]

\[ = g(x + ax + b) + a \]

\[ = g(u(x)) + a \]

\[ u' = g(u) + a \]

\[ \frac{du}{dx} = g(u) + a \]

Fri. Oct. 19
Second Exam