Extra Office Hours

Tues 3:30 – 4:30

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3c.
5.2

$$\frac{1}{7}\int e^{7x-49x^2}$$

$$y' - 7y = 14t \qquad y(0)=0$$

$$e^{-\int 7} = e^{-7t}$$

$$e^{-7t}(y' - 7y) = e^{-7t}\, 14t$$

$$(e^{-7t} y)' = e^{-7t}\, 14t$$

$$e^{-7t} y = 14\int e^{-7t}\, t \; dt$$

$$u = t \qquad dv = e^{-7t}$$

$$du = dt \qquad v = -\frac{1}{7} e^{-7t}$$

$$-\frac{1}{7} t e^{-7t} + \frac{1}{7}\int e^{-7t}\, dt$$

5.2 #7 $k = -0.2$

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②

$$\frac{dT}{dt} = k\left[T(t) - T_{out}(t)\right]$$

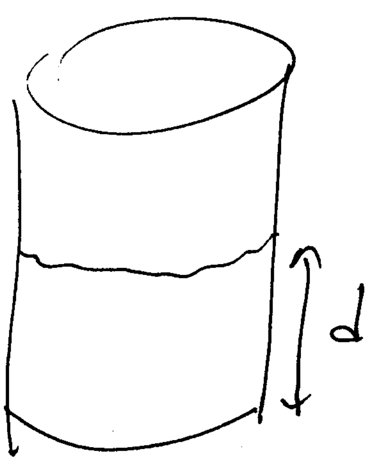
$-$ $>$

$$T_{out}(t) = 45 + 10 \sin\left(\frac{2\pi t}{24}\right)$$

period = 24 t hours

5.2 #17

$$\frac{dV}{dt} = 3 - \frac{k}{5\sqrt{\pi}} \sqrt{V}$$



enter $3 \frac{ft^3}{sec}$

leaves rate prop $\sqrt{\text{depth}}$

$V = \pi\, 25\, d$

leave rate $= k\sqrt{d}$

§5.3 Bernoulli's Equation

$$y' + p(x)\, y = q(x)\, y^n$$

n is a real number $\neq 1$

Trick $\quad u = y^{1-n}$

will get a linear eq. for u.

Example ($x > 0$) $\quad y' - \frac{1}{2x} y = \frac{1}{2y} \quad \cancel{= \frac{1}{2} y^{-1}}$

$n = -1 \qquad$ Try $\quad u = y^2$

$$u' = 2 y y'$$
$$= 2y \left(\frac{1}{2x} y + \frac{1}{2y} \right)$$

$$u' = \frac{1}{x} y^2 + 1$$

$$u' = \frac{1}{x} u + 1$$

$$u' - \frac{1}{x} u = 1$$

$$\text{I.F.} = \exp(-\ln|x|) = \frac{1}{|x|} = \frac{1}{x}$$

$$\frac{1}{x}u' - \frac{1}{x^2}u = \frac{1}{x}$$

$$\left(\frac{1}{x}u\right)' = \frac{1}{x}$$

$$\frac{1}{x}u = \int \frac{1}{x}\,dx = \ln x + C$$

$$u = x\ln x + Cx$$

$$y = \pm\sqrt{u} = \pm\sqrt{x\ln x + Cx}$$

Example

$$\frac{dP}{dt} = kP\left(1 - \frac{P}{b}\right) \qquad ||| \quad P(t)$$

k, b constants

$$\frac{dP}{dt} = kP - \frac{k}{b}P^2$$

$$\frac{dP}{dt} - kP = -\frac{k}{b}P^2$$

$$\cancel{u = y^m \qquad u' = m y^{m-1} y'}$$

$$u = P^m \qquad u' = m P^{m-1} P'$$

$$u' = m P^{m-1}\left(kP - \frac{k}{b}P^2\right)$$

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$$u' = mkp^m - \frac{mk}{b} p^{m+1}$$

$$u' = mku - \frac{mk}{b}$$

$m+1=0$

$m=-1$

$$u' - mku = \frac{-mk}{b}$$

$$\left(ue^{-mkt}\right)' = -\frac{mk}{b} e^{-mkt}$$

$$ue^{-mkt} = -\frac{mk}{b}\int e^{-mkt}\,dt$$

$$= -\frac{mk}{b}\,\frac{-1}{mk}\,e^{-mkt} +$$

$$= \frac{1}{b} e^{-mkt} + C$$

$$ue^{kt} = \frac{1}{b} e^{kt} + C$$

$$u = \frac{1}{b} + Ce^{-kt}$$

$$u = p^m = p^{-1}$$

$$p(t) = \frac{1}{\frac{1}{b} + Ce^{-kt}}$$

Seasonal Capacity

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$$b(t) = 7 + \sin 6t$$

$$\frac{dP}{dt} = kP\left(1 - \frac{P}{b(t)}\right)$$

Eq. $P = 0$ $\qquad$ $P = b(t) = 7 + \sin 6t$
not solution

No other eq.

Bernolli :

$$\frac{dP}{dt} = kP - \frac{k}{b(t)} P^2$$

$n = 2 \qquad 1 - n = -1$

$u = P^{-1} \qquad u' = -P^{-2} P'$

$$u' = -P^{-2}\left(kP - \frac{k}{b(t)} P^2\right)$$

$$u' = -kP^{-1} + \frac{k}{b(t)}$$

$$= -ku + \frac{k}{b(t)}$$

$$u' + ku = \frac{k}{b(t)}$$

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$$\left(u e^{kt}\right)' = e^{kt} \frac{k}{b(t)}$$

$$u e^{kt} = \int e^{kt} \frac{k}{7 + \sin 6t} \, dt$$

Approximate

$$\frac{1}{1+x} \approx 1 - x \quad \text{if } x \text{ is small}$$

$$f(x) = \frac{1}{1+x} \qquad \text{tangent line at } 0$$

$$f'(x) = \frac{-1}{(1+x)^2}, \quad f'(0) = -1$$

$$f(0) = 1 \qquad (0,1), \quad m = -1$$

tangent line $\quad 1 + (-1)x = 1 - x$

$$\frac{1}{1+x} = 1 - x + x^2 - x^3 + x^4 - \ldots$$

$$\frac{1}{7 + \sin 6t} = \frac{1}{7\left(1 + \frac{1}{7}\sin 6t\right)}$$

$$= \frac{1}{7} \frac{1}{1 + \frac{1}{7}\sin 6t}$$

$$\approx \frac{1}{7}\left(1 - \frac{1}{7}\sin 6t\right)$$

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$\frac{1}{7}$ is small

$$u e^{kt} = \int e^{kt} k \frac{1}{7}\left(1 - \frac{1}{7}\sin 6t\right) dt$$

$$= \frac{k}{7}\int e^{kt}\, dt - \frac{k}{49}\int e^{kt}\sin 6t\, dt$$

$$u = \frac{1}{7} - \frac{1}{49}\cdot\frac{1}{37}\left(\sin 6t - 6\cos 6t\right) + C e^{-t}$$

$$p = \frac{1}{u}$$