

Exam covers

Calculus 7.5, 7.6

D.A. Eq. 3.3, 3.4

4.1, 4.2, 4.3, 4.4

# § 5.1 Solving Linear D.E.

An equation of the form

$$y' + p(x)y = q(x)$$

is called a first order linear diff. eq.

1st order:  $y''$ ,  $y'''$  don't appear

linear: No  $y^2$ ,  $\cos y$ ,  $e^y$

$(y')^2$ ,  $\sqrt{y'}$

## Examples

$$y' + x^2 y = \sin x$$

$$y' = e^x + \frac{y}{x+1}$$

$$(x^2 + 1)y' + \frac{1}{x}y = \cos x$$

$$y' + \frac{y}{x(x^2+1)} = \frac{\cos x}{x^2+1}$$

linear

Non-example

$$y' + y^2 = e^x$$

(1/7)  
(2/3)

$$y y' + x^2 - y = \cos x$$

$$y y' + y^2 = \frac{y}{x}$$

Divide by  $y$  (careful  $y \neq 0$ )

$$y' + y = \frac{1}{x} \quad \text{linear.}$$

# Method for solution

11/7  
3

key ~~factor~~ ideas

integrating factor

$$y' + p(x)y = q(x)$$

$$\text{integrating factor} = \exp\left(\int p(x) dx\right)$$

= function of  $x$

multiply the dif. eq. by I.F.

Example

$$y' + xy = x$$

$$p(x) = x$$

$$\int p(x) dx = \frac{1}{2} x^2$$

$$\text{I.F.} = e^{\int p} = e^{\frac{1}{2} x^2}$$

$$y' e^{\frac{1}{2} x^2} + x e^{\frac{1}{2} x^2} y = x e^{\frac{1}{2} x^2}$$

$$\left( e^{\frac{1}{2} x^2} y \right)' = x e^{\frac{1}{2} x^2}$$

$$(e^{\frac{1}{2}x^2} y)' = x e^{\frac{1}{2}x^2} \quad \left( \begin{array}{l} 1 \\ 4 \end{array} \right)$$

Integrate this

$$e^{\frac{1}{2}x^2} y = \int x e^{\frac{1}{2}x^2} dx$$

$$e^{\frac{1}{2}x^2} y = e^{\frac{1}{2}x^2} + C$$

$$y = 1 + C e^{-\frac{1}{2}x^2}$$

check

$$y' + xy = x$$

$$y' = -x C e^{-\frac{1}{2}x^2}$$

$$y' + xy = -x C e^{-\frac{1}{2}x^2} +$$

$$x(1 + C e^{-\frac{1}{2}x^2})$$

$$= x \quad \text{yeah!}$$

Example

11/7  
⑤

$$y' + \frac{1}{x} y - \sin x = 0, \quad x > 0$$

$$y' + \frac{1}{x} y = \sin x$$

$$p(x) = \frac{1}{x} \quad \int p(x) = \ln x$$

$$e^{\int p} = e^{\ln x} = x$$

multiply diff eq by  $x$

$$\underbrace{xy' + y}_{(xy)'} = x \sin x$$

$$(xy)' = x \sin x$$

$u = x$
$v' = \sin x$

Integrate

$$xy = \int x \sin x \, dx \quad \text{parts}$$

...

~~$y = -\cos x + \frac{\sin x}{x} + \frac{1}{x}$~~

$$y = -\cos x + \frac{\sin x}{x} + \frac{1}{x}$$

Example

~~Problem~~

11/7  
⑥

$$(1 + \sin x) y' + 2 \cos(x) y = 2$$

$$\star y' + \frac{2 \cos x}{1 + \sin x} y = \frac{2}{1 + \sin x}$$

$$p(x) = \frac{2 \cos x}{1 + \sin x}$$

$$u = 1 + \sin x \\ du = \cos x dx$$

$$\int p(x) dx = 2 \ln(1 + \sin x)$$

$$e^{\int p} = e^{2 \ln(1 + \sin x)}$$

$$= \left( e^{\ln(1 + \sin x)} \right)^2$$

$$= (1 + \sin x)^2$$

$\star$   $\times$  integrating factor

$$(1 + \sin x)^2 y' + 2 \cos x (1 + \sin x) y = 2 (1 + \sin x)$$

$$\left( (1 + \sin x)^2 y \right)' = 2 (1 + \sin x) \quad (17)$$

∫ it

$$(1 + \sin x)^2 y = \int 2 (1 + \sin x) dx$$

$$(1 + \sin x)^2 y = 2x - 2 \cos x + C$$

$$y = \frac{2x - 2 \cos x + C}{(1 + \sin x)^2}$$