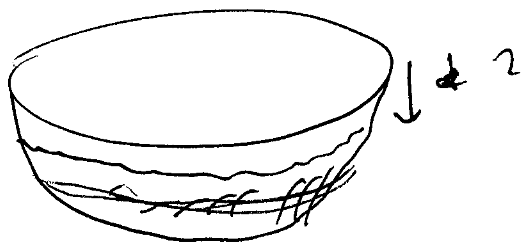
Example

Swimming pool is a hemisphere.



Radius = 20 ft.

~~Fall~~

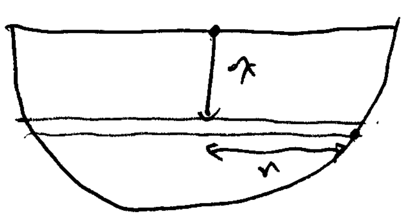
62.4 lbs/ft³

water level is ~~3~~ 2 ft below top (ground level)

How much work to pump the water out (to ground level)

Slice : Disc of radius r

let x be distance below ground level. water: x = 2 to x = 20



$x^2 + r^2 = (20)^2$

Slice vol $= \pi r^2 \Delta x$

Slice weight $= \pi r^2 \Delta x \, 62.4$

Slice work $= \pi \, 62.4 \, r^2 x \, \Delta x$

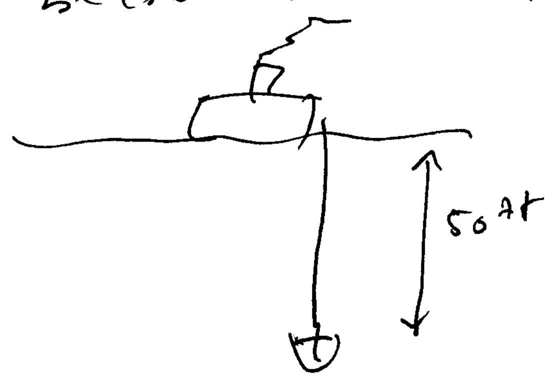
total work $= \int_2^{20} \pi \, 62.4 \, r^2 x \, dx$

$$= \int_2^{20} \pi \; 62.4 \; (400 - x^2) \; x \; dx$$

$$\frac{\text{lbs}}{\text{ft}^3} \quad \text{ft}^2 \quad \text{ft} \quad \text{ft}$$

ft-lb ~~ft~~

(2) A boat has anchor which weighs 100 lbs. The chain weighs 2 lbs/ft. The chain is 50 ft long and the anchor is directly below the boat.



How much work to raise anchor?

Anchor itself:

50 lbs 100 ft = 5,000 ft-lbs

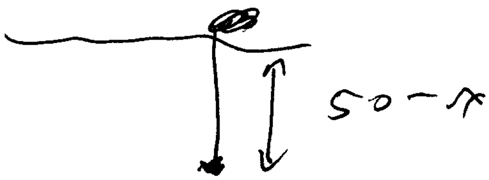
Chain: Let x be the length of chain already raised.

$x = 0$ to 50.

Still $50 - x$ ft of chain in water. For Δx, we need to [raise] this piece by distance Δx.

12/3

(3)



height of remaining chain is

$2(50-x)$

Lift it Δx

slice work $= 2(50-x)\,\Delta x$

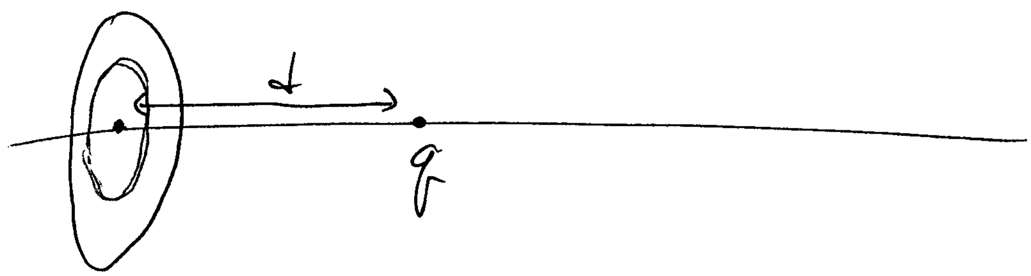
total work $= \int_0^{50} 2(50-x)\,dx$

$= 2 \cdot 50 \cdot 25 = 2{,}500$ ft·lbs

Example

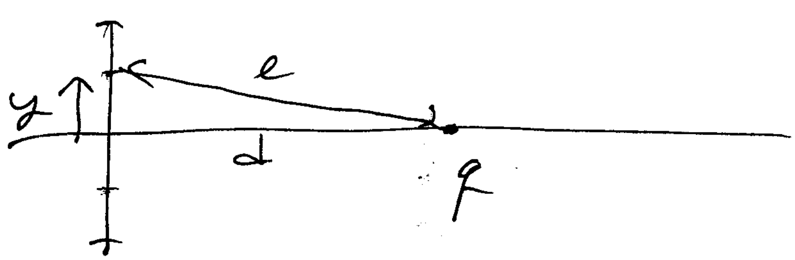
Electrostatic potential between charges q and Q ~~at a~~ separated a distance ℓ is

$$U = c\,\frac{qQ}{\ell}$$



Circular plate of radius R has total charge of Q. Look at a point along axis of plate at a distance d from plate. Find U there. Charge is uniform on plate.

Slice disc into thin rings.



RC3 5

$$\ell^2 = y^2 + d^2$$

Distance from ring to q is ℓ

Ring potential is

$$c \frac{q \, \Delta Q}{\ell}$$

ΔQ = charge on ring

$$\Delta Q = \frac{\text{area of ring}}{\text{area of plate}} \, Q$$

$$= \frac{2\pi y \, \Delta y}{\pi R^2} \, Q$$

Total $U = \int c \frac{q}{\ell} \frac{2\pi y}{\pi R^2} \, Q \, dy$

$$= \frac{2cqQ}{R^2} \int_0^R \frac{y}{\sqrt{y^2 + d^2}} \, dy$$