Example

Swimming pool is a hemisphere.

Radius = 20 ft.

62.4 lb/ft$^3$

Water level is 2 ft below top (ground level)

How much work to pump the water out (to ground level)

Slice: Disc of radius $r$

Let $x$ be distance below ground level, water: $x = 0$ to $x = 20$

$x^2 + r^2 = (20)^2$

Slice vol = $\pi r^2 \Delta x$

Slice weight = $\pi r^2 \Delta x \times 62.4$

Slice work = $\pi 62.4 r^2 x \Delta x$

Total work = $\int_{2}^{20} \pi 62.4 r^2 x \, dx$
\[ \int_{\pi/2}^{\pi} 62.4 \left(400 - x^2\right) \, dx \]

\[ \frac{1}{f} \frac{f^2}{f + \frac{1}{3}} \]

\[ f + 16 \text{ ft} \]

2) A boat has an anchor which weighs 150 lbs. The chain weighs 2 lbs / ft. The chain is 50 ft long and the anchor is directly below the boat.

How much work to raise the anchor?

Anchor chain:

50 lbs / 50 ft = 1 lbs / ft

Chain: Let \( x \) be the length of chain already raised.

\[ x = 0 \text{ to } 50 \text{ ft} \]

Still 50 - \( x \) ft of chain in water. For a \( \Delta x \), we need to lift this piece by distance \( f(x) \).
Weight of remaining chain is

\[ 2 \ (50-x) \]

Lift if \( \Delta z \)

Slice work \( = 2 \ (50-x) \ \Delta x \)

Total work \( = \int_0^{50} 2 \ (50-x) \ \text{d}x \)

\[ = 2 \cdot 50 \cdot 25 = 2500 \ \text{ft-lbs} \]
Example

Electrostatic potential between charges $q$ and $Q$ at a separated a distance $r$ is

$$U = k \frac{qQ}{r}$$

Circular plate of radius $R$ has total charge $Q$. Look at a point along axis of plate at a distance $d$ from plate. Find $U$ there. Charge is uniform on plate.

Slice disc into thin rings.
\[ l^2 = y^2 + d^2 \]

Distance from ring to \( z \) is \( l \).

Ring potential is:

\[
C \frac{\Delta Q}{l}
\]

\( \Delta Q = \text{charge on ring} \)

\[
\Delta Q = \frac{\text{area of ring}}{\text{area of plate}} \cdot Q
\]

\[
= \frac{2\pi y \Delta y}{\pi R^2} \cdot Q
\]

Total:

\[
U = \int C \frac{\Delta Q}{l} \frac{2\pi y}{\pi R^2} \cdot Q \, dy
\]

\[
= \frac{2C \pi Q}{R^2} \int_0^R \frac{y}{\sqrt{y^2 + d^2}} \, dy
\]