

$$\textcircled{1} \int x \sin(x^2) dx$$

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p. 1

$$\frac{d}{dx} \cos(x^2) = -\sin(x^2) \cdot 2x \\ = -2x \sin(x^2)$$

$$\int x \sin(x^2) dx = -\frac{1}{2} \cos(x^2) + C$$

Formal sub:

$$u = x^2$$

$$u' = 2x, \quad du = 2x dx$$

$$\int \sin(x^2) x dx = \int \sin(u) \frac{1}{2} du$$

$$= \frac{1}{2} (-\cos u) = -\frac{1}{2} \cos(x^2) + C$$

$$\textcircled{2} \int \sec^2(x^{1/2}) \underbrace{x^{-1/2}} dx$$

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p. 2

$$u = x^{1/2}, \quad du = \frac{1}{2} \underbrace{x^{-1/2}} dx$$

$$\rightarrow = \int \sec^2(u) \cdot 2 du$$

$$= 2 \tan(u) + C$$

$$= 2 \tan(x^{1/2}) + C$$

$$\textcircled{3} \int \frac{\cos t}{1 + \sin^2 t} dt$$

$$u = \cancel{1 + \sin^2 t}$$

$$u = \sin t$$

$$du = \cos t dt$$

$$\rightarrow = \int \frac{du}{1 + u^2} = \tan^{-1} u + C$$

$$= \tan^{-1}(\sin t) + C$$

(4)

$$\int \frac{x+3}{x^2+6x+2} dx$$

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p. 3

$$u = x^2 + 6x + 2$$

$$u' = 2x + 6$$

$$du = (2x+6) dx$$

$$= \int \frac{\frac{1}{2} du}{u} = \frac{1}{2} \ln |u| + C$$

$$= \frac{1}{2} \ln |x^2 + 6x + 2| + C$$

$$\int \frac{dx}{x} = \ln |x|$$

(5)

$$\int \frac{x}{x^2+6x+2} dx$$

$$= \int \frac{x+3}{x^2+6x+2} dx - \int \frac{3}{x^2+6x+2} dx$$

$$= \int \frac{x+3}{x^2+6x+2} dx - 3 \int \frac{dx}{x^2+6x+2}$$

$$\int \frac{dx}{x^2 + 6x + 2}$$

$$b^2 - 4ac = \frac{36 - 8}{\cancel{27}} > 0$$

= partial fractions (later)

Non-obvious subs

$$\int \frac{dx}{1 + x^{1/3}}$$

$$u = 1 + x^{1/3}$$

$$u' = \frac{1}{3} x^{-2/3}$$

$$du = \frac{1}{3} x^{-2/3} dx \Rightarrow dx = 3x^{2/3} du$$

$$\int \frac{3x^{2/3} du}{u}$$

$$u-1 = x^{1/3}$$

$$(u-1)^3 = x$$

$$= \int \frac{3(u-1)^2 du}{u}$$

$$= 3 \int \frac{u^2 - 2u + 1}{u} du = 3 \int (u - 2 + \frac{1}{u}) du$$

$$= 3 \left[ \frac{1}{2} (1 + x^{1/3})^2 - 2(1 + x^{1/3}) + \ln|1 + x^{1/3}| \right] + C$$

# Definite integrals

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$$\int_a^b f(x) dx = F(b) - F(a)$$

$$f' = f$$

↑ ↑  
"x-values"

①  $\int_0^2 x e^{x^2} dx$

$$\int x e^{x^2} dx = \frac{1}{2} e^{x^2} + C$$

$$\left. \frac{1}{2} e^{x^2} \right|_{x=0}^{x=2} = \frac{1}{2} e^{2^2} - \frac{1}{2} e^0$$

$$= \frac{1}{2} e^4 - \frac{1}{2}$$

2<sup>nd</sup> method

$$u = x^2$$

$$du = 2x dx$$

$$\int_0^2 x e^{x^2} dx = \int_0^4 e^u \frac{1}{2} du$$

$$= \frac{1}{2} e^u \Big|_{u=0}^{u=4}$$

$$= \frac{1}{2} e^4 - \frac{1}{2} e^0$$

## Sophisticated subs

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$$\text{Let } g(a) = \int_{-a}^a \frac{e^{\pi/a} dx}{x^2}$$

How does  $g(a)$  depend on  $a$ ?

$$u = \frac{x}{a} \quad du = \frac{1}{a} dx$$

$$g(a) = \int_{-1}^1 \frac{e^u a du}{x^2} \quad \left. \begin{array}{l} \\ \\ \\ \end{array} \right\} x = ua$$

$$= \int_{-1}^1 \frac{e^u a du}{u^2 a^2}$$

$$= \frac{1}{a} \int_{-1}^1 \frac{e^u du}{u^2} = \frac{c}{a}$$

$$c = \int_{-1}^1 \frac{e^u du}{u^2}$$

Given that

$$\int_{-\infty}^{\infty} e^{-x^2} dx = \sqrt{\pi}$$

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Find

$$\int_{-\infty}^{\infty} e^{-cx^2} dx$$

$$u^2 = cx^2$$

$$u = \sqrt{c} x$$

$$u' = \sqrt{c}$$

$$du = \sqrt{c} dx$$

$$\int_{-\infty}^{\infty} e^{-u^2} \frac{1}{\sqrt{c}} du$$

$$= \frac{1}{\sqrt{c}} \int_{-\infty}^{\infty} e^{-u^2} du = \frac{\sqrt{\pi}}{\sqrt{c}}$$